one example of a situation where the two equations look exactly alike.

**Q6** Consider the acceleration of a car on dry pavement, if there is no slipping. The axle moves at speed $v$, and the outside of the tire moves at speed $v$ relative to the axle. The instantaneous velocity of the bottom of the tire is zero. How much work is done by the force exerted on the tire by the road? What is the source of the energy that increases the car's translational kinetic energy?

**Q7** Two people with different masses but equal speeds slide toward each other with little friction on ice with their arms extended straight out to the side (so each has the shape of a "T"). Her right hand meets his right hand, they hold hands and spin 90°, then release their holds and slide away. Make a rough sketch of the path of the center of mass of the system consisting of the two people, and explain briefly. (It helps to mark equal time intervals along the paths of the two people and of their center of mass.)

## PROBLEMS

### Section 9.1

**•P8** Three uniform-density spheres are positioned as follows:

- A 3 kg sphere is centered at $\langle 10, 20, -5\rangle$ m.
- A 5 kg sphere is centered at $\langle 4, -15, 8\rangle$ m.
- A 6 kg sphere is centered at $\langle -7, 10, 9\rangle$ m.

What is the location of the center of mass of this three-sphere system?

**•P9** Relative to an origin at the center of the Earth, where is the center of mass of the Earth–Moon system? The mass of the Earth is $6\times 10^{24}$ kg, the mass of the Moon is $7\times 10^{22}$ kg, and the distance from the center of the Earth to the center of the Moon is $4\times 10^{8}$ m. The radius of the Earth is 6400 km. One can show that the Earth and Moon orbit each other around this center of mass.

**•P10** A meter stick whose mass is 300 g lies on ice (Figure 9.49). You pull at one end of the meter stick, at right angles to the stick, with a force of 6 N. The ensuing motion of the meter stick is quite complicated, but what are the initial magnitude and direction of the rate of change of the momentum of the stick, $d\vec{p}_{sys}/dt$, when you first apply the force? What is the magnitude of the initial acceleration of the center of the stick?


***Figure 9.49***

**•P11** Determine the location of the center of mass of an L-shaped object whose thin vertical and horizontal members have the same length $L$ and the same mass $M$. Use the formal definition to find the $x$ and $y$ coordinates, and check your result by doing the calculation with respect to two different origins, one in the lower left corner at the intersection of the horizontal and vertical members and one at the right end of the horizontal member.

**••P12** A man whose mass is 80 kg and a woman whose mass is 50 kg sit at opposite ends of a canoe 5 m long, whose mass is 30 kg. **(a)** Relative to the man, where is the center of mass of the system consisting of man, woman, and canoe? (*Hint:* Choose a specific coordinate system with a specific origin.) **(b)** Suppose that the man moves quickly to the center of the canoe and sits down there. How far does the canoe move in the water? Explain your work and your assumptions.

**•P13** If an object has a moment of inertia 19 kg · m$^2$ and rotates with an angular speed of 70 rad/s, what is its rotational kinetic energy?

**•P14** A group of particles of total mass 35 kg has a total kinetic energy of 340 J. The kinetic energy relative to the center of mass is 85 J. What is the speed of the center of mass?

**••P15** By calculating numerical quantities for a multiparticle system, one can get a concrete sense of the meaning of the relationships $\vec{p}_{sys} = M_{tot}\vec{v}_{CM}$ and $K_{tot} = K_{trans} + K_{rel}$. Consider an object consisting of two balls connected by a spring, whose stiffness is 400 N/m. The object has been thrown through the air and is rotating and vibrating as it moves. At a particular instant the spring is stretched 0.3 m, and the two balls at the ends of the spring have the following masses and velocities:

