PROBLEMS

Section 11.1

•P13 Evaluate the cross product $(5\hat{\imath} + 3\hat{\jmath}) \times (-4\hat{\imath} + 2\hat{\jmath})$, which expands to $-20\hat{\imath} \times \hat{\imath} + 10\hat{\imath} \times \hat{\jmath} - 12\hat{\jmath} \times \hat{\imath} + 6\hat{\jmath} \times \hat{\jmath}$.

•P14 What is the angular momentum $\vec{L}_A$ if $\vec{r}_A = \langle 9, -9, 0\rangle$ m and $\vec{p} = \langle 12, 10, 0\rangle$ kg·m/s?

•P15 At a particular instant the location of an object relative to location A is given by the vector $\vec{r}_A = \langle 6, 6, 0\rangle$ m. At this instant the momentum of the object is $\vec{p} = \langle -11, 13, 0\rangle$ kg·m/s. What is the angular momentum of the object about location A?

•P16 Figure 11.75 shows seven particles, each with the same magnitude of momentum $|\vec{p}| = 25$ kg·m/s but with different directions of momentum and different positions relative to location A. The distances shown in the diagram have these values: $w = 18$ m, $h = 28$ m, and $d = 27$ m.

Calculate the z component of angular momentum L_{Az} for each particle (x to the right, y up, z out of the page). Make sure you give the correct sign.

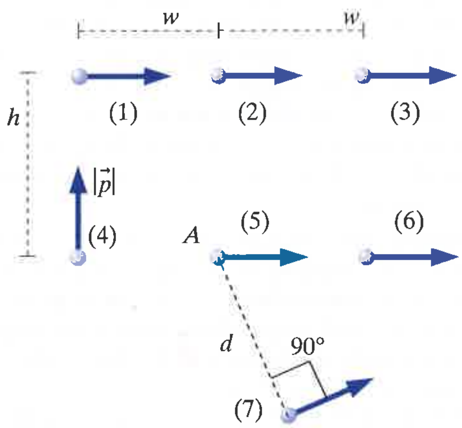

Figure 11.75

•P17 A comet orbits the Sun (Figure 11.76). When it is at location 1 it is a distance d_1 from the Sun, and has magnitude of momentum p_1. Location A is at the center of the Sun. When the comet is at location 2, it is a distance d_2 from the Sun, and has magnitude of momentum p_2. **(a)** When the comet is at location 1, what is the direction of $\vec{L}_A$? **(b)** When the comet is at location 1, what is the magnitude of $\vec{L}_A$? **(c)** When the comet is at location 2, what is the direction of $\vec{L}_A$? **(d)** When the comet is at location 2, what is the magnitude of $\vec{L}_A$? Later we'll see that the Angular Momentum Principle tells us that the angular momentum at location 1 must be equal to the angular momentum at location 2.

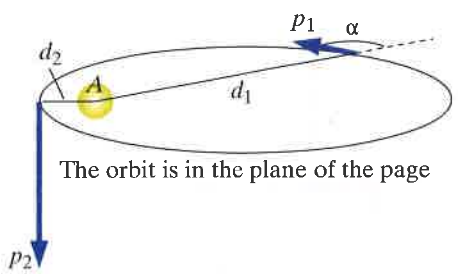

Figure 11.76

•P18 A common amusement park ride is a Ferris wheel (see Figure 11.77, which is not drawn to scale). Riders sit in chairs that are on pivots so they remain level as the wheel turns at a constant rate. A particular Ferris wheel has a radius of 24 meters, and it makes one complete revolution around its axle (at location A) in 20 s. In all of the following questions, consider location A (at the center of the axle) as the location around which we will calculate the angular momentum. At the instant shown in the diagram, a child of mass 40 kg, sitting at location F, is traveling with velocity $\langle 7.5, 0, 0\rangle$ m/s.

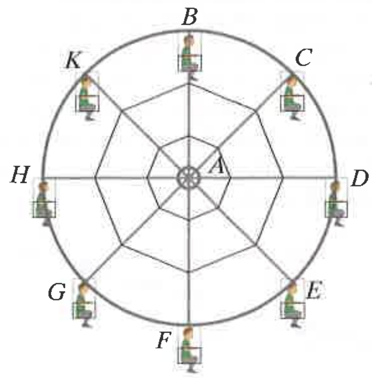

Figure 11.77

(a) What is the linear momentum of the child? **(b)** In the definition $\vec{L} = \vec{r} \times \vec{p}$, what is the vector $\vec{r}$? **(c)** What is $\vec{r}_\perp$? **(d)** What is the magnitude of the angular momentum of the child about location A? **(e)** What is the plane defined by $\vec{r}$ and $\vec{p}$ (that is, the plane containing both of these vectors)? **(f)** Use the right-hand rule to determine the z component of the angular momentum of the child about location A. **(g)** You used the right-hand rule to determine the z component of the angular momentum, but as a check, calculate in terms of position and momentum: What is xp_y? What is yp_x? Therefore, what is the z component of the angular momentum of the child about location A? **(h)** The Ferris wheel keeps turning, and at a later time, the same child is at location E, with coordinates $\langle 16.971, -16.971, 0\rangle$ m relative to location A, moving with velocity $\langle 5.303, 5.303, 0\rangle$ m/s. Now what is the magnitude of the angular momentum of the child about location A?

Section 11.2

•P19 The moment of inertia of a sphere of uniform density rotating on its axis is $\frac{2}{5}MR^2$. Use data given at the end of this book to calculate the magnitude of the rotational angular momentum of the Earth.

•P20 Calculate the angular momentum for a rotating disk, sphere, and rod: **(a)** A uniform disk of mass 13 kg, thickness 0.5 m, and radius 0.2 m is located at the origin, oriented with its axis along the y axis. It rotates clockwise around its axis when viewed from above (that is, you stand at a point on the $+y$ axis and look toward the origin at the disk). The disk makes one complete rotation every 0.6 s. What is the rotational angular momentum of the disk? What is the rotational kinetic energy of the disk? **(b)** A sphere of uniform density, with mass 22 kg and radius 0.7 m, is located at the origin and rotates around an axis parallel with the x axis. If you stand somewhere on the $+x$ axis and look toward the origin at the sphere, the sphere spins counterclockwise. One complete revolution takes 0.5 s. What is the rotational angular momentum of the sphere? What is the rotational kinetic energy of the sphere? **(c)** A cylindrical rod of uniform density is located with its center at the origin, and its axis along the z axis. Its radius is 0.06 m, its length is 0.7 m, and its mass is 5 kg. It makes one revolution every 0.03 s. If you stand on the $+x$ axis and look toward the origin at the rod, the rod spins clockwise. What is the rotational angular momentum of the rod? What is the rotational kinetic energy of the rod?

•P21 If an object has a moment of inertia 19 $kg \cdot m^2$ and the magnitude of its rotational angular momentum is 36 $kg \cdot m^2/s$, what is its rotational kinetic energy?

•P22 Mounted on a low-mass rod of length 0.32 m are four balls (Figure 11.78). Two balls (shown in red on the diagram), each of mass 0.82 kg, are mounted at opposite ends of the rod. Two other balls, each of mass 0.29 kg (shown in blue on the diagram), are each mounted a distance 0.08 m from the center of the rod. The rod rotates on an axle through the center of the rod (indicated by the "x" in the diagram), perpendicular to the rod, and it takes 0.9 s to make one full rotation.

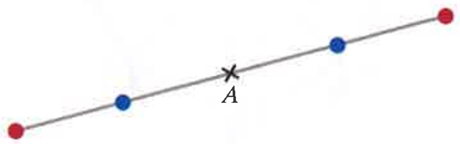

Figure 11.78

(a) What is the moment of inertia of the device about its center? **(b)** What is the angular speed of the rotating device? **(c)** What is the magnitude of the angular momentum of the rotating device?

•P23 The moment of inertia of a uniform-density disk rotating about an axle through its center can be shown to be $\frac{1}{2}MR^2$. This result is obtained by using integral calculus to add up the contributions of all the atoms in the disk. The factor of 1/2 reflects the fact that some of the atoms are near the center and some are far from the center; the factor of 1/2 is an average of the square distances. A uniform-density disk whose mass is 16 kg and radius is 0.15 m makes one complete rotation every 0.5 s. **(a)** What is the moment of inertia of this disk? **(b)** What is its rotational kinetic energy? **(c)** What is the magnitude of its rotational angular momentum?

•P24 In Figure 11.79 a barbell spins around a pivot at its center at A. The barbell consists of two small balls, each with mass 500 g (0.5 kg), at the ends of a very low mass rod of length $d = 20$ cm (0.2 m; the radius of rotation is 0.1 m). The barbell spins clockwise with angular speed 80 rad/s.

