quanta? **(e)** In which block is the number of possible ways of arranging this 1 J of energy greater? **(f)** Which block now has the larger entropy? **(g)** Which block experienced a greater entropy change? **(h)** Which block experienced the larger temperature change? **(i)** Which metal has the larger specific heat at low temperatures? **(j)** Does your conclusion agree with the actual data given in Figure 12.33? (The numerical data are given in a table accompanying Problem P64.)

PROBLEMS

Section 12.2

•P16 List explicitly all the ways to arrange 2 quanta among 4 one-dimensional oscillators.

•P17 How many different ways are there to get 5 heads in 10 throws of a true coin? How many different ways are there to get no heads in 10 throws of a true coin?

•P18 How many different ways are there to arrange 4 quanta among 3 atoms in a solid?

•P19 A carbon nanoparticle (very small particle) contains 6000 carbon atoms. According to the Einstein model of a solid, how many oscillators are in this block?

•P20 In order to calculate the number of ways of arranging a given amount of energy in a tiny block of copper, the block is modeled as containing 8.7×10^5 independent oscillators. How many atoms are in the copper block?

•P21 In Chapter 4 you determined the stiffness of the interatomic "spring" (chemical bond) between atoms in a block of lead to be 5 N/m, based on the value of Young's modulus for lead. Since in our model each atom is connected to two springs, each half the length of the interatomic bond, the effective "interatomic spring stiffness" for an oscillator is 4×5 N/m = 20 N/m. The mass of one mole of lead is 207 g (0.207 kg). What is the energy, in joules, of one quantum of energy for an atomic oscillator in a block of lead?

•P22 Consider an object containing 6 one-dimensional oscillators (this object could represent a model of 2 atoms in an Einstein solid). There are 4 quanta of vibrational energy in the object. **(a)** How many microstates are there, all with the same energy? **(b)** If you examined a collection of 48,000 objects of this kind, each containing 4 quanta of energy, about how many of these objects would you expect to find in the microstate 000004?

•P23 Suppose that you look once every second at a system with 300 oscillators and 100 energy quanta, to see whether your favorite oscillator happens to have all the energy (all 100 quanta) at the instant you look. You expect that just once out of 1.7×10^{96} times you will find all of the energy concentrated on your favorite oscillator. On the average, about how many years will you have to wait? Compare this to the age of the Universe, which is thought to be about 1×10^{10} years. ($1 \text{ y} \approx \pi \times 10^7$ s.)

••P24 The reasoning developed for counting microstates applies to many other situations involving probability. For example, if you flip a coin 5 times, how many different sequences of 3 heads and 2 tails are possible? Answer: 10 different sequences, such as HTHHT or TTHHH. In contrast, how many different sequences of 5 heads and 0 tails are possible? Obviously only one, HHHHH, and our equation gives $5!/[5!0!] = 1$, using the standard definition that 0! is defined to equal 1.

If the coin is equally likely on a single throw to come up heads or tails, any specific sequence like HTHHT or HHHHH is equally likely. However, there is only one way to get HHHHH, while there are 10 ways to get 3 heads and 2 tails, so this is 10 times more probable than getting all heads.

Use the expression $5!/[N!(5-N)!]$ to calculate the number of ways to get 0 heads, 1 head, 2 heads, 3 heads, 4 heads, or 5 heads in a sequence of 5 coin tosses. Make a graph of the number of ways vs. the number of heads.

Section 12.3

•P25 Object A and object B are two identical microscopic objects. Figure 12.55 below shows the number of ways to arrange energy in one of these objects, as a function of the amount of energy in the object.

E, J	0.8 E–20	1.0 E–20	1.2 E–20	1.4 E–20
# ways	37	60	90	122

Figure 12.55

(a) When there are 1.0×10^{-20} J of energy in object A, what is the entropy of this object? **(b)** When there are 1.4×10^{-20} J of energy in object B, what is the entropy of this object? **(c)** Now the two objects are placed in contact with each other. At this moment, before there is time for any energy flow between the objects, what is the entropy of the combined system of objects A and B?