1: 5 kg, $\langle 8, 14, 0\rangle$ m/s
2: 3 kg, $\langle -5, 9, 0\rangle$ m/s

**(a)** For this system, calculate $\vec{p}_{sys}$. **(b)** Calculate $\vec{v}_{CM}$. **(c)** Calculate $K_{tot}$. **(d)** Calculate $K_{trans}$. **(e)** Calculate $K_{rel}$. **(f)** Here is a way to check your result for $K_{rel}$. The velocity of a particle relative to the center of mass is calculated by subtracting $\vec{v}_{CM}$ from the particle's velocity. To take a simple example, if you're riding in a car that's moving with $v_{CM,x} = 20$ m/s, and you throw a ball with $v_{rel,x} = 35$ m/s, relative to the car, a bystander on the ground sees the ball moving with $v_x = 55$ m/s. So $\vec{v} = \vec{v}_{CM} + \vec{v}_{rel}$, and therefore we have $\vec{v}_{rel} = \vec{v} - \vec{v}_{CM}$. Calculate $\vec{v}_{rel} = \vec{v} - \vec{v}_{CM}$ for each mass and calculate the corresponding $K_{rel}$. Compare with the result you obtained in part (e).

**••P16** Consider a system consisting of three particles:

$m_1 = 2$ kg, $\vec{v}_1 = \langle 8, -6, 15\rangle$ m/s
$m_2 = 6$ kg, $\vec{v}_2 = \langle -12, 9, -6\rangle$ m/s
$m_3 = 4$ kg, $\vec{v}_3 = \langle -24, 34, 23\rangle$ m/s

What is $K_{rel}$, the kinetic energy of this system relative to the center of mass?

**••P17** Binary stars: **(a)** About half of the visible "stars" are actually binary star systems, two stars that orbit each other with no other objects nearby (the small, dim stars called "red dwarfs" however tend to be single). Describe the motion of the center of mass of a binary star system. Briefly explain your reasoning. **(b)** For a particular binary star system, telescopic observations repeated over many years show that one of the stars (whose unknown mass we'll call $M_1$) has a circular orbit with radius $R_1 = 6\times 10^{11}$ m, while the other star (whose unknown mass we'll call $M_2$) has a circular orbit of radius $R_2 = 9\times 10^{11}$ m about the same point. Make a sketch of the orbits, and show the positions of the two stars on these orbits at some instant. Label the two stars as to which is which, and label their orbital radii. Indicate on your sketch the location of the center of mass of the system, and explain how you know its location, using the concepts and results of this chapter. **(c)** This double-star system is observed to complete one revolution in 40 years. What are the masses of the two stars? (For comparison, the distance from Sun to Earth is about $1.5 \times 10^{11}$ m, and the mass of the Sun is about $2 \times 10^{30}$ kg.) This method is often used to determine the masses of stars. The mass of a star largely determines many of the other properties of a star, which is why astrophysicists need a method for measuring the mass.

## Section 9.2

**•P18** If an object's rotational kinetic energy is 50 J and it rotates with an angular speed of 12 rad/s, what is its moment of inertia?

**•P19** A uniform-density disk of mass 13 kg, thickness 0.5 m, and radius 0.2 m makes one complete rotation every 0.6 s. What is the rotational kinetic energy of the disk?

**•P20** A sphere of uniform density with mass 22 kg and radius 0.7 m is spinning, making one complete revolution every 0.5 s. The center of mass of the sphere has a speed of 4 m/s. **(a)** What is the rotational kinetic energy of the sphere? **(b)** What is the total kinetic energy of the sphere?

**•P21** A uniform-density disk whose mass is 10 kg and radius is 0.4 m makes one complete rotation every 0.2 s. What is the rotational kinetic energy of the disk?

**•P22** A cylindrical rod of uniform density is located with its center at the origin, and its axis along the $x$ axis. It rotates about its center in the $xy$ plane, making one revolution every 0.03 s. The rod has a radius of 0.08 m, length of 0.7 m, and mass of 5 kg. It makes one revolution every 0.03 s. What is the rotational kinetic energy of the rod?

**•P23** A uniform-density 6 kg disk of radius 0.3 m is mounted on a nearly frictionless axle. Initially it is not spinning. A string is wrapped tightly around the disk, and you pull on the string with a constant force of 25 N through a distance of 0.6 m. Now what is the angular speed?