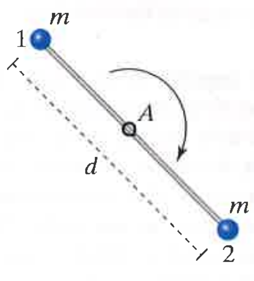

Figure 11.79

We can calculate the angular momentum and kinetic energy of this object in two different ways, by treating the object as two separate balls or as one barbell. Use the usual coordinate system, with x to the right, y toward the top of the page, and z out of the page, toward you.

I: Treat the object as two separate balls. Calculate the following quantities:
(a) The speed of ball 1, **(b)** $\vec{L}_{trans,1,A}$ of ball 1, **(c)** $\vec{L}_{trans,2,A}$ of ball 2, **(d)** $\vec{L}_{tot,A}$, **(e)** the translational kinetic energy of ball 1, **(f)** the translational kinetic energy of ball 2, **(g)** the total kinetic energy of the barbell.

II: Treat the object as one barbell. Calculate the following quantities:
(h) The moment of inertia I of the barbell, **(i)** $\vec{\omega}$, expressed as a vector, **(j)** $\vec{L}_{rot}$ of the barbell, **(k)** K_{rot}.

III: Compare the two approaches:
1. Compare your result for $\vec{L}_{tot,A}$ in part I to your result for $\vec{L}_{rot}$ in part II. Should these quantities be the same, or different?
2. Compare your result for K_{total} in part I to your result for K_{rot} in part II. Should these quantities be the same, or different?

•P25 A low-mass rod of length 0.30 m has a metal ball of mass 1.7 kg at each end. The center of the rod is located at the origin, and the rod rotates in the yz plane about its center. The rod rotates clockwise around its axis when viewed from a point on the $+x$ axis, looking toward the origin. The rod makes one complete rotation every 0.5 s. **(a)** What is the moment of inertia of the object (rod plus two balls)? **(b)** What is the rotational angular momentum of the object? **(c)** What is the rotational kinetic energy of the object?

••P26 Calculate the angular momentum of the Earth: **(a)** Calculate the magnitude of the translational angular momentum of the Earth relative to the center of the Sun. See the data on inside back cover. **(b)** Calculate the magnitude of the rotational angular momentum of the Earth. How does this compare to your result in part (a)?
(The angular momentum of the Earth relative to the center of the Sun is the sum of the translational and rotational angular momenta. The rotational axis of the Earth is tipped 23.5° away from a perpendicular to the plane of its orbit.)

••P27 In Figure 11.80 two small objects each of mass $m = 0.3$ kg are connected by a lightweight rod of length $d = 1.5$ m. At a particular instant they have velocities whose magnitudes are $v_1 = 38$ m/s and $v_2 = 60$ m/s and are subjected to external forces whose magnitudes are $F_1 = 41$ N and $F_2 = 26$ N. The distance $h = 0.3$ m, and the distance $w = 0.7$ m. The system is moving in outer space. Assuming the usual coordinate system with $+x$ to the right, $+y$ toward the top of the page, and $+z$ out of the page toward you, calculate these quantities for this system:

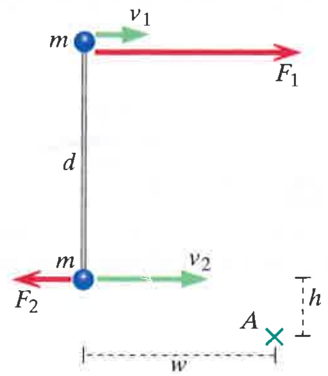

Figure 11.80

(a) $\vec{p}_{\text{total}}$, **(b)** $\vec{v}_{\text{CM}}$, **(c)** $\vec{L}_{\text{tot},A}$, **(d)** $\vec{L}_{\text{rot}}$, **(e)** $\vec{L}_{\text{trans},A}$, **(f)** $\vec{p}_{\text{total}}$ at a time 0.23 s after the initial time.

Section 11.3

•P28 A barbell consists of two small balls, each with mass $m = 0.4$ kg, at the ends of a very low mass rod of length $d = 0.6$ m. It is mounted on the end of a low-mass rigid rod of length $b = 0.9$ m (Figure 11.81). The apparatus is set in motion in such a way that although the rod rotates clockwise with angular speed $\omega_1 = 15$ rad/s, the barbell maintains its vertical orientation. Calculate these vector quantities: **(a)** $\vec{L}_{\text{rot}}$, **(b)** $\vec{L}_{\text{trans},B}$, **(c)** $\vec{L}_{\text{tot},B}$.

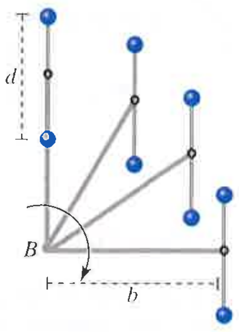

Figure 11.81

•P29 A barbell consists of two small balls, each with mass $m = 0.4$ kg, at the ends of a very low mass rod of length $d = 0.6$ m. It is mounted on the end of a low-mass rigid rod of length $b = 0.9$ m. The apparatus is set in motion in such a way that it again rotates clockwise with angular speed $\omega_1 = 15$ rad/s, but in addition, the barbell rotates clockwise about its center, with an angular speed $\omega_2 = 20$ rad/s (Figure 11.82). Calculate these vector quantities: **(a)** $\vec{L}_{\text{rot}}$, **(b)** $\vec{L}_{\text{trans},B}$, **(c)** $\vec{L}_{\text{tot},B}$.

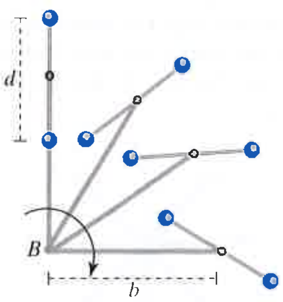

Figure 11.82

Section 11.5

•P30 As shown in Figure 11.83, seven forces all with magnitude $|\vec{F}| = 25$ N are applied to an irregularly shaped object. Each force is applied at a different location on the object, indicated by the tail of the arrow; the directions of the forces differ. The distances shown in the diagram have these values: $w = 9$ m, $h = 14$ m, and $d = 13$ m. For each force, calculate the z component of the torque due to that force, relative to location A (x to the right, y up, z out of the page). Make sure you give the correct sign. Relative to location A, what is the z component of the net torque acting on this object?

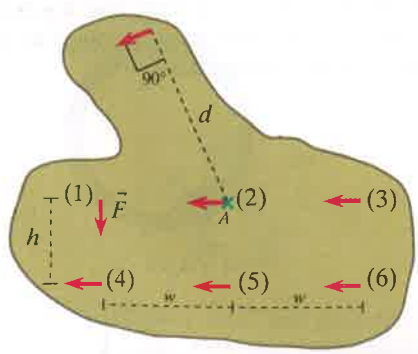

Figure 11.83

•P31 At $t = 15$ s, a particle has angular momentum $\langle 3, 5, -2 \rangle$ kg·m²/s relative to location A. A constant torque $\langle 10, -12, 20 \rangle$ N·m relative to location A acts on the particle. At $t = 15.1$ s, what is the angular momentum of the particle?

•P32 Calculating torque in Figure 11.84: **(a)** If $r_A = 3$ m, $F = 8$ N, and $\theta = 51°$, what is the magnitude of the torque about location A, including units? **(b)** If the force were perpendicular to $\vec{r}_A$ but gave the same torque as in the preceding question, what would be its magnitude?

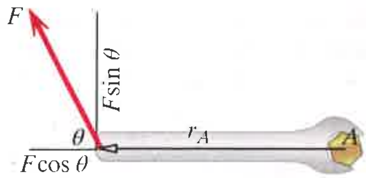

Figure 11.84

••P33 Let's compare the Momentum Principle and the Angular Momentum Principle in a simple situation. Consider a mass m falling near the Earth (Figure 11.85). Neglecting air resistance, the Momentum Principle gives $dp_y/dt = -mg$, yielding $dv_y/dt = -g$ (nonrelativistic). Choose a location A off to the side, on the ground. Apply the Angular Momentum Principle to find an algebraic expression for the rate of change of angular momentum of the mass about location A.

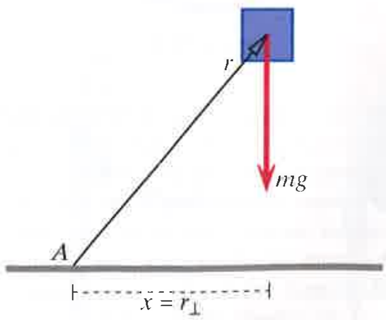

Figure 11.85

••P34 A small rock passes a massive star, following the path shown in red on the diagram. When the rock is a distance 4.5×10^{13} m (indicated as d_1 in Figure 11.86) from the center of the star, the magnitude p_1 of its momentum is 1.35×10^{17} kg·m/s, and the angle is 126°. At a later time, when the rock is a distance $d_2 = 1.3 \times 10^{13}$ m from the center of the star, it is heading in the $-y$ direction. There are no other massive objects nearby. What is the magnitude p_2 of the final momentum?