••P26 For a certain metal the stiffness of the interatomic bond and the mass of one atom are such that the spacing of the quantum oscillator energy levels is 1.5×10^{-23} J. A nanoparticle of this metal consisting of 10 atoms has a total thermal energy of 18×10^{-23} J. **(a)** What is the entropy of this nanoparticle? **(b)** The temperature of the nanoparticle is 87 K. Next we add 18×10^{-23} J to the nanoparticle. By how much does the entropy increase?

Section 12.5

•P27 A block of copper at a temperature of 50 °C is placed in contact with a block of aluminum at a temperature of 45 °C in an insulated container. As a result of a transfer of 2500 J of energy from the copper to the aluminum, the final equilibrium temperature of the two blocks is 48 °C. **(a)** What is the approximate change in the entropy of the aluminum block? **(b)** What is the approximate change in the entropy of the copper block? **(c)** What is the approximate change in the entropy of the Universe? **(d)** What is the change in the energy of the Universe?

•P28 It takes about 335 J to melt one gram of ice. During the melting, the temperature stays constant. Which has higher entropy, a gram of liquid water at 0 °C or a gram of ice at 0 °C? Does this make sense? How large is the entropy difference?

•**P29** Suppose that the entropy of a certain substance (not an Einstein solid) is given by $S = a\sqrt{E}$, where a is a constant. What is the energy E as a function of the temperature T?

••**P30** A nanoparticle consisting of four iron atoms (object 1) initially has 1 quantum of energy. It is brought into contact with a nanoparticle consisting of two iron atoms (object 2), which initially has 2 quanta of energy. The mass of one mole of iron is 56 g. **(a)** Using the Einstein model of a solid, calculate and plot $\ln \Omega_1$ vs. q_1 (the number of quanta in object 1), $\ln \Omega_2$ vs. q_1, and $\ln \Omega_{\text{total}}$ vs. q_1 (put all three plots on the same graph). Show your work and explain briefly. **(b)** Calculate the approximate temperature of the objects at equilibrium. State what assumptions or approximations you made.

Section 12.6

•**P31** Suppose that the entropy of a certain substance (not an Einstein solid) is given by $S = a\sqrt{E}$, where a is a constant. What is the specific heat C as a function of the temperature T?

••**P32** A 100-g block of metal at a temperature of 20 °C is placed into an insulated container with 400 g of water at a temperature of 0 °C. The temperature of the metal and water ends up at 2 °C. What is the specific heat of this metal, per gram? Start from the Energy Principle. The specific heat of water is 4.2 J/K/g.

••**P33** A nanoparticle containing 6 atoms can be modeled approximately as an Einstein solid of 18 independent oscillators. The evenly spaced energy levels of each oscillator are 4×10^{-21} J apart. **(a)** When the nanoparticle's energy is in the range $5 \times 4 \times 10^{-21}$ J to $6 \times 4 \times 10^{-21}$ J, what is the approximate temperature? (In order to keep precision for calculating the specific heat, give the result to the nearest tenth of a kelvin.) **(b)** When the nanoparticle's energy is in the range $8 \times 4 \times 10^{-21}$ J to $9 \times 4 \times 10^{-21}$ J, what is the approximate temperature? (In order to keep precision for calculating the specific heat, give the result to the nearest tenth of a degree.) **(c)** When the nanoparticle's energy is in the range $5 \times 4 \times 10^{-21}$ J to $9 \times 4 \times 10^{-21}$ J, what is the approximate heat capacity per atom? Note that between parts (a) and (b) the average energy increased from 5.5 quanta to 8.5 quanta. As a check, compare your result with the high temperature limit of $3k_B$.

••**P34** The entropy S of a certain object (not an Einstein solid) is the following function of the internal energy E: $S = bE^{1/2}$, where b is a constant. **(a)** Determine the internal energy of this object as a function of the temperature. **(b)** What is the specific heat of this object as a function of the temperature?

••**P35** The interatomic spring stiffness for tungsten is determined from Young's modulus measurements to be 90 N/m. The mass of one mole of tungsten is 0.185 kg. If we model a block of tungsten as a collection of atomic "oscillators" (masses on springs), note that since each oscillator is attached to two "springs," and each "spring" is half the length of the interatomic bond, the effective interatomic spring stiffness for one of these oscillators is 4 times the calculated value given above.