**••P24** A high diver tucks himself into a ball and spins rapidly (Figure 9.50). Make estimates of the relevant parameters and calculate $K_{rot}$, the rotational kinetic energy of the diver.

***Figure 9.50***

**••P25** The Earth is $1.5 \times 10^{11}$ m from the Sun and takes a year to make one complete orbit. It rotates on its own axis once per day. It can be treated approximately as a uniform-density sphere of mass $6 \times 10^{24}$ kg and radius $6.4 \times 10^{6}$ m (actually, its center has higher density than the rest of the planet, and the Earth bulges out a bit at the equator). Using this crude approximation, calculate the following: **(a)** What is $v_{CM}$? **(b)** What is $K_{trans}$? **(c)** What is $\omega$, the angular speed of rotation around its own axis? **(d)** What is $K_{rot}$? **(e)** What is $K_{tot}$?

**••P26** Show that the moment of inertia of a disk of mass $M$ and radius $R$ is $\frac{1}{2}MR^2$. Divide the disk into narrow rings, each of radius $r$ and width $dr$. The contribution to $I$ by one of these rings is simply $r^2 dm$, where $dm$ is the amount of mass contained in that particular ring. The mass of any ring is the total mass times the fraction of the total area occupied by the area of the ring. The area of this ring is approximately $2\pi r dr$. Use integral calculus to add up all the contributions.

## Section 9.3

**•P27** You pull straight up on the string of a yo-yo with a force 0.235 N, and while your hand is moving up a distance 0.18 m, the yo-yo moves down a distance 0.70 m. The mass of the yo-yo is 0.025 kg, and it was initially moving downward with speed 0.5 m/s and angular speed 124 rad/s. **(a)** What is the increase in the translational kinetic energy of the yo-yo? **(b)** What is the new speed of the yo-yo? **(c)** What is the increase in the rotational kinetic energy of the yo-yo? **(d)** The yo-yo is approximately a uniform-density disk of radius 0.02 m. What is the new angular speed of the yo-yo?

**••P28** A string is wrapped around a disk of mass 2.1 kg (its density is not necessarily uniform). Starting from rest, you pull the string with a constant force of 9 N along a nearly frictionless surface. At the instant when the center of the disk has moved a distance 0.11 m, your hand has moved a distance of 0.28 m (Figure 9.51).

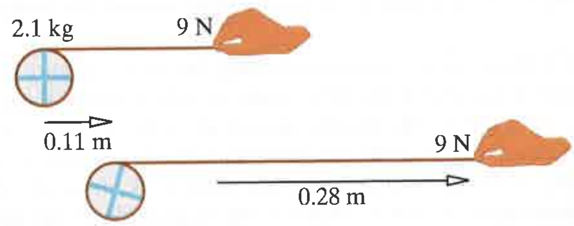


***Figure 9.51***

**(a)** At this instant, what is the speed of the center of mass of the disk? **(b)** At this instant, how much rotational kinetic energy does the disk have relative to its center of mass? **(c)** At this instant, the angular speed of the disk is 7.5 rad/s. What is the moment of inertia of the disk?

**••P29** A chain of metal links with total mass $M = 7$ kg is coiled up in a tight ball on a low-friction table (Figure 9.52). You pull on a link at one end of the chain with a constant force $F = 50$ N. Eventually the chain straightens out to its full length $L = 2.6$ m, and you keep pulling until you have pulled your end of the chain a total distance $d = 4.5$ m.

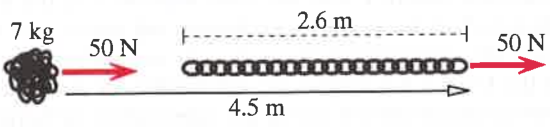


***Figure 9.52***

**(a)** Consider the point particle system. What is the speed of the chain at this instant? **(b)** Consider the extended system. What is the change in energy of the chain? **(c)** In straightening out, the links of the chain bang against each other, and their temperature rises. Assume that the process is so fast that there is insufficient time for significant transfer of energy from the chain to the table due to the temperature difference, and ignore the small amount of energy radiated away as sound produced in the collisions among the links. Calculate the increase in internal energy of the chain.