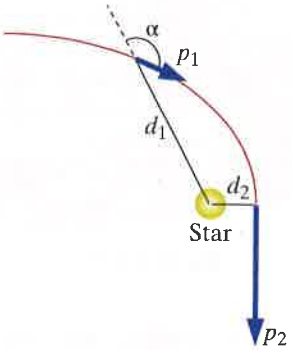

Figure 11.86

Section 11.7

•**P35** A stationary bicycle wheel of radius 0.9 m is mounted in the vertical plane (Figure 11.87). The axle is held up by supports that are not shown, and the wheel is free to rotate on the nearly frictionless axle. The wheel has mass 4.8 kg, all concentrated in the rim (the spokes have negligible mass). A lump of clay with mass 0.5 kg falls and sticks to the outer edge of the wheel at the location shown. Just before the impact the clay has speed 5 m/s, and the wheel is rotating clockwise with angular speed 0.33 rad/s.

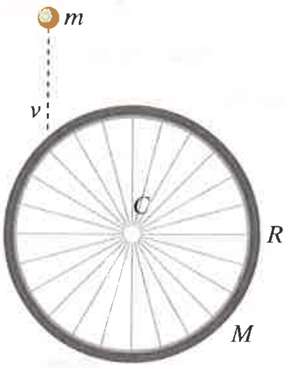

Figure 11.87

(a) Just before the impact, what is the angular momentum (magnitude and direction) of the combined system of wheel plus clay about the center C? (As usual, x is to the right, y is up, and z is out of the screen, toward you.) **(b)** Just after the impact, what is the angular momentum (magnitude and direction) of the combined system of wheel plus clay about the center C? **(c)** Just after the impact, what is the angular velocity (magnitude and direction) of the wheel? **(d)** Qualitatively, what happens to the linear momentum of the combined system? Why? (1) The downward linear momentum decreases because the axle exerts an upward force. (2) Some of the linear momentum is changed into angular momentum. (3) Some of the linear momentum is changed into energy. (4) There is no change because linear momentum is always conserved.

•**P36** A rotating uniform-density disk of radius 0.6 m is mounted in the vertical plane, as shown in Figure 11.88. The axle is held up by supports that are not shown, and the disk is free to rotate on the nearly frictionless axle. The disk has mass 5 kg. A lump of clay with mass 0.4 kg falls and sticks to the outer edge of the wheel at the location $\langle -0.36, 0.480, 0 \rangle$ m, relative to an origin at the center of the axle. Just before the impact the clay has speed 8 m/s, and the disk is rotating clockwise with angular speed 0.51 radians/s.

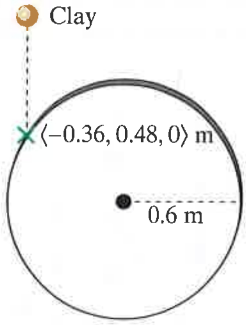

Figure 11.88

(a) Just before the impact, what is the angular momentum (magnitude and direction) of the combined system of wheel plus clay about the center C? (As usual, x is to the right, y is up, and z is out of the screen, toward you.) **(b)** Just after the impact, what is the angular momentum (magnitude and direction) of the combined system of wheel plus clay about the center C? **(c)** Just after the impact, what is the angular velocity (magnitude and direction) of the wheel? **(d)** Qualitatively, what happens to the linear momentum of the combined system? Why? (A) There is no change because linear momentum is always conserved. (B) Some of the linear momentum is changed into angular momentum. (C) Some of the linear momentum is changed into energy. (D) The downward linear momentum decreases because the axle exerts an upward force.

•**P37** Figure 11.89 depicts a device that can rotate freely with little friction with the axle. The radius is 0.4 m, and each of the eight balls has a mass of 0.3 kg. The device is initially not rotating. A piece of clay falls and sticks to one of the balls as shown in the figure. The mass of the clay is 0.066 kg and its speed just before the collision is 10 m/s.

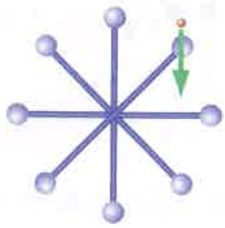

Figure 11.89

(a) Which of the following statements are true, for angular momentum relative to the the axle of the wheel? (1) Just before the collision, $r_\perp = 0.4\sqrt{2}/2 = 0.4\cos(45°)$ (for the clay). (2) The angular momentum of the wheel is the same before and after the collision. (3) Just before the collision, the angular momentum of the wheel is 0. (4) The angular momentum of the wheel is the sum of the angular momenta of all eight balls. (5) The angular momentum of the wheel + clay after the collision is equal to the initial angular momentum of the clay. (6) The angular momentum of the falling clay is zero because the clay is moving in a straight line. **(b)** Just after the collision, what is the speed of one of the balls?

••P38 A stick of length L and mass M hangs from a low-friction axle (Figure 11.90). A bullet of mass m traveling at a high speed v strikes near the bottom of the stick and quickly buries itself in the stick.

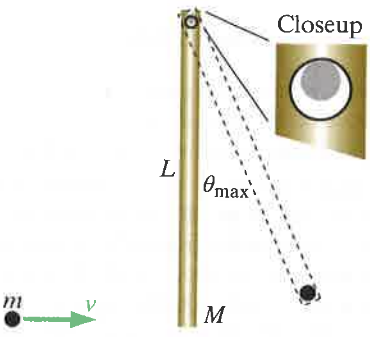

Figure 11.90

(a) During the brief impact, is the linear momentum of the stick + bullet system constant? Explain why or why not. Include in your explanation a sketch of how the stick shifts on the axle during the impact. **(b)** During the brief impact, around what point does the angular momentum of the stick + bullet system remain constant? **(c)** Just after the impact, what is the angular speed ω of the stick (with the bullet embedded in it)? (Note that the center of mass of the stick has a speed $\omega L/2$. The moment of inertia of a uniform rod about its center of mass is $\frac{1}{12}ML^2$.) **(d)** Calculate the change in kinetic energy from just before to just after the impact. Where has this energy gone? **(e)** The stick (with the bullet embedded in it) swings through a maximum angle θ_{max} after the impact, then swings back. Calculate θ_{max}.

•P39 **(a)** What is the period of small-angle oscillations of a simple pendulum with a mass of 0.1 kg at the end of a string of length 1 m? **(b)** What is the period of small-angle oscillations of a meter stick suspended from one end, whose mass is 0.1 kg?

••P40 A disk of mass 3 kg and radius 0.15 m hangs in the xy plane from a horizontal low-friction axle. The axle is 0.09 m from the center of the disk. What is the frequency f of small-angle oscillations of the disk? What is the period?

••P41 Design a decorative "mobile" to consist of a low-mass rod of length 0.49 m suspended from a string so that the rod is horizontal, with two balls hanging from the ends of the rod. At the left end of the rod hangs a ball with mass 0.484 kg. At the right end of the rod hangs a ball with mass 0.273 kg. You need to decide how far from the left end of the rod you should attach the string that will hold up the mobile, so that the mobile hangs motionless with the rod horizontal ("equilibrium"). You also need to determine the tension in the string supporting the mobile. **(a)** What is the tension in the string that supports the mobile? **(b)** How far from the left end of the rod should you attach the support string?

••P42 A space station has the form of a hoop of radius $R =$ 14 m, with mass $M = 6250$ kg (Figure 11.91). Initially its center of mass is not moving, but it is spinning with angular speed $\omega_0 = 0.0013$ rad/s. Then a small package of mass $m = 6$ kg is thrown by a spring-loaded gun toward a nearby spacecraft as shown; the package has a speed $v = 40$ m/s after launch. Calculate the center-of-mass velocity of the space station and its rotational speed ω after the launch.

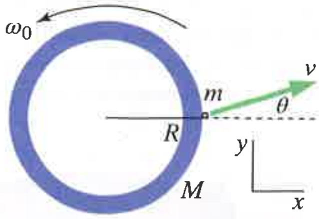

Figure 11.91

••P43 You sit on a rotating stool and hold barbells in both hands with your arms fully extended horizontally. You make one complete turn in 2 s. You then pull the barbells in close to your body. **(a)** Estimate how long it now takes you to make one complete turn. Be clear and explicit about the principles you apply and about your assumptions and approximations. **(b)** About how much energy did you expend?

••P44 A certain comet of mass m at its closest approach to the Sun is observed to be at a distance r_1 from the center of the Sun, moving with speed v_1 (Figure 11.92). At a later time the comet is observed to be at a distance r_2 from the center of the Sun, and the angle between $\vec{r}_2$ and the velocity vector is measured to be θ. What is v_2? Explain briefly.