Use these precise values for the constants: $\hbar = 1.0546 \times 10^{-34}$ J·s (Planck's constant divided by 2π), Avogadro's number $= 6.0221 \times 10^{23}$ molecules/mole, $k_B = 1.3807 \times 10^{-23}$ J/K (the Boltzmann constant). **(a)** What is one quantum of energy for one of these atomic oscillators? **(b)** Figure 12.56 contains the number of ways to arrange a given number of quanta of energy in a particular block of tungsten. Fill in the blanks to complete the table, including calculating the temperature of the block. The energy E is measured from the ground state. Nothing goes in the shaded boxes. Be sure to give the temperature to the nearest 0.1 kelvin. **(c)** There are about 60 atoms in this object. What is the heat capacity on a per-atom basis? (Note that at high temperatures the heat capacity on a per-atom basis approaches the classical limit of $3k_B = 4.2 \times 10^{-23}$ J/K/atom.)

q	# ways	E, J	S, J/K	ΔE, J	ΔS, J/K	T, K
20	4.91 E26					
21	4.44 E27					
22	3.85 E28					

Figure 12.56

••**P36** A 50-g block of copper (one mole has a mass of 63.5 g) at a temperature of 35 °C is put in contact with a 100-g block of aluminum (molar mass 27 g) at a temperature of 20 °C. The blocks are inside an insulated enclosure, with little contact with the walls. At these temperatures, the high-temperature limit is valid for the specific heat. Calculate the final temperature of the two blocks. Do NOT look up the specific heats of aluminum and copper; you should be able to figure them out on your own.

••**P37** Young's modulus for copper is measured by stretching a copper wire to be about 1.2×10^{11} N/m^2. The density of copper is about 9 g/cm^3, and the mass of a mole is 63.5 g. Starting from a very low temperature, use these data to estimate roughly the temperature T at which we expect the specific heat for copper to approach $3k_B$. Compare your estimate with the data shown on a graph in this chapter.

••**P38** Figure 12.57 shows a one-dimensional row of 5 microscopic objects each of mass 4×10^{-26} kg, connected by forces that can be modeled by springs of stiffness 15 N/m. These objects can move only along the x axis.

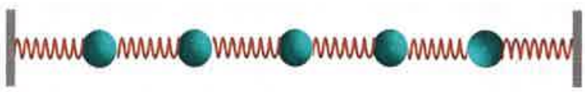

Figure 12.57

(a) Using the Einstein model, calculate the approximate entropy of this system for total energy of 0, 1, 2, 3, 4, and 5 quanta. Think carefully about what the Einstein model is, and apply those concepts to this one-dimensional situation. **(b)** Calculate the approximate temperature of the system when the total energy is 4 quanta. **(c)** Calculate the approximate specific heat on a per-object basis when the total energy is 4 quanta. **(d)** If the temperature is raised very high, what is the approximate specific heat on a per-object basis? Give a numerical value and compare with your result in part (c).

••**P39** In an insulated container a 100-W electric heating element of small mass warms up a 300-g sample of copper for 6 s. The initial temperature of the copper was 20 °C (room temperature). Predict the final temperature of the copper, using the $3k_B$ specific heat per atom.

••P40 The goal of this experiment is to understand, in a concrete way, what specific heat is and how it can be measured. You will need a microwave oven, a styrofoam coffee cup, and a clock or watch.

In the range of temperature where water is a liquid (0 C to 100 °C), it is approximately true that it takes 4.2 J of energy (1 calorie) to raise the temperature of 1 g of water through 1 Kelvin. To measure this specific heat of water, we need some way to raise a known mass of water from a known initial temperature to a final temperature that can also be measured, while we keep track of the energy supplied to the water. One way to do this, as discussed in the text, is to put water in a well-insulated container within which a heater, whose power output is known, warms up the water. In this experiment, instead of a well-insulated box with a heater, we will use microwave power, which preferentially warms up water by exciting rotational modes of the water molecules, as opposed to burners or heaters that warm up water in a pan by first warming up the pan.

