However, fast neutrons have low probability of capture and usually scatter off uranium nuclei without triggering fission. In order to sustain a chain reaction, the fast neutrons must be slowed down in some material, called a "moderator." For reasons having to do with the details of nuclear physics, slow neutrons have a high probability of being captured by uranium nuclei.

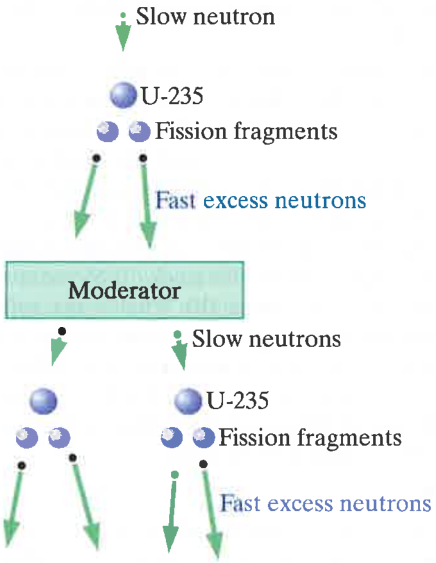

Figure 10.41

A slow neutron induces fission of U-235, with the emission of additional (fast) neutrons. The moderator is some material that slows down the fast neutrons, enabling a chain reaction.

In the following analyses, remember that neutrons have almost no interaction with electrons. Neutrons do, however, interact strongly with nuclei, either by scattering or by being captured and made part of the nucleus. Therefore you should think about neutrons interacting with nuclei (through the strong force), not with entire atoms. **(a)** Based on what you now know about collisions, explain why fast neutrons moving through a block of uranium experience little change in speed. **(b)** Explain why carbon should be a much better moderator of fast neutrons than uranium. **(c)** Should water be a better or worse moderator of fast neutrons than carbon? Explain briefly.

Background: The first fission reactor was constructed in 1941 in a squash court under the stands of Stagg Field at the University of Chicago by a team led by the physicist Enrico Fermi. The moderator consisted of blocks of graphite, a form of carbon. The graphite had to be exceptionally pure because certain kinds of impurities have nuclei that capture neutrons with high probability, removing them from contributing to the chain reaction. Many reactors use ordinary "light" water as a moderator, though sometimes a proton captures a neutron and forms a stable deuterium nucleus, in which case the neutron is lost to contributing to the chain reaction. Heavy water, D_2O, in which the hydrogen atoms are replaced by deuterium atoms, actually works better as a moderator than light water, because the probability of a deuterium nucleus capturing a neutron to form tritium is quite small.

PROBLEMS

Section 10.1

•P14 A ball whose mass is 0.2 kg hits the floor with a speed of 8 m/s and rebounds upward with a speed of 7 m/s. The ball was in contact with the floor for 0.5 ms (0.5×10^{-3} s). **(a)** What was the average magnitude of the force exerted on the ball by the floor? **(b)** Calculate the magnitude of the gravitational force that the Earth exerts on the ball. **(c)** In a collision, for a brief time there are forces between the colliding objects that are much greater than external forces. Compare the magnitudes of the forces found in parts (a) and (b).

Section 10.3

••P15 A projectile of mass m_1 moving with speed v_1 in the $+x$ direction strikes a stationary target of mass m_2 head-on. The collision is elastic. Use the Momentum Principle and the Energy Principle to determine the final velocities of the projectile and target, making no approximations concerning the masses. After obtaining your results, see what your equations would predict if $m_1 \gg m_2$, or if $m_2 \gg m_1$. Verify that these predictions are in agreement with the analysis in this chapter of the Ping-Pong ball hitting the bowling ball, and of the bowling ball hitting the Ping-Pong ball.