**••P30** Tarzan, whose mass is 100 kg, is hanging at rest from a tree limb. Then he lets go and falls to the ground. Just before he lets go, his center of mass is at a height 2.9 m above the ground and the bottom of his dangling feet are at a height 2.1 above the ground. When he first hits the ground he has dropped a distance 2.1, so his center of mass is (2.9 − 2.1) above the ground. Then his knees bend and he ends up at rest in a crouched position with his center of mass a height 0.5 above the ground. **(a)** Consider the point particle system. What is the speed $v$ at the instant just before Tarzan's feet touch the ground? **(b)** Consider the extended system. What is the net change in internal energy for Tarzan from just before his feet touch the ground to when he is in the crouched position?

**••P31** Here is an experiment on jumping up you can do. **(a)** Crouch down and jump straight up, as high as you can. Estimate the location of your center of mass, and measure its height at three stages in this process: in the crouch, at lift-off, and at the top of the jump. Report your measurements. You may need to have a friend help you make the measurements. **(b)** Analyze this process as fully as possible, using all the theoretical tools now available to you, especially the concepts in this chapter. Include a calculation of the average force of the floor on your feet, the change in your internal energy, and the approximate time of contact from the beginning of the jump to lift-off. Be sure to explain clearly what approximations and simplifying assumptions you made in modeling the process.

**••P32** Consider an ice skater who pushes away from a wall. **(a)** Estimate the speed she can achieve just after pushing away from the wall. Then estimate the average acceleration during this process. How many g's is this? (That is, what fraction or multiple of 9.8 $m/s^2$ is your estimate?) Be sure to explain clearly what approximations and simplifying assumptions you made in modeling the process. **(b)** For this process, choose the woman as the system of interest and discuss the energy transfers, and the changes in the various forms of energy. Estimate the amount of each of these, including the correct signs.

**••P33** A hoop of mass $M$ and radius $R$ rolls without slipping down a hill, as shown in Figure 9.53. The lack of slipping means that when the center of mass of the hoop has speed $v$, the tangential speed of the hoop relative to the center of mass is also equal to $v_{CM}$, since in that case the instantaneous speed is zero for the part of the hoop that is in contact with the ground ($v - v = 0$). Therefore, the angular speed of the rotating hoop is $\omega = v_{CM}/R$.

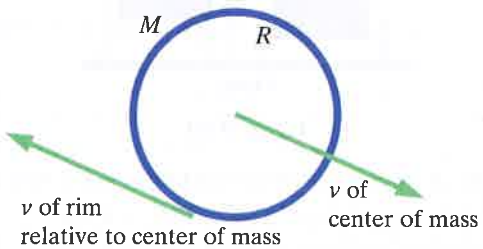


*Figure 9.53*

**(a)** The initial speed of the hoop is $v_i$, and the hill has a height $h$. What is the speed $v_f$ at the bottom of the hill? **(b)** Replace the hoop with a bicycle wheel whose rim has mass $M$ and whose hub has mass $m$, as shown in Figure 9.54. The spokes have negligible mass. What would be the bicycle wheel's speed at the bottom of the hill?

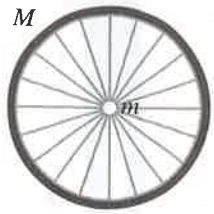


*Figure 9.54*

**••P34** A sphere or cylinder of mass $M$, radius $R$, and moment of inertia $I$ rolls without slipping down a hill of height $h$, starting from rest. As explained in Problem P33, if there is no slipping $\omega = v_{CM}/R$. **(a)** In terms of the given variables ($M$, $R$, $I$, and $h$), what is $v_{CM}$ at the bottom of the hill? **(b)** If the object is a thin hollow cylinder, what is $v_{CM}$ at the bottom of the hill? **(c)** If the object is a uniform-density solid cylinder, what is $v_{CM}$ at the bottom of the hill? **(d)** If the object is a uniform-density sphere, what is $v_{CM}$ at the bottom of the hill? An interesting experiment that you can perform is to roll various objects down an inclined board and see how much time each one takes to reach the bottom.