Use a styrofoam coffee cup of known volume in which water can be warmed up. The density of water is 1 gram/cm^3.

It is a good idea not to fill the cup completely full, because this makes it more likely to spill.

One method of recording the initial temperature of the water is to get water from the faucet and wait for it to equilibrate with room temperature (which can either be read off a thermostat or estimated based on past experience). After waiting about a half hour for this to happen, place the cup in the microwave oven and turn on the oven at maximum power. The cup needs to be watched as it warms up, so that when the water starts to boil, the elapsed time can be noted accurately.

BE CAREFUL! A styrofoam cup full of hot liquid can buckle if you hold it near the rim. Hold the cup near the bottom. If the cup is full, do not attempt to move the cup while the water is hot. A spill can cause a painful burn.

On the back of the microwave oven (or inside the front door), there is usually a sticker with specifications that says "Output Power= ...Watts" which can be used to calculate the energy supplied. If there is no indication, use a typical value of 600 W for a standard microwave oven. Using all the quantities measured above and knowing the temperature interval over which you have warmed up the water, you can calculate the specific heat of water. **(a)** Show and explain all your data and calculations, and compare with the accepted value for water (4.2 J/K/g). **(b)** Discuss why your result might be expected to differ from the accepted value. For each effect that you consider, state whether this effect would lead to a result that is larger or smaller than the accepted value.

••P41 A box contains a uniform disk of mass M and radius R that is pivoted on a low-friction axle through its center (Figure 12.58). A block of mass m is pressed against the disk by a spring, so that the block acts like a brake, making the disk hard to turn. The box and the spring have negligible mass. A string is wrapped around the disk (out of the way of the brake) and passes through a hole in the box. A force of constant magnitude F acts on the end of the string. The motion takes place in outer space. At time t_i the speed of the box is v_i, and the rotational speed of the disk is ω_i. At time t_f the box has moved a distance x, and the end of the string has moved a longer distance d, as shown.

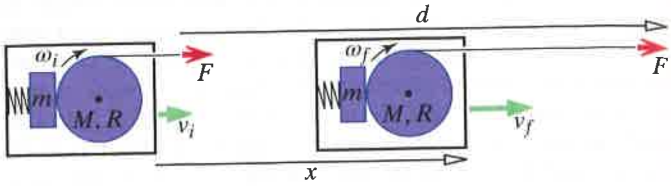

Figure 12.58

(a) At time t_f, what is the speed v_f of the box? **(b)** During this process, the brake exerts a tangential friction force of magnitude f. At time t_f, what is the angular speed ω_f of the disk? **(c)** At time t_f, assume that you know (from part b) the rotational speed ω_f of the disk. From time t_i to time t_f, what is the increase in thermal energy of the apparatus? **(d)** Suppose that the increase in thermal energy in part (c) is 8×10^4 J. The disk and brake are made of iron, and their total mass is 1.2 kg. At time t_i their temperature was 350 K. At time t_f, what is their approximate temperature?

Section 12.8

•P42 Consider the exponential function e^{-x}. Evaluate this function for $x = 1$, 10,000, and 0.01.

•P43 At room temperature (293 K), calculate k_BT in joules and eV.

•P44 A microscopic oscillator has its first and second excited states 0.05 eV and 0.10 eV above the ground-state energy. Calculate the Boltzmann factor for the ground state, first excited state, and second excited state, at room temperature.

Section 12.9

•P45 Suppose that you put one air molecule on your desk, so it is in thermal equilibrium with the desk at room temperature. Suppose that there is no atmosphere to get in the way of this one molecule bouncing up and down on the desk. Calculate the typical height that the air molecule will be above your desk, so that $Mgy \approx k_BT$.

•P46 Approximately what fraction of the sea-level air density is found at the top of Mount Everest, a height of 8848 m above sea level?

•P47 Calculate v_{rms} for a helium atom and for a nitrogen molecule (N_2; molecular mass 28 g per mole) in the room you're in (whose temperature is probably about 293 K).