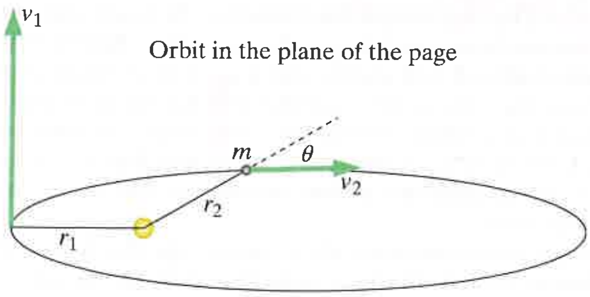

Figure 11.92

••P45 An ice skater whirls with her arms and one leg stuck out as shown on the left in Figure 11.93, making one complete turn in 1 s. Then she quickly moves her arms up above her head and pulls her leg in as shown at the right in Figure 11.93.

Figure 11.93

(a) Estimate how long it now takes for her to make one complete turn. Explain your calculations, and state clearly what approximations and estimates you make. **(b)** Estimate the minimum amount of chemical energy she must expend to change her configuration.

••P46 A playground ride consists of a disk of mass $M = 43$ kg and radius $R = 1.7$ m mounted on a low-friction axle (Figure 11.94). A child of mass $m = 25$ kg runs at speed $v = 2.3$ m/s on a line tangential to the disk and jumps onto the outer edge of the disk.

Figure 11.94

(a) If the disk was initially at rest, now how fast is it rotating? **(b)** What is the change in the kinetic energy of the child plus the disk? **(c)** Where has most of this kinetic energy gone? **(d)** Calculate the change in *linear* momentum of the system consisting of the child plus the disk (but not including the axle), from just before to just after impact. What caused this change in the linear momentum? **(e)** The child on the disk walks inward on the disk and ends up standing at a new location a distance 0.85 m from the axle. Now what is the angular speed? **(f)** What is the change in the kinetic energy of the child plus the disk, from the beginning to the end of the walk on the disk? **(g)** What was the source of this increased kinetic energy?

••P47 In Figure 11.95 two small objects each of mass m_1 are connected by a lightweight rod of length L. At a particular instant the center of mass speed is v_1 as shown, and the object is rotating counterclockwise with angular speed ω_1. A small object of mass m_2 traveling with speed v_2 collides with the rod at an angle θ_2 as shown, at a distance b from the center of the rod. After being struck, the mass m_2 is observed to move with speed v_4, at angle θ_4. All the quantities are positive magnitudes. This all takes place in outer space.

For the object consisting of the rod with the two masses, write equations that, in principle, could be solved for the center of mass speed v_3, direction θ_3, and angular speed ω_3 in terms of the given quantities. State clearly what physical principles you use to obtain your equations.

Don't attempt to solve the equations; just set them up.

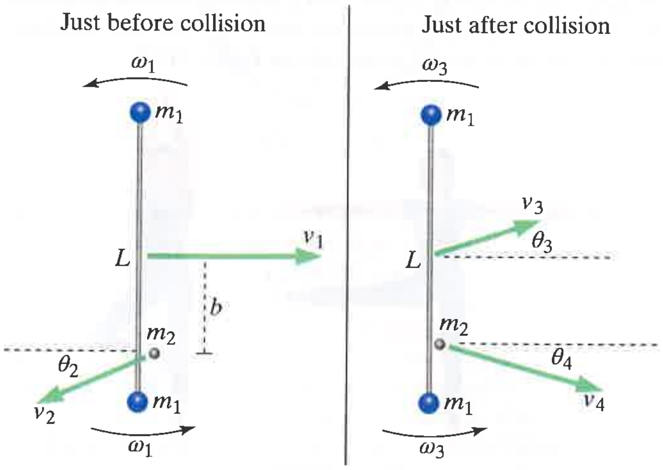

Figure 11.95

••P48 In Figure 11.96 a spherical nonspinning asteroid of mass M and radius R moving with speed v_1 to the right collides with a similar nonspinning asteroid moving with speed v_2 to the left, and they stick together. The impact parameter is d. Note that $I_{\text{sphere}} = \frac{2}{5}MR^2$.

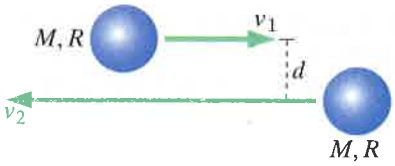

Figure 11.96

After the collision, what is the velocity v_{CM} of the center of mass and the angular velocity ω about the center of mass? (Note that each asteroid rotates about its own center with this same ω.)

••P49 A spherical satellite of approximately uniform density with radius 4.8 m and mass 205 kg is originally moving with velocity $\langle 2600, 0, 0\rangle$ m/s, and is originally rotating with an angular speed 2 rad/s, in the direction shown in the diagram. A small piece of space junk of mass 4.1 kg is initially moving toward the satellite with velocity $\langle -2200, 0, 0\rangle$ m/s. The space junk hits the edge of the satellite at location C as shown in Figure 11.97, and moves off with a new velocity $\langle -1300, 480, 0\rangle$ m/s. Both before and after the collision, the rotation of the space junk is negligible.

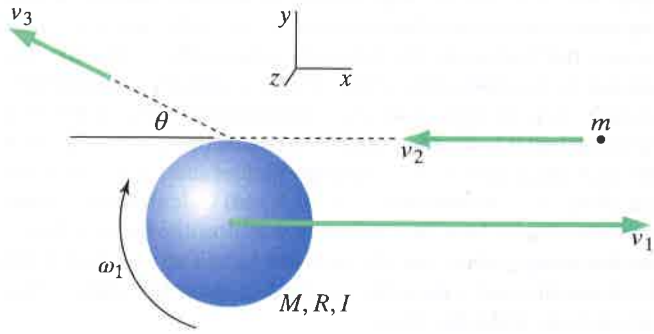

Figure 11.97

(a) Just after the collision, what are the components of the center-of-mass velocity of the satellite (v_x and v_y) and its rotational speed ω? **(b)** Calculate the rise in the internal energy of the satellite and space junk combined.

••P50 A device consisting of four heavy balls connected by low-mass rods is free to rotate about an axle, as shown in Figure 11.98. It is initially not spinning. A small bullet traveling very fast buries itself in one of the balls. $m = 0.002$ kg, $v = 550$ m/s, $M_1 = 1.2$ kg, $M_2 = 0.4$ kg, $R_1 = 0.6$ m, and $R_2 = 0.2$ m. The axle of the device is at the origin $\langle 0, 0, 0\rangle$, and the bullet strikes at location $\langle 0.155, 0.580, 0\rangle$ m. Just after the impact, what is the angular speed of the device? Note that this is an inelastic collision; the system's temperature increases.

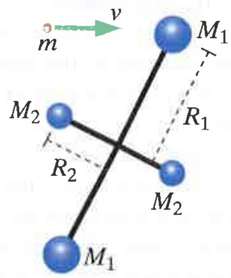

Figure 11.98

••**P51** A thin metal rod of mass 1.3 kg and length 0.4 m is at rest in outer space, near a space station (Figure 11.99). A tiny meteorite with mass 0.06 kg traveling at a high speed of 200 m/s strikes the rod a distance 0.2 m from the center and bounces off with speed 60 m/s as shown in the diagram. The magnitudes of the initial and final angles to the x axis of the small mass's velocity are $\theta_i = 26°$ and $\theta_f = 82°$. **(a)** Afterward, what is the velocity of the center of the rod? **(b)** Afterward, what is the angular velocity $\vec{\omega}$ of the rod? **(c)** What is the increase in internal energy of the objects?

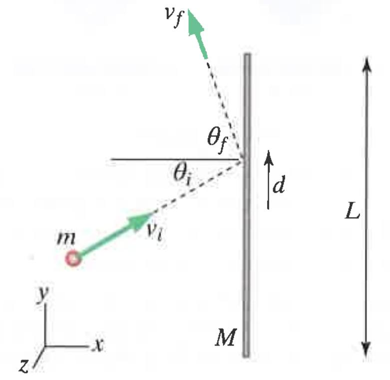

Figure 11.99

••**P52** A diver dives from a high platform (Figure 11.100). When he leaves the platform, he tucks tightly and performs three complete revolutions in the air, then straightens out with his body fully extended before entering the water. He is in the air for a total time of 1.4 s. What is his angular speed ω just as he enters the water? Give a numerical answer. Be explicit about details of your model, and include (brief) explanations. You will need to estimate some quantities.