Section 10.6

•P16 Object A has mass $m_A = 7$ kg and initial momentum $\vec{p}_{A,i} = \langle 17,-5,0\rangle$ kg·m/s, just before it strikes object B, which has mass $m_B = 11$ kg. Object B has initial momentum $\vec{p}_{B,i} = \langle 4,6,0\rangle$ kg·m/s. After the collision, object A is observed to have final momentum $\vec{p}_{A,f} = \langle 13,3,0\rangle$ kg·m/s. In the following questions, "initial" refers to values before the collisions, and "final" refers to values after the collision. Consider a system consisting of both objects A and B. Calculate the following quantities: **(a)** The total initial momentum of this system. **(b)** The final momentum of object B. **(c)** The initial kinetic energy of object A. **(d)** The initial kinetic energy of object B. **(e)** The final kinetic energy of object A. **(f)** The final kinetic energy of object B. **(g)** The total initial kinetic energy of the system. **(h)** The total final kinetic energy of the system. **(i)** The increase of internal energy of the two objects. **(j)** What assumption did you make about Q (energy flow from surroundings into the system due to a temperature difference)?

•P17 In outer space a rock whose mass is 3 kg and whose velocity was $\langle 3900,-2900,3000\rangle$ m/s struck a rock with mass 13 kg and velocity $\langle 220,-260,300\rangle$ m/s. After the collision, the 3 kg rock's velocity is $\langle 3500,-2300,3500\rangle$ m/s. **(a)** What is the final velocity of the 13 kg rock? **(b)** What is the change in the internal energy of the rocks? **(c)** Which of the following statements about Q (transfer of energy into the system because of a temperature difference between system and surroundings) are correct? (1) $Q \approx 0$ because the duration of the collision was very short. (2) $Q = \Delta E_{thermal}$ of the rocks. (3) $Q \approx 0$ because there are no significant objects in the surroundings. (4) $Q = \Delta K$ of the rocks.

••P18 A spring has an unstretched length of 0.32 m. A block with mass 0.2 kg is hung at rest from the spring, and the spring becomes 0.4 m long (Figure 10.42). Next the spring is stretched to a length of 0.43 m and the block is released from rest. Air resistance is negligible.

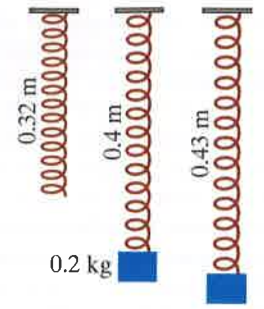

Figure 10.42

(a) How long does it take for the block to return to where it was released? **(b)** Next the block is again positioned at rest, hanging from the spring (0.4 m long) as shown in Figure 10.43. A bullet of mass 0.003 kg traveling at a speed of 200 m/s straight upward buries itself in the block, which then reaches a maximum height h above its original position. What is the speed of the block immediately after the bullet hits? **(c)** Now write an equation that could be used to determine how high the block goes after being hit by the bullet (a height h), but you need not actually solve for h.