**••P35** Two disks are initially at rest, each of mass $M$, connected by a string between their centers, as shown in Figure 9.55. The disks slide on low-friction ice as the center of the string is pulled by a string with a constant force $F$ through a distance $d$. The disks collide and stick together, having moved a distance $b$ horizontally.

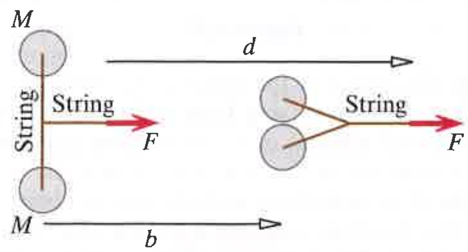


*Figure 9.55*

**(a)** What is the final speed of the stuck-together disks? **(b)** When the disks collide and stick together, their temperature rises. Calculate the increase in internal energy of the disks, assuming that the process is so fast that there is insufficient time for there to be much transfer of energy to the ice due to a temperature difference. (Also ignore the small amount of energy radiated away as sound produced in the collisions between the disks.)

**••P36** You hang by your hands from a tree limb that is a height $L$ above the ground, with your center of mass a height $h$ above the ground and your feet a height $d$ above the ground, as shown in Figure 9.56. You then let yourself fall. You absorb the shock by bending your knees, ending up momentarily at rest in a crouched position with your center of mass a height $b$ above the ground. Your mass is $M$. You will need to draw labeled physics diagrams for the various stages in the process.

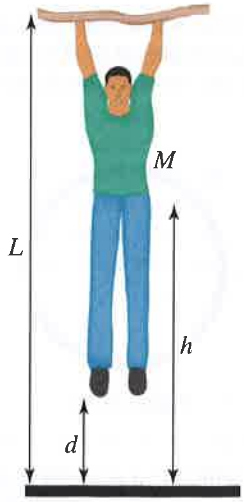


*Figure 9.56*

**(a)** What is the net internal energy change $\Delta E_{int}$ in your body (chemical plus thermal)? **(b)** What is your speed $v$ at the instant your feet first touch the ground? **(c)** What is the approximate average force $F$ exerted by the ground on your feet during the time when your knees are bending? **(d)** How much work is done by this force $F$?

••**P37** A box and its contents have a total mass $M$. A string passes through a hole in the box (Figure 9.57), and you pull on the string with a constant force $F$ (this is in outer space—there are no other forces acting).


*Figure 9.57*

**(a)** Initially the speed of the box was $v_i$. After the box had moved a long distance $w$, your hand had moved an additional distance $d$ (a total distance of $w+d$), because additional string of length $d$ came out of the box. What is now the speed $v_f$ of the box? **(b)** If we could have looked inside the box, we would have seen that the string was wound around a hub that turns on an axle with negligible friction, as shown in Figure 9.58. Three masses, each of mass $m$, are attached to the hub at a distance $r$ from the axle. Initially the angular speed relative to the axle was $\omega_i$. In terms of the given quantities, what is the final angular speed relative to the axis, $\omega_f$?

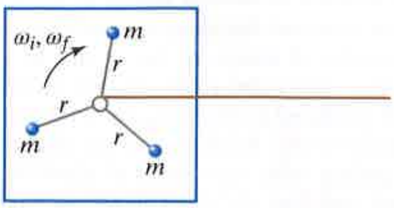


*Figure 9.58*

••**P38** Two identical 0.4 kg blocks (labeled 1 and 2) are initially at rest on a nearly frictionless surface, connected by an unstretched spring, as shown in the upper portion of Figure 9.59.