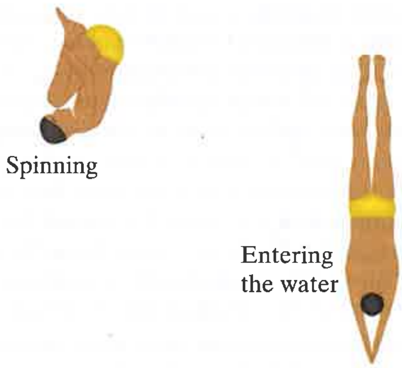

Figure 11.100

•••**P53** Suppose that an asteroid of mass 2×10^{21} kg is nearly at rest outside the solar system, far beyond Pluto. It falls toward the Sun and crashes into the Earth at the equator, coming in at an angle of 30° to the vertical as shown, against the direction of rotation of the Earth (Figure 11.101; not to scale). It is so large that its motion is barely affected by the atmosphere.

Figure 11.101

(a) Calculate the impact speed. **(b)** Calculate the change in the length of a day due to the impact.

Section 11.8

•**P54** A disk of radius 8 cm is pulled along a frictionless surface with a force of 10 N by a string wrapped around the edge (Figure 11.102). 24 cm of string has unwound off the disk. What are the magnitude and direction of the torque exerted about the center of the disk at this instant?

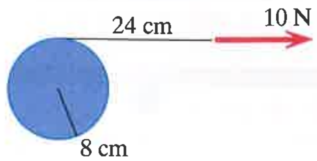

Figure 11.102

•**P55** In Figure 11.102, the uniform solid disk has mass 0.4 kg (moment of inertia $I = \frac{1}{2}MR^2$). At the instant shown, the angular velocity is 20 rad/s into the page. **(a)** At this instant, what are the magnitude and direction of the angular momentum about the center of the disk? **(b)** At a time 0.2 s later, what are the magnitude and direction of the angular momentum about the center of the disk? **(c)** At this later time, what are the magnitude and direction of the angular velocity?

•**P56** Two people of different masses sit on a seesaw (Figure 11.103). M_1, the mass of person 1, is 90 kg, M_2 is 42 kg, $d_1 = 0.8$ m, and $d_2 = 1.3$ m. The people are initially at rest. The mass of the board is negligible.

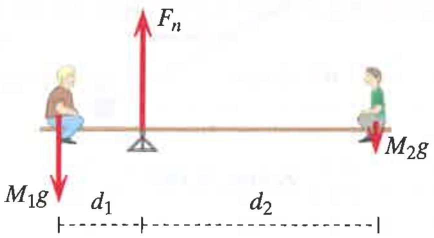

Figure 11.103

(a) What are the magnitude and direction of the torque about the pivot due to the gravitational force on person 1? **(b)** What are the magnitude and direction of the torque about the pivot due to the gravitational force on person 2? **(c)** Since at this instant the linear momentum of the system may be changing, we don't know the magnitude of the "normal" force exerted by the pivot. Nonetheless, it is possible to calculate the torque due to this force. What are the magnitude and direction of the torque about the pivot due to the force exerted by the pivot on the board? **(d)** What are the magnitude and direction of the net torque on the system (board + people)? **(e)** Because of this net torque, what will happen? (A) The seesaw will begin to rotate clockwise. (B) The seesaw will begin to rotate counterclockwise. (C) The seesaw will not move. **(f)** Person 2 moves to a new position, in which the magnitude of the net torque about the pivot is now 0, and the seesaw is balanced. What is the new value of d_2 in this situation?

•**P57** A board of length $2d = 6$ m rests on a cylinder (the "pivot"). A ball of mass 5 kg is placed on the end of the board. Figure 11.104 shows the objects at a particular instant. **(a)** On a free-body diagram, show the forces acting on the ball + board system, in their correct locations. **(b)** Take the point at which the board touches the cylinder as location A. What is the magnitude of the torque on the system of (ball + board) about location A? **(c)** Which of the following statements are correct? (1) Because there is a torque, the angular momentum of the system will change in the next tenth of a second. (2) The forces balance, so the angular momentum of the system about location A will not change. (3) The force by the cylinder on the board contributes nothing to the torque about location A.

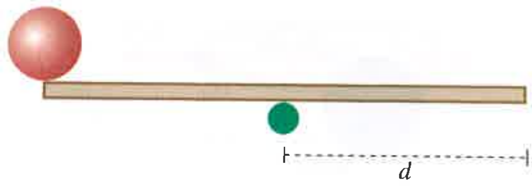

Figure 11.104

•P58 A barbell is mounted on a nearly frictionless axle through its center (Figure 11.105). At this instant, there are two forces of equal magnitude applied to the system as shown, with the directions indicated, and at this instant the angular velocity is 60 rad/s, counterclockwise. In the next 0.001 s, the angular momentum relative to the center increases by an amount $2.5 \times 10^{-4}\,\text{kg}\cdot\text{m}^2/\text{s}$. What is the magnitude of each force? What is the net force?

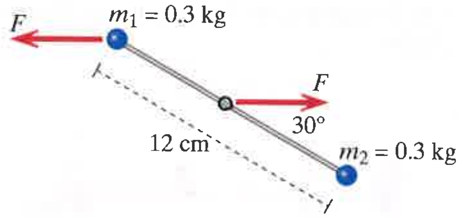

Figure 11.105

••P59 A disk of radius 0.2 m and moment of inertia $1.5\,\text{kg}\cdot\text{m}^2$ is mounted on a nearly frictionless axle (Figure 11.106). A string is wrapped tightly around the disk, and you pull on the string with a constant force of 25 N. After a while the disk has reached an angular speed of 2 rad/s. What is its angular speed 0.1 seconds later? Explain briefly.

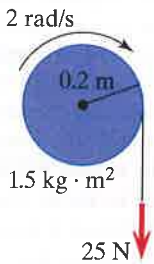

Figure 11.106

••P60 String is wrapped around an object of mass 1.2 kg, radius 0.06 m, and moment of inertia $0.0015\,\text{kg}\cdot\text{m}^2$ (the density of the object is not uniform). With your hand you pull the string straight up with some constant force F such that the center of the object does not move up or down, but the object spins faster and faster (Figure 11.107). This is like a yo-yo; nothing but the vertical string touches the object.

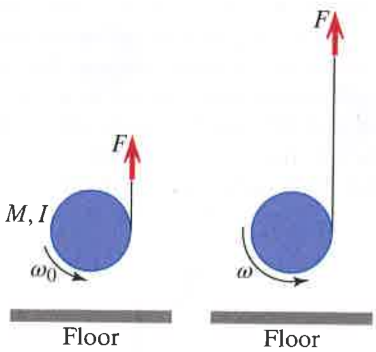

Figure 11.107

(a) Initially the object has an angular speed $\omega_0 = 12$ rad/s. After a time interval of 0.1 s, what is the angular speed ω of the object? **(b)** Your answer must be numeric and not contain the symbol F. Explain the physics principles you are using.

••P61 A solid object of uniform density with mass M, radius R, and moment of inertia I rolls without slipping down a ramp at an angle θ to the horizontal. The object could be a hoop, a disk, a sphere, etc. **(a)** Carefully follow the complete analysis procedure explained in earlier chapters, but with the addition of the Angular Momentum Principle about the center of mass. Note that in your force diagram you must include a small frictional force f that points *up* the ramp. Without that force the object will slip. Also note that the condition of nonslipping implies that the instantaneous velocity of the atoms of the object that are momentarily in contact with the ramp is zero, so $f < \mu F_N$ (no slipping). This zero-velocity condition also implies that $v_{CM} = \omega R$, where ω is the angular speed of the object, since the instantaneous speed of the contact point is $v_{CM} - \omega R$. **(b)** The moment of inertia about the center of mass of a uniform hoop is MR^2, for a uniform disk it is $(1/2)MR^2$, and for a uniform sphere it is $(2/5)MR^2$. Calculate the acceleration dv_{CM}/dt for each of these objects. **(c)** If two hoops of different mass are started from rest at the same time and the same height on a ramp, which will reach the bottom first? If a hoop, a disk, and a sphere of the same mass are started from rest at the same time and the same height on a ramp, which will reach the bottom first? **(d)** Write the energy equation for the object rolling down the ramp, and for the point-particle system. Show that the time derivatives of these equations are compatible with the force and torque analyses.

••P62 A yo-yo is constructed of three disks: two outer disks of mass M, radius R, and thickness d, and an inner disk (around which the string is wrapped) of mass m, radius r, and thickness d. The yo-yo is suspended from the ceiling and then released with the string vertical (Figure 11.108).

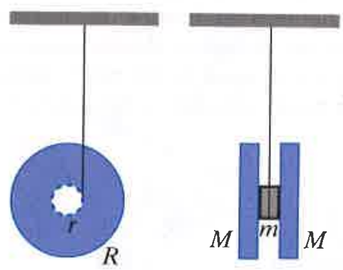

Figure 11.108

Calculate the tension in the string as the yo-yo falls. Note that when the center of the yo-yo moves down a distance y, the yo-yo turns through an angle y/r, which in turn means that the angular speed ω is equal to v_{CM}/r. The moment of inertia of a uniform disk is $\frac{1}{2}MR^2$.