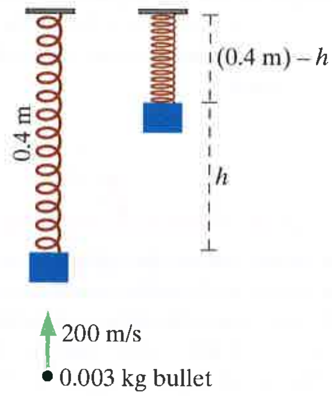

Figure 10.43

•**P19** In outer space, rock 1 whose mass is 5 kg and whose velocity was $\langle 3300, -3100, 3400\rangle$ m/s struck rock 2, which was at rest. After the collision, rock 1's velocity is $\langle 2800, -2400, 3700\rangle$ m/s. **(a)** What is the final momentum of rock 2? **(b)** Before the collision, what was the kinetic energy of rock 1? **(c)** Before the collision, what was the kinetic energy of rock 2? **(d)** After the collision, what is the kinetic energy of rock 1? **(e)** Suppose that the collision was elastic (that is, there was no change in kinetic energy and therefore no change in thermal or other internal energy of the rocks). In that case, after the collision, what is the kinetic energy of rock 2? **(f)** On the other hand, suppose that in the collision some of the kinetic energy is converted into thermal energy of the two rocks, where $\Delta E_{\text{thermal},1} + \Delta E_{\text{thermal},2} = 7.16 \times 10^6$ J. What is the final kinetic energy of rock 2? **(g)** In this case (some of the kinetic energy being converted to thermal energy), what was the transfer of energy Q (microscopic work) from the surroundings into the two-rock system during the collision? (Remember that Q represents energy transfer due to a temperature difference between a system and its surroundings.)

•**P20** A bullet of mass 0.102 kg traveling horizontally at a speed of 300 m/s embeds itself in a block of mass 3.5 kg that is sitting at rest on a nearly frictionless surface. **(a)** What is the speed of the block after the bullet embeds itself in the block? **(b)** Calculate the kinetic energy of the bullet plus the block before the collision. **(c)** Calculate the kinetic energy of the bullet plus the block after the collision. **(d)** Was this collision elastic or inelastic? **(e)** Calculate the rise in internal energy of the bullet plus block as a result of the collision.

••**P21** A car of mass 2300 kg collides with a truck of mass 4300 kg, and just after the collision the car and truck slide along, stuck together, with no rotation. The car's velocity just before the collision was $\langle 38, 0, 0\rangle$ m/s, and the truck's velocity just before the collision was $\langle -16, 0, 27\rangle$ m/s. **(a)** Your first task is to determine the velocity of the stuck-together car and truck just after the collision. What system and principle should you use? (1) Energy Principle (2) Car plus truck (3) Momentum Principle (4) Car alone (5) Truck alone **(b)** What is the velocity of the stuck-together car and truck just after the collision? **(c)** In your analysis in part (b), why can you neglect the effect of the force of the road on the car and truck? **(d)** What is the increase in internal energy of the car and truck (thermal energy and deformation)? **(e)** Is this collision elastic or inelastic?

Section 10.7

••**P22** A gold nucleus contains 197 nucleons (79 protons and 118 neutrons) packed tightly against each other. A single nucleon (proton or neutron) has a radius of about 1×10^{-15} m. Remember that the volume of a sphere is $\frac{4}{3}\pi r^3$. **(a)** Calculate the approximate radius of the gold nucleus. **(b)** Calculate the approximate radius of the alpha particle, which consists of 4 nucleons, 2 protons and 2 neutrons. **(c)** What kinetic energy must alpha particles have in order to make contact with a gold nucleus?

Rutherford correctly predicted the angular distribution for 10-MeV (kinetic energy) alpha particles colliding with gold nuclei. He was lucky: if the alpha particle had been able to touch the gold nucleus, the strong interaction would have been involved and the angular distribution would have deviated from that predicted by Rutherford, which was based solely on electric interactions.

•••**P23** An alpha particle (a helium nucleus, containing 2 protons and 2 neutrons) starts out with kinetic energy of 10 MeV (10×10^6 eV), and heads in the $+x$ direction straight toward a gold nucleus (containing 79 protons and 118 neutrons). The particles are initially far apart, and the gold nucleus is initially at rest. Assuming that all speeds are small compared to the speed of light, answer the following questions about the collision. **(a)** What is the final momentum of the alpha particle, long after it interacts with the gold nucleus? **(b)** What is the final momentum of the gold nucleus, long after it interacts with the alpha particle? **(c)** What is the final kinetic energy of the alpha particle? **(d)** What is the final kinetic energy of the gold nucleus? **(e)** Assuming that the movement of the gold nucleus is negligible, calculate how close the alpha particle will get to the gold nucleus in this head-on collision.