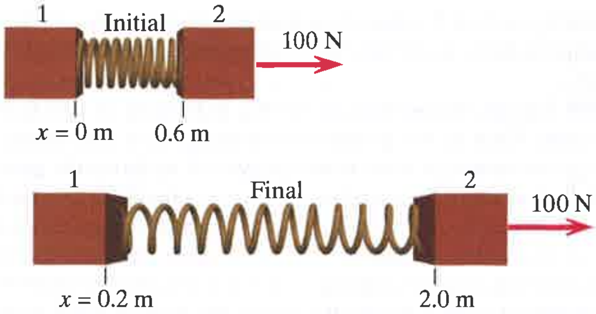


*Figure 9.59*

Then a constant force of 100 N to the right is applied to block 2, and at a later time the blocks are in the new positions shown in the lower portion of Figure 9.59. At this final time, the system is moving to the right and also vibrating, and the spring is stretched. **(a)** The following questions apply to the system modeled as a point particle. (i) What is the initial location of the point particle? (ii) How far does the point particle move? (iii) How much work was done on the particle? (iv) What is the change in translational kinetic energy of this system? **(b)** The following questions apply to the system modeled as an extended object. (1) How much work is done on the right-hand block? (2) How much work is done on the left-hand block? (3) What is the change of the total energy of this system? **(c)** Combine the results of both models to answer the following questions. (1) Assuming that the object does not get hot, what is the final value of $K_{vib} + U_{spring}$ for the extended system? (2) If the spring stiffness is 50 N/m, what is the final value of the vibrational kinetic energy?

••**P39** You hold up an object that consists of two blocks at rest, each of mass $M = 5$ kg, connected by a low-mass spring. Then you suddenly start applying a larger upward force of constant magnitude $F = 167$ N (which is greater than $2Mg$). Figure 9.60 shows the situation some time later, when the blocks have moved upward, and the spring stretch has increased.

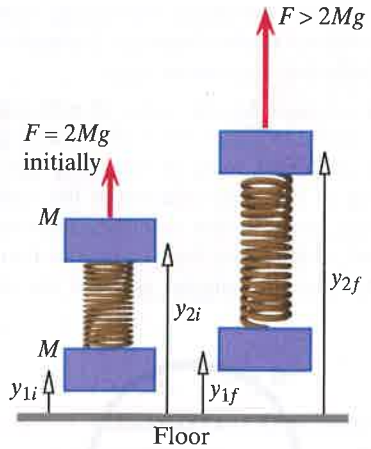


*Figure 9.60*

The heights of the centers of the two blocks are as follows:

Initial and final positions of block 1: $y_{1i} = 0.3$ m, $y_{1f} = 0.5$ m
Initial and final positions of block 2: $y_{2i} = 0.7$ m, $y_{2f} = 1.2$ m

It helps to show these heights on a diagram. Note that the initial center of mass of the two blocks is $(y_{1i} + y_{2i})/2$, and the final center of mass of the two blocks is $(y_{1f} + y_{2f})/2$. **(a)** Consider the point particle system corresponding to the two blocks and the spring. Calculate the increase in the total translational kinetic energy of the two blocks. It is important to draw a diagram showing all of the forces that are acting, and through what distance each force acts. **(b)** Consider the extended system corresponding to the two blocks and the spring. Calculate the increase of $(K_{vib} + U_s)$, the vibrational kinetic energy of the two blocks (their kinetic energy relative to the center of mass) plus the potential energy of the spring. It is important to draw a diagram showing all of the forces that are acting, and through what distance each force acts.

**••P40** A box contains machinery that can rotate. The total mass of the box plus the machinery is 7 kg. A string wound around the machinery comes out through a small hole in the top of the box. Initially the box sits on the ground, and the machinery inside is not rotating (left side of Figure 9.61). Then you pull upward on the string with a force of constant magnitude 130 N. At an instant when you have pulled 0.6 m of string out of the box (indicated on the right side of Figure 9.61), the box has risen a distance of 0.2 m and the machinery inside is rotating.