•P48 Calculate the escape speed from the Moon and compare with typical speeds of gas molecules. The mass of the Moon is 7×10^{22} kg, and its radius is 1.75×10^6 m.

•P49 Sketch and label graphs of specific heat vs. temperature for hydrogen gas (H_2) and oxygen gas (O_2), using the same temperature scale. Explain briefly.

•P50 Marbles of mass $M = 10$ g are lying on the floor. They are of course in thermal equilibrium with their surroundings. What is a typical height above the floor for one of these marbles? That is, for what value of y is $Mgy \approx k_BT$?

•P51 Viruses of mass $M = 2 \times 10^{-20}$ kg are lying on the floor at room temperature (about 20 °C = 293 K). They are of course in thermal equilibrium with their surroundings. What is a typical height above the floor for one of these viruses? That is, for what value of y is $Mgy \approx k_BT$?

•P52 The temperature of the surface of a certain star is 7000 K. Most hydrogen atoms at the surface of the star are in the electronic ground state. What is the approximate fraction of the hydrogen atoms that are in the first excited state (and therefore could emit a photon)? The energy of the first excited state above the ground state is $(-13.6/2^2 eV) - (-13.6 eV) = 10.2 eV = 1.632 \times 10^{-18}$ J.

(In this estimate we are ignoring the fact that there may be several excited states with the same energies—for example, the 2s and 2p states in hydrogen—because this makes only a small difference in the answer.)

•P53 Figure 12.59 shows the distribution of speeds of atoms in a particular gas at a particular temperature. Approximately what is the average speed? Is the rms (root-mean-square) speed bigger or smaller than this? Approximately what fraction of the molecules have speeds greater than 1000 m/s?

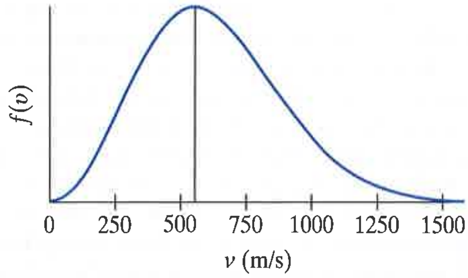

Figure 12.59

••P54 Calculate the temperature rise of a gas. **(a)** You have a bottle containing a mole of a monatomic gas such as helium or neon. You warm up this monatomic gas with an electrical heater, which inputs $Q = 580$ J of energy. How much does the temperature of the gas increase? **(b)** You have a bottle containing a mole of a diatomic gas such as nitrogen (N_2) or oxygen (O_2). The initial temperature is in the range where many rotational energy levels are excited but essentially no vibrational energy levels are excited. You warm up this diatomic gas with an electrical heater, which inputs $Q = 580$ J of energy. How much does the temperature of the gas increase? **(c)** You have a bottle containing a mole of a diatomic gas such as nitrogen (N_2) or oxygen (O_2). The initial temperature is in the range where not only many rotational energy levels are excited but also many vibrational energy levels are excited. You warm up this diatomic gas with an electrical heater, which inputs $Q = 580$ J of energy. How much does the temperature of the gas increase?

••P55 In studying a voyage to the Moon in Chapter 3 we somewhat arbitrarily started at a height of 50 km above the surface of the Earth. **(a)** At this altitude, what is the density of the air as a fraction of the density at sea level? **(b)** Approximately how many air molecules are there in one cubic centimeter at this altitude? **(c)** At what altitude is air density one-millionth (1×10^{-6}) that at sea level?

••P56 At sufficiently high temperatures, the thermal speeds of gas molecules may be high enough that collisions may ionize a molecule (that is, remove an outer electron). An ionized gas in which each molecule has lost an electron is called a "plasma." Determine approximately the temperature at which air becomes a plasma.

••P57 100 J of energy transfer due to a temperature difference are given to air in a 50-L rigid container and to helium in a 50-L rigid container, both initially at STP (standard temperature and pressure). **(a)** Which gas experiences a greater temperature rise? **(b)** What is the temperature rise of the helium gas?