Section 10.10

••**P24** A Fe-57 nucleus is at rest and in its first excited state, 14.4 keV above the ground state (14.4×10^3 eV, where 1 eV $= 1.6 \times 10^{-19}$ J). The nucleus then decays to the ground state with the emission of a gamma ray (a high-energy photon). **(a)** What is the recoil speed of the nucleus? **(b)** Calculate the slight difference in eV between the gamma-ray energy and the 14.4 keV difference between the initial and final nuclear states. **(c)** The "Mössbauer effect" is the name given to a related phenomenon discovered by Rudolf Mössbauer in 1957, for which he received the 1961 Nobel Prize for physics. If the Fe-57 nucleus is in a solid block of iron, occasionally when the nucleus emits a gamma ray the entire solid recoils as one object. This can happen due to the fact that neighboring atoms and nuclei are connected by the electric interatomic force. In this case, repeat the calculation of part (b) and compare with your previous result. Explain briefly.

••P25 There is an unstable particle called the "sigma-minus" (Σ^-), which can decay into a neutron and a negative pion (π^-): $\Sigma^- \to n + \pi^-$. The mass of the Σ^- is 1196 MeV/c^2, the mass of the neutron is 939 MeV/c^2, and the mass of the π^- is 140 MeV/c^2. Write equations that could be used to calculate the momentum and energy of the neutron and the pion. You do not need to solve the equations, which would involve some messy algebra. However, be clear in showing that you have enough equations that you could in principle solve for the unknown quantities in your equations.

It is advantageous to write the equations not in terms of v but rather in terms of E and p; remember that $E^2 - (pc)^2 = (mc^2)^2$.

••P26 A beam of high-energy π^- (negative pions) is shot at a flask of liquid hydrogen, and sometimes a pion interacts through the strong interaction with a proton in the hydrogen, in the reaction $\pi^- + p^+ \to \pi^- + X^+$, where X^+ is a positively charged particle of unknown mass.

The incoming pion momentum is 3 GeV/c (1 GeV = 1000 MeV = 1×10^9 electron-volts). The pion is scattered through 40°, and its momentum is measured to be 1510 MeV/c (this is done by observing the radius of curvature of its circular trajectory in a magnetic field). A pion has a rest energy of 140 MeV, and a proton has a rest energy of 938 MeV.

What is the rest mass of the unknown X^+ particle, in MeV/c^2? Explain your work carefully.

It is advantageous to write the equations not in terms of v but rather in terms of E and p; remember that $E^2 - (pc)^2 = (mc^2)^2$.

••P27 A particle of mass m, moving at speed $v = (4/5)c$, collides with an identical particle that is at rest. The two particles react to produce a new particle of mass M and nothing else. **(a)** What is the speed V of the composite particle? **(b)** What is its mass M?

••P28 A charged pion ($m_\pi c^2 = 139.6$ MeV) at rest decays into a muon ($m_\mu = 105.7$ MeV) and a neutrino (whose mass is very nearly zero). Find the kinetic energy of the muon and the kinetic energy of the neutrino, in MeV (1 MeV = 1×10^6 eV).

••P29 A hydrogen atom is at rest, in the first excited state, when it emits a photon of energy 10.2 eV. **(a)** What is the speed of the ground-state hydrogen atom when it recoils due to the photon emission? Remember that the magnitude of the momentum of a photon of energy E is $p = E/c$. Make the initial assumption that the kinetic energy of the recoiling atom is negligible compared to the photon energy. **(b)** Calculate the kinetic energy of the recoiling atom. Is this kinetic energy indeed negligible compared to the photon energy?

•••P30 At the PEP II facility at the Stanford Linear Accelerator Center (SLAC) in California and at the KEKB facility in Japan, electrons with momentum 9.03 GeV/c were made to collide head-on with positrons whose momentum is 3.10 GeV/c (1 GeV = 10^9 eV); see Figure 10.44. That is, pc for the electron is 9.03 GeV and pc for the positron is 3.10 GeV. The values of pc and the corresponding energies are so large with respect to the electron or positron rest energy (0.5 MeV = 0.0005 GeV) that for the purposes of this analysis you may, if you wish, safely consider the electron and positron to be massless.