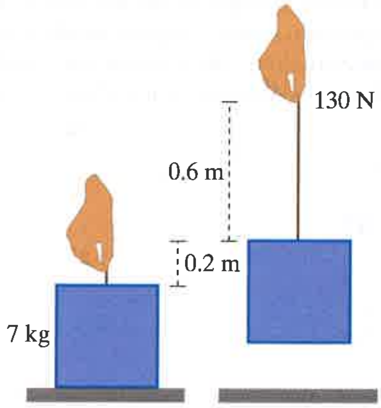


***Figure 9.61***

**POINT PARTICLE SYSTEM (a)** List all the forms of energy that change for the point particle system during this process. **(b)** What is the $y$ component of the displacement of the point particle system during this process? **(c)** What is the $y$ component of the net force acting on the point particle system during this process? **(d)** What is the distance through which the net force acts on the point particle system? **(e)** How much work is done on the point particle system during this process? **(f)** What is the speed of the box at the instant shown in the right side of Figure 9.61? **(g)** Why is it not possible to find the rotational kinetic energy of the machinery inside the box by considering only the point particle system?

**EXTENDED SYSTEM (h)** The extended system consists of the box, the machinery inside the box, and the string. List all the forms of energy that change for the extended system during this process. **(i)** What is the translational kinetic energy of the extended system, at the instant shown in the right side of Figure 9.61? **(j)** What is the distance through which the gravitational force acts on the extended system? **(k)** How much work is done on the system by the gravitational force? **(l)** What is the distance through which your hand moves? **(m)** How much work do you do on the extended system? **(n)** At the instant shown in the right side of Figure 9.61, what is the total kinetic energy of the extended system? **(o)** What is the rotational kinetic energy of the machinery inside the box?

**••P41** String is wrapped around an object of mass $M$ and moment of inertia $I$ (the density of the object is not uniform). With your hand you pull the string straight up with some constant force $F$ such that the center of the object does not move up or down, but the object spins faster and faster (Figure 9.62). This is like a yo-yo; nothing but the vertical string touches the object.

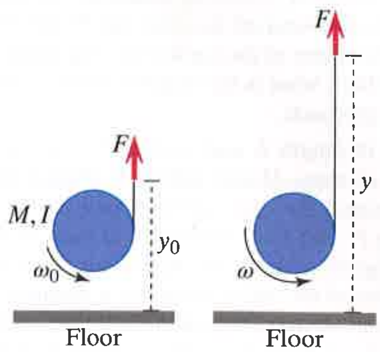


***Figure 9.62***

When your hand is a height $y_0$ above the floor, the object has an angular speed $\omega_0$. When your hand has risen to a height $y$ above the floor, what is the angular speed $\omega$ of the object?

Your result should not contain $F$ or the (unknown) radius of the object. Explain the physics principles you are using.

**••P42** String is wrapped around an object of mass 1.2 kg and moment of inertia 0.0015 kg · m$^2$ (the density of the object is not uniform). With your hand you pull the string straight up with some constant force $F$ such that the center of the object does not move up or down, but the object spins faster and faster (Figure 9.62). This is like a yo-yo; nothing but the vertical string touches the object. When your hand is a height $y_0 = 0.25$ m above the floor, the object has an angular speed $\omega_0 = 12$ rad/s. When your hand has risen to a height $y = 0.35$ m above the floor, what is the angular speed $\omega$ of the object? Your answer must be numeric and not contain the symbol $F$.

**••P43** A string is wrapped around a uniform disk of mass $M$ and radius $R$. Attached to the disk are four low-mass rods of radius $b$, each with a small mass $m$ at the end (Figure 9.63).