••P63 A string is wrapped around a uniform disk of mass $M = 1.2$ kg and radius $R = 0.11$ m (Figure 11.109). Attached to the disk are four low-mass rods of radius $b = 0.14$ m, each with a small mass $m = 0.4$ kg at the end. The device is initially at rest on a nearly frictionless surface. Then you pull the string with a constant force $F = 21$ N for a time of 0.2 s. Now what is the angular speed of the apparatus?

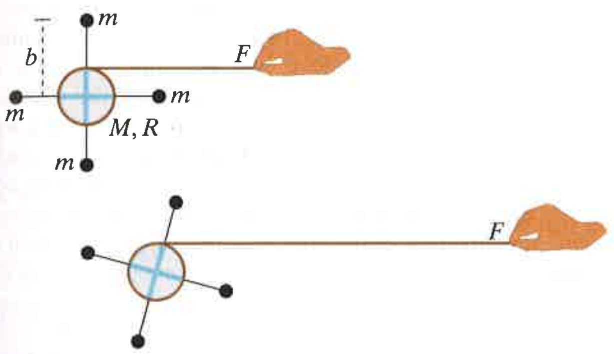

Figure 11.109

•••P64 A small solid rubber ball of radius r hits a rough horizontal floor such that its speed v just before striking the floor at location A makes an angle of 60° with the horizontal and also has back spin with angular speed ω. It is observed that the ball repeatedly bounces from A to B, then from B back to A, etc. Assuming perfectly elastic impact determine **(a)** the required magnitude of ω of the back spin in terms of v and r, and **(b)** the minimum magnitude of co-efficient of static friction μ_s to enable this motion. *Hint:* Notice that the direction of $\vec{\omega}$ flips in each collision.

Section 11.9

•P65 A wheel rotates at a rate of 200 rpm (revolutions per minute). **(a)** What is the angular speed in radians per second? **(b)** What is the angle in radians through which the wheel rotates in 5 s? **(c)** What is this angle in degrees?

•P66 A wheel is mounted on a stationary axle that lies along the z axis. The center of the wheel is at $\langle 0,0,0 \rangle$ m. It is rotating with a constant angular speed of 30 rad/s. At time $t = 4.20$ s a red dot painted on the rim of the wheel is located at $\langle 0.25,0,0 \rangle$ m. At time $t = 4.26$ s, what is the location of the red dot?

•P67 A rotating wheel accelerates at a constant rate from an angular speed of 24 rad/s to 36 rad/s in a time interval of 3 s. **(a)** What is the angular acceleration in rad/s/s? **(b)** What is the average angular speed? **(c)** What is the angle in radians through which the wheel rotates? **(d)** What is this angle in degrees?

•P68 A string is wrapped around a wheel of radius 25 cm mounted on a stationary axle. The wheel is initially not rotating. You pull the string with a constant force through a distance of 40 cm. What is the angle in radians and degrees through which the wheel rotates?

••P69 A wheel with radius 30 cm is rotating at a rate of 10 rev/s. **(a)** What is the angular speed in radians per second? **(b)** In a time interval of 6 s, what is the angle in radians through which the wheel rotates? **(c)** At $t = 10$ s the angular speed begins to increase at a rate of 1.4 rad/s/s. At $t = 15$ s, what is the angular speed in radians per second? **(d)** Through what angle in radians did the wheel rotate during the time between $t = 10$ s and $t = 15$ s? **(e)** If the wheel rolls along the ground without slipping, the instantaneous velocity of the atoms of the object that are momentarily in contact with the ground is zero. This zero-velocity condition implies that $v_{CM} = \omega R$, where ω is the angular speed of the object, since the instantaneous speed of the contact point is $v_{CM} - \omega R$. During the time between $t = 10$ s and $t = 15$ s, how far did the center of the wheel move, in meters?

••P70 If you did not already do Problem P59, do it now. Also calculate the angle through which the disk turns, in radians and degrees.

••P71 If you did not already do Problem P60, do it now. Also calculate numerically the angle through which the yo-yo turns, in radians and degrees.

••P72 If you did not already do Problem P63, do it now. Also calculate numerically the angle through which the apparatus turns, in radians and degrees.

•••P73 Two balls of equal radius and mass, free to roll on a horizontal plane, are separated by a distance L large compared to their radius. One ball is solid, the other hollow with a thickness small compared to its radius. They are attracted by an electric force. How far will the solid ball roll before it collides with the hollow ball?

•••P74 An amusing trick is to press a finger down on a marble on a horizontal table top, in such a way that the marble is projected along the table with an initial linear speed v and an initial backward rotational speed ω about a horizontal axis perpendicular to v. The coefficient of sliding friction between marble and top is constant. The marble has radius R. **(a)** If the marble slides to a complete stop, what was ω in terms of v and R? **(b)** If the marble skids to a stop and then starts returning toward its initial position, with a final constant speed of (3/7)v, what was ω in terms of v and R? *Hint for part (b):* When the marble rolls without slipping, the relationship between speed and angular speed is $v = \omega R$.

Section 11.11

•P75 According to the Bohr model of the hydrogen atom, what is the magnitude of the translational angular momentum of the electron (relative to the location of the proton) when the atom is in the 2nd excited state above the ground state ($N = 3$)?

••P76 Calculate rotational energy levels and photon emission energies for a carbon monoxide molecule (CO). **(a)** Calculate the energies of the quantized rotational energy levels for the CO molecule. Estimate any quantities you need. See the discussion of diatomic molecules in the section on quantized angular momentum; the parameter l has values $0,1,2,3\ldots$. **(b)** Describe the emission spectrum for electromagnetic radiation emitted in transitions among the rotational CO energy levels. Include a calculation of the lowest-energy emission in electron volts (1.6×10^{-19} J). **(c)** It is transitions among "electronic" states of atoms that produce visible light, with photon energies on the order of a couple of electron-volts. Each electronic energy level has quantized rotational and vibrational (harmonic oscillator) energy sublevels. Explain why this leads to a visible spectrum that contains "bands" rather than individual energies.

••P77 Review the derivation of the Bohr model of the hydrogen atom and apply this reasoning to predict the energy levels of ionized helium He^+ (a helium atom with only one electron, and a nucleus containing two protons and two neutrons). What are the energies in eV of the ground state and the first excited state? What is the energy of a photon emitted in a transition from the first excited state to the ground state? How do these results differ from those for a hydrogen atom?

••P78 The Bohr model correctly predicts the main energy levels not only for atomic hydrogen but also for other "one-electron" atoms where all but one of the atomic electrons has been removed, such as in He^+ (one electron removed) or Li^{++} (two electrons removed). **(a)** Predict the energy levels in eV for a system consisting of a nucleus containing Z protons and just one electron. You need not recapitulate the entire derivation for the Bohr model, but do explain the changes you have to make to take into account the factor Z. **(b)** The negative muon (μ^-) behaves like a heavy electron, with the same charge as the electron but with a mass 207 times as large as the electron mass. As a moving μ^- comes to rest in matter, it tends to knock electrons out of atoms and settle down onto a nucleus to form a "one-muon" atom. For a system consisting of a lead nucleus (Pb^{208} has 82 protons and 126 neutrons) and just one negative muon, predict the energy in eV of a photon emitted in a transition from the first excited state to the ground state. The high-energy photons emitted by transitions between energy levels in such "muonic atoms" are easily observed in experiments with muons. **(c)** Calculate the radius of the smallest Bohr orbit for a μ^- bound to a lead nucleus (Pb^{208} has 82 protons and 126 neutrons). Compare with the approximate radius of the lead nucleus (remember that the radius of a proton or neutron is about 1×10^{-15} m, and the nucleons are packed closely together in the nucleus).

Comments: This analysis in terms of the simple Bohr model hints at the result of a full quantum-mechanical analysis, which shows that in the ground state of the lead–muon system there is a rather high probability for finding the muon *inside* the lead nucleus. Nothing in quantum mechanics forbids this penetration, especially since the muon does not participate in the strong interaction. Electrons in an atom can also be found inside the nucleus, but the probability is very low, because on average the electrons are very far from the nucleus, unlike the muon.

The eventual fate of the μ^- in a muonic atom is that it either decays into an electron, neutrino, and antineutrino, or it reacts through the weak interaction with a proton in the nucleus to produce a neutron and a neutrino. This "muon capture" reaction is more likely if the probability is high for the muon to be found inside the nucleus, as is the case with heavy nuclei such as lead.

••P79 The nucleus dysprosium-160 (containing 160 nucleons) acts like a spinning object with quantized angular momentum, $L^2 = l(l+1)\hbar^2$, and for this nucleus it turns out that l must be an even integer (0,2,4...). When a Dy-160 nucleus drops from the $l = 2$ state to the $l = 0$ state, it emits an 87 keV photon (87×10^3 eV). **(a)** What is the moment of inertia of the Dy-160 nucleus? **(b)** Given your result from part (a), find the approximate radius of the Dy-160 nucleus, assuming it is spherical. (In fact, these and similar experimental observations have shown that some nuclei are not quite spherical.) **(c)** The radius of a (spherical) nucleus is given approximately by $(1.3 \times 10^{-15}\,\text{m})A^{1/3}$, where A is the total number of protons and neutrons. Compare this prediction with your result in part (b).