•••P58 It is possible to estimate some properties of a diatomic molecule from the temperature dependence of the specific heat. **(a)** Below about 80 K the specific heat at constant volume for hydrogen gas (H_2) is $\frac{3}{2}k_B$ per molecule, but at higher temperatures the specific heat increases to $\frac{5}{2}k_B$ per molecule due to contributions from rotational energy states. Use these observations to estimate the distance between the hydrogen nuclei in an H_2 molecule. **(b)** At about 2000 K the specific heat at constant volume for hydrogen gas (H_2) increases to $\frac{7}{2}k_B$ per molecule due to contributions from vibrational energy states. Use these observations to estimate the stiffness of the "spring" that approximately represents the interatomic force.

•••P59 In 1988, telescopes viewed Pluto as it crossed in front of a distant star. As the star emerged from behind the planet, light from the star was slightly dimmed as it went through Pluto's atmosphere. The observations indicated that the atmospheric density at a height of 50 km above the surface of Pluto is about one-third the density at the surface. The mass of Pluto is known to be about 1.5×10^{22} kg and its radius is about 1200 km. Spectroscopic data indicate that the atmosphere is mostly nitrogen (N_2). Estimate the temperature of Pluto's atmosphere. State what approximations and/or simplifying assumptions you made.

•••P60 Buckminsterfullerene, C_{60}, is a large molecule consisting of 60 carbon atoms connected to form a hollow sphere. The diameter of a C_{60} molecule is about 7×10^{-10} m. It has been hypothesized that C_{60} molecules might be found in clouds of interstellar dust, which often contain interesting chemical compounds. The temperature of an interstellar dust cloud may be very low, around 3 K. Suppose you are planning to try to detect the presence of C_{60} in such a cold dust cloud by detecting photons emitted when molecules undergo transitions from one rotational energy state to another.

Approximately, what is the highest-numbered rotational level from which you would expect to observe emissions? Rotational levels are $l = 0, 1, 2, 3, \ldots$.

COMPUTATIONAL PROBLEMS

More detailed and extended versions of some computational modeling problems may be found in the lab activities included in the *Matter & Interactions, 4th Edition*, resources for instructors.

Working through Problems P61–P64 can be very informative and can help make the models discussed in this chapter concrete and clear.

••P61 Write a program to calculate the number of ways to arrange energy in an Einstein solid. **(a)** Model a system consisting of two atoms (three oscillators each), among which 4 quanta of energy are to be distributed. Write a program to display a histogram showing the total number of possible microstates of the two-atom system vs. the number of quanta assigned to atom

1. Compare your histogram to the one shown in Figure 12.16 (you should get the same distribution). **(b)** Model a system consisting of two solid blocks, block 1 containing 300 oscillators and block 2 containing 200 oscillators. Find the possible distributions of 100 quanta among these blocks, and plot number of microstates vs. number of quanta assigned to block 1. Compare your histogram to the one shown in Figure 12.22. Determine the distribution of quanta for which the probability is half as large as the most probable 60-40 distribution. **(c)** Do a series of calculations distributing 100 quanta between two blocks whose total number of oscillators is 500, but whose relative number of atoms varies. For example, consider equal numbers of oscillators, and ratios of 2:1, 5:1, and so on. Describe your observations.

••P62 Start with your solution to Problem P61. For the same system of two blocks, with $N_1 = 300$ oscillators and $N_2 = 200$ oscillators, plot $\ln(\Omega_1)$, $\ln(\Omega_2)$, and $\ln(\Omega_1\Omega_2)$, for q_1 running from 0 to 100 quanta. Your graph should look like the one in Figure 12.26. Determine the maximum value of $\ln(\Omega_1\Omega_2)$ and the value of q_1 where this maximum occurs. What is the significance of this value of q_1?

••P63 Modify your program from Problem P62 to plot the temperature of block 1 in kelvins as a function of the number of quanta q_1 present in the first block. On the same graph, plot the temperature of block 2 in kelvins as a function of q_1 (of course, $q_2 = q_{\text{tot}} - q_1$).