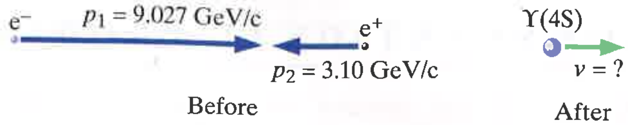

Figure 10.44

(a) The electron–positron collision produces in an intermediate state a particle called the $\Upsilon(4S)$ ("Upsilon 4S"), in the reaction $e^- + e^+ \to \Upsilon(4S)$. Show that the rest energy of the $\Upsilon(4S)$ is 10.58 GeV. **(b)** What is the speed of the $\Upsilon(4S)$ produced in the collision? **(c)** The $\Upsilon(4S)$ decays almost immediately into two "B" mesons: $\Upsilon(4S) \to B^0 + \overline{B^0}$. The $\overline{B^0}$ is the antiparticle of the B^0 and has the same rest energy $Mc^2 = 5.28$ GeV as the B^0. Consider the case in which both "B" mesons are emitted at the same angle θ to the direction of the moving $\Upsilon(4S)$, as shown in Figure 10.45. Calculate this angle θ.

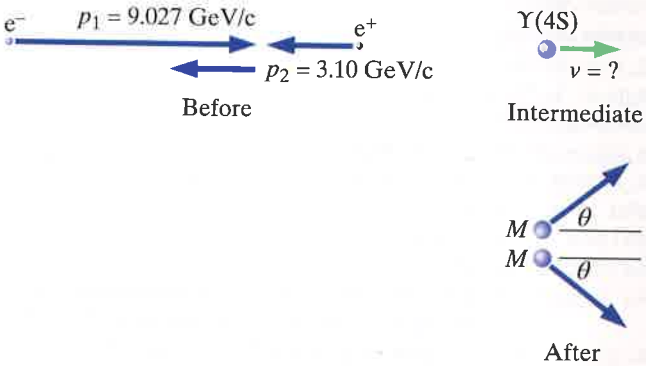

Figure 10.45

•••P31 In a reference frame where a Δ^+ particle is at rest, it decays into a proton and a high-energy photon (a "gamma ray"): $\Delta^+ \to p^+ + \gamma$. The mass of the Δ^+ particle is 1232 MeV/c^2 and the mass of the proton is 938 MeV/c^2 (1 MeV = 1×10^6 eV). Calculate the energy of the gamma ray and the speed of the proton.

Section 10.12

••P32 Redo Problem P21, this time using the concept of the center-of-momentum reference frame.

••P33 Redo the analysis of the Rutherford experiment, this time using the concept of the center-of-momentum reference frame. Let m = the mass of the alpha particle and M = the mass of the gold nucleus. Consider the specific case of the alpha particle rebounding straight back. The incoming alpha particle has a momentum p_1, the outgoing alpha particle has a momentum p_3, and the gold nucleus picks up a momentum p_4. **(a)** Determine the velocity of the center of momentum of the system. **(b)** Transform the initial momenta to that frame (by subtracting the center-of-momentum velocity from the original velocities). **(c)** Show that if the momenta in the center-of-momentum frame simply turn around (180°), with no change in their magnitudes, both momentum and energy conservation are satisfied, whereas no other possibility satisfies both conservation principles. (Try drawing some other momentum diagrams.) **(d)** After the collision, transform back to the original reference frame (by adding the center-of-momentum velocity to the velocities of the particles in the center-of-mass frame). Although using the center-of-momentum frame may be conceptually more difficult, the algebra for solving for the final speeds is much simpler.

COMPUTATIONAL PROBLEMS

More detailed and extended versions of some of these computational modeling problems may be found in the lab activities included in the *Matter & Interactions*, 4th Edition, resources for instructors.