Section 11.12

•P80 Two gyroscopes are made exactly alike except that the spinning disk in one is made of low-density aluminum, whereas the disk in the other is made of high-density lead. If they have the same spin angular speeds and the same torque is applied to both, which gyroscope precesses faster?

••P81 This problem requires that you have a toy gyroscope available. The purpose of this problem is to make as concrete as possible the unusual motions of a gyroscope and their analysis in terms of fundamental principles. In all of the following studies, the effects are most dramatic if you give the gyroscope as large a spin angular speed as possible. **(a)** Hold the spinning gyroscope firmly in your hand, and try to rotate the spin axis quickly to point in a new direction. Explain qualitatively why this feels "funny." Also explain why you *don't* feel anything odd when you move the spinning gyroscope in any direction *without* changing the direction of the spin axis. **(b)** Support one end of the spinning gyroscope (on a pedestal or in an open loop of the string) so that the gyroscope precesses *counterclockwise* as seen from above. Explain this counterclockwise precession direction; include sketches of top and side views of the gyroscope. **(c)** Again support one end of the spinning gyroscope so that the gyroscope precesses *clockwise* as seen from above. Explain this clockwise precession direction; include sketches of top and side views of the gyroscope.

••P82 This problem requires that you have a toy gyroscope available. The purpose of this problem is to make as concrete as possible the unusual motions of a gyroscope and their analysis in terms of fundamental principles. In all of the following studies, the effects are most dramatic if you give the gyroscope as large a spin angular speed as possible. **(a)** If you knew the spin angular speed of your gyroscope, you could predict the precession rate. Invent an appropriate experimental technique and determine the spin angular speed approximately. Explain your experimental method and your calculations. Then predict the corresponding precession rate, and compare with your measurement of the precession rate. You will have to measure and estimate some properties of the gyroscope and how it is constructed. **(b)** Make a *quick* measurement of the precession rate with the spin axis horizontal, then make another quick measurement of the precession rate with the spin axis nearly vertical. (If you make quick measurements, friction on the spin axis doesn't have much time to change the spin angular speed.) Repeat, this time with the spin axis initially nearly vertical, then horizontal. Making all four of these measurements gives you some indication of how much the spin unavoidably changes due to friction while you are quickly changing the angle. What do you conclude about the dependence of the precession rate on the angle, assuming the same spin rate at these different angles? What is the theoretical prediction for the dependence of the precession rate on angle (for the same spin rate)?

••P83 The axis of a gyroscope is tilted at an angle of 30° to the vertical (Figure 11.110). The rotor has a radius of 15 cm, mass 3 kg, moment of inertia $0.06\,\text{kg}\cdot\text{m}^2$, and spins on its axis at 30 rad/s. It is supported in a cage (not shown) in such a way that without an added weight it does not precess. Then a mass of 0.2 kg is hung from the axis at a distance of 18 cm from the center of the rotor.

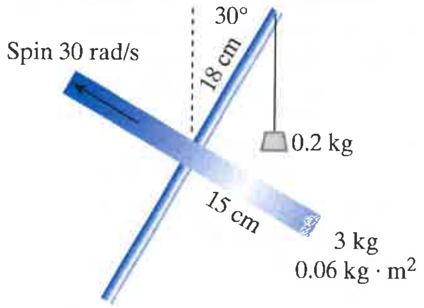

Figure 11.110

(a) Viewed from above, does the gyroscope precess in a (1) clockwise or a (2) counterclockwise direction? That is, does the top end of the axis move (1) out of the page or (2) into the page in the next instant? Explain your reasoning. **(b)** How long does it take for the gyroscope to make one complete precession?

••P84 A bicycle wheel with a heavy rim is mounted on a lightweight axle, and one end of the axle rests on top of a post. The wheel is observed to precess in the horizontal plane. With the spin direction shown in Figure 11.111, does the wheel precess clockwise or counterclockwise? Explain in detail, including appropriate diagrams.

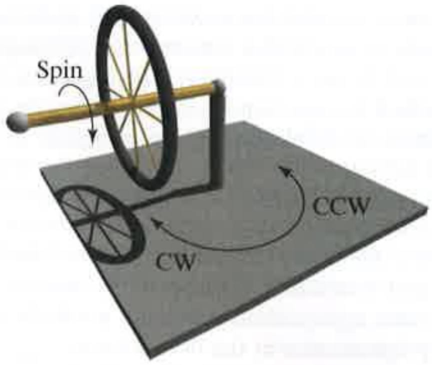

Figure 11.111

••P85 A solid wood top spins at high speed on the floor, with a spin direction shown in Figure 11.112.

Figure 11.112

(a) Using appropriately labeled diagrams, explain the direction of motion of the top (you do not need to explain the magnitude). **(b)** How would the motion change if the top had a higher spin rate? Explain briefly. **(c)** If the top were made of solid steel instead of wood, explain how this would affect the motion (for the same spin rate).

COMPUTATIONAL PROBLEMS

•P86 A rod on a low-friction axle is initially at rest when a constant torque begins to be applied. **(a)** Complete the skeleton program shown below and make sure that it runs properly. See Section 11.10 for suggestions. After exiting the loop, print the values of the final angle `theta` and the final angular speed `mag(omega)`. Given the fact that the torque is constant, what should be the value of the ratio `mag(omega)/theta`? Is this true for your program? **(b)** Make graphs of `theta` vs. time and `mag(omega)` vs. t. What should these graphs look like? Do they? **(c)** Change the torque to oscillate: $\tau_{\text{net},z} = 3\cos(5t)$. Graph `theta` vs. time and measure the period. Is it consistent with the angular frequency of 5 rad/s of the torque? **(d)** Plot on the same graph the rotational kinetic energy of the rod vs. time and the magnitude of the torque as a function of time. Is the rotational kinetic energy large when the torque is large?

```
from visual import *
from visual.graph import *

M = 2
Lrod = 1
R = 0.1
Laxle = 4*R
I = (1/12)*M*Lrod**2 + (1/4)*M*R**2

rod = cylinder(pos=vector(-1,0,0),
               radius=R, color=color.orange,
               axis=vector(Lrod,0,0))
axle = cylinder(pos=vector(-1+Lrod/2,0,-Laxle/2),
                radius=R/6, color=color.red,
                axis=vector(0,0,4*R))

L = vector(0,0,0) # angular momentum
deltat = 0.0001   # for accuracy in later parts
t = 0
theta = 0
dtheta = 0

while t < 2:
    rate(10000)
    torque = vector(0,0,2) # constant torque
    # Apply Angular Momentum Principle
    # Update angle and rod position
    t = t + deltat
```

•P87 Start with the program for part **(c)** of Problem P86. Add a constant force $\vec{F} = \langle 0.1,0,0\rangle$ N applied to the axle. Let the while loop run to 7 s. Describe what you see.

••P88 Start with the program from Chapter 3 Problem P68 to model the motion of a planet going around a fixed star. In this problem you will build on that program. **(a)** Use initial conditions that produce an elliptical orbit. At each step calculate the translational angular momentum $\vec{L}_{\text{trans},A}$ of the planet with respect to a location A chosen to be in the orbital plane but outside the orbit. Display this in two ways (i and ii below), and briefly describe in words what you observe: (i) Display $\vec{L}_{trans,A}$ as an arrow with its tail at location A, throughout the orbit. Since the magnitude of $\vec{L}_{trans,A}$ is quite different from the magnitudes of the distances involved, you will need to scale the arrow by some factor to fit it on the screen. (ii) Graph the component of $\vec{L}_{trans,A}$ perpendicular to the orbital plane as a function of time. **(b)** Repeat part (a), but this time choose a different location B at the center of the fixed star, and calculate and display $\vec{L}_{trans,B}$ relative to that location B. As in part (a), display $\vec{L}_{trans,B}$ as an arrow (scaled appropriately), and also graph the component of $\vec{L}_{trans,B}$ perpendicular to the orbital plane, as a function of time, and briefly describe in words what you observe. **(c)** Choose a location C which is not in the orbital plane, and calculate and display $\vec{L}_{trans,C}$ as an arrow throughout the orbit. (You do not need to make a graph.) Briefly describe in words what you observe.