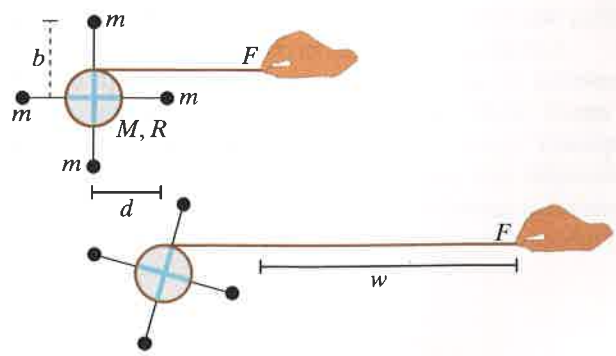


***Figure 9.63***

The apparatus is initially at rest on a nearly frictionless surface. Then you pull the string with a constant force $F$. At the instant when the center of the disk has moved a distance $d$, an additional length $w$ of string has unwound off the disk. **(a)** At this instant, what is the speed of the center of the apparatus? Explain your approach. **(b)** At this instant, what is the angular speed of the apparatus? Explain your approach.

**••P44** A string is wrapped around a uniform disk of mass $M = 1.2$ kg and radius $R = 0.11$ m (Figure 9.63). Attached to the disk are four low-mass rods of radius $b = 0.14$ m, each with a small mass $m = 0.4$ kg at the end. The device is initially at rest on a nearly frictionless surface. Then you pull the string with a constant force $F = 21$ N. At the instant that the center of the disk has moved a distance $d = 0.026$ m, an additional length $w = 0.092$ m of string has unwound off the disk. **(a)** At this instant, what is the speed of the center of the apparatus? Explain your approach. **(b)** At this instant, what is the angular speed of the apparatus? Explain your approach.

**••P45** A rod of length $L$ and negligible mass is attached to a uniform disk of mass $M$ and radius $R$ (Figure 9.64). A string is wrapped around the disk, and you pull on the string with a constant force $F$. Two small balls each of mass $m$ slide along the rod with negligible friction. The apparatus starts from rest, and when the center of the disk has moved a distance $d$, a length of string $s$ has come off the disk, and the balls have collided with the ends of the rod and stuck there. The apparatus slides on a nearly frictionless table. Here is a view from above:

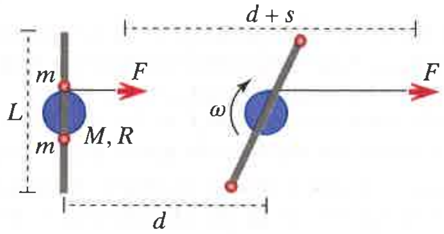


*Figure 9.64*

**(a)** At this instant, what is the speed $v$ of the center of the disk? **(b)** At this instant the angular speed of the disk is $\omega$. How much internal energy change has there been?

## Section 9.4

**•••P46** It is sometimes claimed that friction forces always slow an object down, but this is not true. If you place a box of mass $M$ on a moving horizontal conveyor belt, the friction force of the belt acting on the bottom of the box speeds up the box. At first there is some slipping, until the speed of the box catches up to the speed $v$ of the belt. The coefficient of friction between box and belt is $\mu$. **(a)** What is the distance $d$ (relative to the floor) that the box moves before reaching the final speed $v$? Use energy arguments, and explain your reasoning carefully. **(b)** How much time does it take for the box to reach its final speed? **(c)** The belt and box of course get hot. Is the effective distance through which the friction force acts on the box greater than or less than $d$? Give as quantitative an argument as possible. You can assume that the process is quick enough that you can neglect transfer of energy $Q$ due to a temperature difference between the belt and the box. Do not attempt to use the *results* of the friction analysis in this chapter; rather, apply the *methods* of that analysis to this different situation. **(d)** Explain the result of part (c) qualitatively from a microscopic point of view, including physics diagrams.

## ANSWERS TO CHECKPOINTS

**1** 39.2 J
**2 (a)** 61.3 J; **(b)** 99.7 J
**3** 316 J
**4 (a)** $v/d$; **(b)** $\frac{1}{2}(\frac{2}{5}MR^2)(v/d)^2$;
**(c)** $\frac{1}{2}(Md^2 + \frac{2}{5}MR^2)(v/d)^2$

**5 (a)** 167 N; **(b)** zero displacement, so no work done on the extended system, but on the point particle system there is 2500 J of work done; **(c)** internal energy (especially chemical energy) decreases by 2500 J