••P34 The following code is the skeleton of a computational model of a collision between a single moving alpha particle and a single gold nucleus which is initially stationary. Some pieces of the physical model are missing. An alpha particle consists of 2 protons and 2 neutrons. A gold nucleus contains 79 protons and 118 neutrons.

```
from visual import *
from visual.graph import *
scene.width = 1024
scene.height = 600
q_e = 1.6e-19
m_p = 1.7e-27
oofpez = 9e9
m_Au = (79+118) * m_p
m_Alpha = (2+2) * m_p
qAu = 2 * q_e
qAlpha = 79 * q_e
deltat = 1e-23
Au = sphere(pos=vector(0,0,0), radius=4e-15,
            color=color.yellow, make_trail=True)
Alpha = sphere(pos=vector(-1e-13,5e-15,0),
               radius=1e-15, color=color.magenta,
               make_trail=True)
p_Au = m_Au*vector(0,0,0)
p_Alpha = vector(1.043e-19,0,0)
t = 0
while t<1.3e-20:
    rate(100)
    Alpha.pos = Alpha.pos+(p_Alpha/m_Alpha) *
deltat
    t = t + deltat
```

(a) Read the code, and predict what will happen when the unmodified skeleton program is run. Then run the program to check your prediction. **(b)** Modify the program so that it represents a reasonable physical model of this interaction. **(c)** Does the gold nucleus move during these collisions? Should it? You can see the trail left by the gold nucleus better if you make this object slightly transparent by adding this line of code before the loop: `Au.opacity = 0.7` **(d)** What is the value of the impact parameter b in this program? Experiment with different impact parameters, and report what you observe. **(e)** After the loop, add to your program a calculation of the scattering angle θ (the angle between the final and initial momenta of the alpha particle). The VPython function for the dot product is `dot(A,B)`. The function `acos(D)` returns the angle (in radians) whose cosine is D. Check to make sure the angle you calculate makes sense in terms of what you observe on the screen. **(f)** Find values of the impact parameter that lead to the following scattering angles: (1) 90°, (2) 168°, (3) 38°, (4) 13°.

•P35 Start with the program you wrote for Problem P34. Add graphs to display the values of the x and y components of the momentum of each particle, and the sums of all x components and of all y components, as a function of time. Is momentum conserved during the collision? What should you change in your program if you find that momentum is not conserved? By adding the following code before the loop you can create two different graphing windows, and plot all curves simultaneously:

```
gdx = gdisplay(x=0,y=600, width=500,
               title='p_x')
p_Au_x_graph = gcurve(color=color.yellow)
p_Alpha_x_graph = gcurve(color=color.magenta)
## other gcurves if needed
gdy = gdisplay(x=500,y=600,width=500,title='p_y')
p_Au_y_graph = gcurve(color=color.yellow)
## other gcurves if needed
```

•••P36 The *Voyager 1* spacecraft was launched in 1977 and has recently left the Solar System and entered interstellar space. No existing rocket is powerful enough to launch a spacecraft with enough speed to coast that far against the Sun's gravitational pull, so a "gravity boost" was obtained from Jupiter (and another from Saturn). The basic idea is that it was possible to arrange *Voyager*'s trajectory in such a way that in a brief noncontact collision with Jupiter the spacecraft gained momentum, with a corresponding loss of the giant planet's momentum, which due to Jupiter's huge mass corresponded to a tiny change in Jupiter's speed.