••P89 Figure 11.113 shows a glass disk mounted on a low-friction axle and held stationary. The center of the disk is at the origin, ⟨0,0,0⟩ m. A peg in the disk is connected by two identical stretched springs to a wall. The disk is released from rest. **(a)** Run the skeleton program shown below and rotate the camera to understand the apparatus. Then complete the program so that the disk rotates. **(b)** Make a graph of the angle `theta` and determine the period. Is this system a harmonic oscillator? Try varying the initial angle and/or the relaxed length of the spring. **(c)** Show on a second graph the kinetic energy of the disk, the spring energy, and their sum, as a function of time. What should the sum look like? Try a smaller time step. (The setting `material=materials.rough` for the disk makes it easier to see its rotation; this may not have a visible effect if your computer graphics card is not up to date.)

Figure 11.113

```
from visual import *
from visual.graph import *
M = 2          # mass of uniform-density disk
R = 0.2        # radius of disk
thick = 0.02   # thickness of disk
I = M*R**2     # moment of inertia of disk
r = 0.9*R      # peg in disk at this radius
Lpeg = 5*thick # length of peg
wall = box(pos=vector(-1.2*R,0,0),
           size=(0.01,2.4*R,0.8*R),
           color=color.green)
# Center of disk is at <0,0,0>:
disk = cylinder(pos=vector(0,0,-thick/2),
           radius=R, axis=vector(0,0,thick),
           color=color.white, opacity=0.7,
           material=materials.rough)
axle = cylinder(pos=vector(0,0,-Lpeg/2),
           radius=0.05*R,color=color.red,
           axis=vector(0,0,Lpeg))
# Place peg in the disk on x axis:
peg = cylinder(pos=(r,0,-Lpeg/2),
          radius=0.03*R, color=color.red,
          axis=vector(0,0,Lpeg))
# Rotate to initial position:
theta = pi/6     # radians; CCW from x axis
peg.rotate(angle=theta, axis=axle.axis,
          origin=axle.pos)
rspring = 0.05*R  # spring radius
# Front spring:
springF = helix(pos=(wall.x,r,1.5*thick),
           radius=0.05*R, color=color.orange,
           coils=15, thickness=0.4*rspring)
# Back spring:
springB = helix(pos=(wall.x,r,-1.5*thick),
           radius=0.05*R, color=color.orange,
           coils=15, thickness=0.4*rspring)
# Attach springs to peg:
end = peg.pos+vector(0,0,Lpeg/2+springF.pos.z)
springB.axis = springF.axis = end - springF.pos

t = 0
deltat = 0.01
dtheta = 0
ks = 1.5          # stiffness of each spring
L0 = 0.26         # relaxed length of each spring
L = vector(0,0,0) # initial angular momentum

while True:
    rate(100)
    # Calculate spring force F acting on peg
    # Calculate torque due to springs
    # Update angular momentum L
    # Calculate angular velocity omega
    # Calculate dtheta and theta
    # Update disk and peg positions:
    disk.rotate(angle=dtheta, axis=axle.axis,
                origin=axle.pos)
    peg.rotate(angle=dtheta, axis=axle.axis,
               origin=axle.pos)
    # Update spring lengths:
    end = peg.pos+vector(0,0,Lpeg/2+springF.pos.z)
    springF.axis = end - springF.pos
    springB.axis = springF.axis
```

•••P90 An electric "dipole" consists of an object that is positively charged at one end and negatively charged at the other. A molecular example is an HCl molecule in which the H end is positive and the Cl end is negative. In Figure 11.114 is a snapshot of a positively charged sphere on the left interacting with a dipole on the right in outer space; the two charges on the right are stuck together. The spheres are uniformly charged spheres of radius $R = 1$ mm which interact electrically as though they were point charges concentrated at the centers. Each sphere has uniform density with a mass of 1×10^{-6} kg. Each of the red spheres has an electric charge of $+2\times10^{-10}$ C and the negative sphere's charge is -2×10^{-10} C.

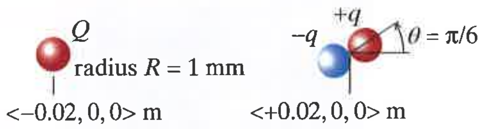

Figure 11.114

The charge on the left is fixed in position at $\langle -0.02, 0, 0\rangle$ m. The center of mass of the dipole is initially located at $\vec{r}_{CM} = \langle 0.02, 0, 0\rangle$ m, with the positive charge at $\vec{r}_{CM}+\vec{r}$ and the negative charge at $\vec{r}_{CM}-\vec{r}$, where $\vec{r} = \langle R\cos\theta, R\sin\theta, 0\rangle$ and $\theta = \pi/6$ rad. The dipole is free to rotate about an axle at its center of mass, but the axle is fixed and cannot move. **(a)** Create a computational model of the situation. Note in calculating the moment of inertia of the rotating dipole that each of the dipole charges has both translational and rotational angular momenta. We suggest using a time step of 0.001 s for accuracy, with a large value in the rate statement for display speed. Use the Angular Momentum Principle to predict and display the motion of the dipole, starting from rest (no rotation). Include a graph of θ vs. time. What is the period of the oscillation? **(b)** Determine the period for starting values of θ of 0.01π and 0.02π. Is this oscillator a harmonic oscillator for small oscillations (that is, is the period independent of the amplitude)? **(c)** Is the period the same if you increase the amplitude to $\theta = \pi/2$? **(d)** Measure the period for a small amplitude with quadruple the charge on the left. What is the ratio of this period to the small-amplitude period you measured in part (b)? How does this effect compare with quadrupling the spring stiffness for a spring–mass oscillator?

•••**P91** This is a continuation of problem P90, in which we allow the center of mass of the dipole to move freely. Restore the charge on the left to $Q = +2\times10^{-10}$ C. **(a)** Use the Momentum Principle and the Angular Momentum Principle to predict and display the motion of the translating, rotating dipole. Stop the animation when the distance between the charge on the left and the center of mass of the dipole is less than 0.02 m. **(b)** Make a graph that shows as a function of time K_{trans}, K_{rot}, U_{elec}, and the total $E = K_{trans} + K_{rot} + U_{elec}$. (We can ignore the constant electric potential energy term associated with the dipole pair of charges; the distance between them does not change.) What does the Energy Principle predict for the graph of E vs. t? Is your graph consistent with the Energy Principle? **(c)** Let the charge on the left move freely. Make sure your graph of E vs. t still makes sense.

In this problem we've seen a situation where we needed to use all three fundamental mechanics principles: The Momentum Principle, the Energy Principle, and the Angular Momentum Principle.

•••**P92** Model the motion of a meter stick suspended from one end on a low-friction axle. Do not make the small-angle approximation but allow the meter stick to swing with large angles. Plot on the same graph both θ and the z component of $\vec{\omega}$ vs. time. Try starting from rest at various initial angles, including nearly straight up (which would be $\theta_i = \pi$ radians). Is this a harmonic oscillator? Is it a harmonic oscillator for small angles?

ANSWERS TO CHECKPOINTS

1 $-y$
2 D: 50 kg · m²/s out of page; E: 50 kg · m²/s out of page; F: 50 kg · m²/s out of page; G: 0; H: 30 kg · m²/s into page
3 **(a)** $L_1 = 0.72$ kg · m²/s and $L_2 = 0.72$ kg · m²/s, both into page; **(b)** $L_{rot} = 1.44$ kg · m²/s into page; **(c)** $I = 0.072$ kg · m²; **(d)** $\vec{\omega}_0$ into page; **(e)** $I\omega_0 = 1.44$ kg · m²/s into page; **(f)** They're the same; **(g)** $K_{rot} = 14.4$ J
4 Nonzero; up (toward the sky); nonzero; up
5 6 N · m; 2 N
6 **(a)** $R\cos(45^\circ)mv$ out of page; **(b)** $(M+m)R^2\omega$ out of page; **(c)** $R\cos(45^\circ)mv/[(M+m)R^2]$ out of page; **(d)** The linear momentum of the clay decreases because the axle exerts an impulsive force upward (the wheel always has zero linear momentum because its center of mass doesn't move).
7 No torques around center of mass means no change in rotational angular momentum, so rotational angular momentum stays constant in magnitude (which determines length of day) and direction (which determines what "North Star" the axis points at). Doesn't matter that Earth is going around Sun; rotational angular momentum is affected solely by torque around center of mass.
8 $dL_z/dt = \frac{d}{dt}[RMv_{CM} - \frac{1}{2}MR^2\omega] = 0$, since there is no torque about a point under the string. Differentiating, we get $d\omega/dt = (2/R)(dv_{CM}/dt) = 2F_T/(MR)$, as before.
9 $dL_z/dt = d_1F_N - (d_1+d_2)M_2g = 0$ ($+z$ out of page), and since $F_N = M_1g + M_2g$, this is equivalent to $d_1M_1g - d_2M_2g$, as before.
10 **(a)** 10 rad/s; **(b)** 5 rad/s; **(c)** 3 rad; **(d)** 0.9 m
12 Same as in horizontal case: $\Omega = RMg/(I\omega)$