In order to plot the temperature in kelvins, you must determine the values of ΔE and ΔS that correspond to a one-quantum change in energy. Consider the model we are using. The energy of one quantum, in joules, is $\Delta E = \hbar\sqrt{k_{s,i}/m_a}$. The increment in entropy corresponding to this increment in energy is $\Delta S = k_B\Delta(\ln\Omega)$. Assume that the blocks are made of aluminum. Based on Young's modulus measurements the interatomic spring constant $k_{s,i}$ for Al is approximately 16 N/m. In Section 12.6 we show that the effective $k_{s,i}$ for oscillations in the Einstein solid is expected to be about 4 times the value obtained from measuring Young's modulus.

What is the significance of the value of q_1 (and of q_2) where the temperature curves for the two blocks cross (the temperatures are equal)?

••P64 Modify your analysis of Problem P63 to determine the specific heat as a function of temperature for a single block of metal. In order to see all of the important effects, consider a single block of 35 atoms (105 oscillators) with up to 300 quanta of energy. Note that in this analysis you are calculating quantities for a *single* block, not two blocks in contact.

To make specific comparison with experimental data, consider the cases of aluminum (Al) and lead (Pb). For each metal, plot the theoretical specific heat C per atom vs. $T(K)$, with dots showing the actual experimental data given in the following table, which you should convert to the same per-atom basis as your theoretical calculations. Adjust the interatomic spring stiffness k_s until your calculations approximately fit the experimental data.

T	C, Al	C, Pb	T	C, Al	C, Pb
20	0.23	11.01	150	18.52	25.27
40	2.09	19.57	200	21.58	25.87
60	5.77	22.43	250	23.25	26.36
80	9.65	23.69	300	24.32	26.82
100	13.04	24.43	400	25.61	27.45

T is in kelvin; C is J/K/mole.

What value of k_s gives a good fit? Based on Young's modulus, estimated values of the interatomic spring stiffnesses are 16 N/m for Al and 5 N/m for lead. However, recall that these values may need modification since the "springs" in the Einstein model are half as long, and there are two of them per oscillator.

Show from your graph that the high-temperature limit of the specific heat is about $3k_B$ per atom. The rise above this limit at high temperatures may be due to the fact that the assumed uniform spacing of the quantized oscillator energy levels isn't a good approximation for highly excited states. See Chapter 8.

••P65 For some examples of your choice, demonstrate by carrying out actual computer calculations that the "square-root" rule holds true for the fractional width of the peak representing the most probable arrangements of the energy. *Warning:* Check to see what is the largest number you can use in your computations; some programs or programming environments won't handle numbers bigger than about 1×10^{307}, for example, and larger numbers are treated as "infinite."

••P66 Create a computational model of one atom of the Einstein solid as shown in Figure 12.4. Let the atom move under the influence of the six spring-like forces that act on the atom. Use the classical Momentum Principle, although the actual behavior is governed by quantum mechanics, with quantized energies. **(a)** Experiment with different initial displacements of the atom away from the equilibrium position. **(b)** In many materials the effective stiffness of the interatomic bonds is different in the x, y, and z directions; see how making the stiffnesses different affects the motion.

ANSWERS TO CHECKPOINTS

1 20
2 123, 132, 213, 231, 312, 321
3 RRRGG, RRGRG, RRGGR, RGRRG, RGRGR, RGGRR, GGRRR, GRGRR, GRRGR, GRRRG
4 See Figure 12.15.
6 This looks *very* improbable; the second law of thermodynamics would be violated by this process—the entropy of the Universe is decreasing.
7 500 K
9 2.50; 2.74
10 Increase, because the average speed of the air molecules increases; 40% (factor of $\sqrt{2}$)
11 (a) $\frac{7}{2}k_B$; (b) $\frac{5}{2}k_B$; (c) $\frac{3}{2}k_B$; (d) transition to $\frac{3}{2}k_B$ occurs at a lower temperature for deuterium because the rotational energy level spacing $L^2/(2I)$ is smaller for deuterium (larger nuclear mass means larger moment of inertia, since the internuclear distance is about the same; $L^2 = l(l+1)\hbar^2$ is the same for the rotational energy levels of hydrogen and deuterium molecules)