Construct a computational model of this maneuver. The collision takes only a few days, so it is okay to approximate Jupiter's motion around the Sun with a straight line (Jupiter takes 12 Earth years to orbit the Sun). Jupiter's mass is 1.9×10^{27} kg. Give your spacecraft the same mass as the *Voyager* spacecraft (722 kg). Jupiter's radius is 7×10^7 m and it is 7.8×10^{11} m from the Sun, with an orbital speed of 1.3×10^4 m/s. **(a)** Place Jupiter at the location $\langle 0,0,0 \rangle$ m, with a velocity of $\langle -1.3 \times 10^4,0,0 \rangle$ m/s. Choose a starting location for the spacecraft at location $\langle x_i,-25R,0 \rangle$, where R is the radius of Jupiter. You will need to find a value for x_i that leads to the spacecraft passing close to the right side of Jupiter. Let the initial velocity of the spacecraft be $\langle 0,1 \times 10^4,0 \rangle$ m/s. (From this perspective you are looking down on the North pole of Jupiter, with the outward bound spacecraft heading toward Jupiter.) **(b)** Exit the computational loop when the spacecraft is as far from Jupiter as it was at the start. (Put this test at the end of your loop, so the loop doesn't stop immediately.) Print the change in the speed, $v_f - v_i$, and the time in Earth days for the process. Modify your starting position x_i until you find that the speed change is approximately 1.6×10^4 m/s. **(c)** Include a graph of the speed of the spacecraft as a function of time. **(d)** Find a different value of x_i for the spacecraft that results in the spacecraft losing a significant amount of speed instead of gaining speed.

Suggestions: If you precede a `gcurve` statement with `gdisplay(y=600)` the graph will be placed conveniently beneath your graphics window. If you specify `scene.autocenter = True` the camera will continually shift to point at the center of the scene, which is useful in this situation.

•••**P37** Start with the program you wrote to model a spacecraft's interaction with Jupiter (Problem P36). Use initial conditions that result in the spacecraft gaining speed. **(a)** Change to a center-of-mass reference frame by subtracting Jupiter's velocity from the initial velocities of the spacecraft and Jupiter. (Because Jupiter is so massive, its velocity is almost exactly the velocity of the center of mass of the combined system.) What do you observe when you re-run your program? **(b)** What is the speed change of the spacecraft in the center-of-mass reference frame?

ANSWERS TO CHECKPOINTS

1 **(a)** 300 J; **(b)** 5 m/s; **(c)** 150 J; **(d)** 150 J

2 **(a)** 2.3×10^{-24} m/s (!), so it is an excellent approximation to say that the Earth doesn't recoil. **(b)** 1.6×10^{-23} J; $K_{baseball} = 150$ J

3 8×10^4 m/s

4 Car 1's momentum was $\sqrt{3}$ greater than the momentum of car 2.

5 We outline the entire solution.

First find $v_{cm,x}$:

$$v_{cm,x} = \frac{(1000\,\text{kg})(30\,\text{m/s}) + (3000\,\text{kg})(-20\,\text{m/s})}{(4000\,\text{kg})} = -7.5\,\text{m/s}$$

Transform to center-of-momentum frame:

$$v_{car,x} = (30\,\text{m/s}) - (-7.5\,\text{m/s}) = 37.5\,\text{m/s}$$
$$v_{truck,x} = (-20\,\text{m/s}) - (-7.5\,\text{m/s}) = -12.5\,\text{m/s}$$

Check to make sure that the momenta are equal and opposite in this frame:

$$p_{car,x} = (1000\,\text{kg})(37.5\,\text{m/s}) = 37500\,\text{kg}\cdot\text{m/s}$$
$$p_{truck,x} = (3000\,\text{kg})(-12.5\,\text{m/s}) = -37500\,\text{kg}\cdot\text{m/s}$$

In this frame the kinetic energy before the collision is this:

$$K = \frac{(37500\,\text{kg}\cdot\text{m/s})^2}{2(1000\,\text{kg})} + \frac{(37500\,\text{kg}\cdot\text{m/s})^2}{2(3000\,\text{kg})} = 9.4 \times 10^5\,\text{J}$$

After the collision, the velocities are zero in this frame, so kinetic energy in this frame goes to zero, and there is an increase in the internal energy of the mangled car and truck of amount 9.4×10^5 J. There must be the same internal energy increase in the original reference frame, so the kinetic energy lost in the original reference frame was 9.4×10^5 J.