Q4 If you try to increase the energy of a quantum harmonic oscillator by adding an amount of energy $\frac{1}{2}\hbar\sqrt{k_s/m}$, the energy doesn't increase. Why not?

Q5 If you double the amplitude, what happens to the frequency in a classical (nonquantum) harmonic oscillator? In a quantum harmonic oscillator?

Q6 What is the energy of the photon emitted by a harmonic oscillator with stiffness k_s and mass m when it drops from energy level 5 to energy level 2?

Q7 Summarize the differences and similarities between different energy levels in a quantum oscillator. Specifically, for the first two levels in Figure 8.26, compare the angular frequency $\sqrt{k_s/m}$, the amplitude A, and the kinetic energy K at the same value of s. (In a full quantum-mechanical analysis the concepts of angular frequency and amplitude require reinterpretation. Nevertheless there remain elements of the classical picture. For example, larger amplitude corresponds to a higher probability of observing a large stretch.)

PROBLEMS

Section 8.1

•P8 A certain laser outputs pure red light (photon energy 1.8 eV) with power 700 mW (0.7 W). How many photons per second does this laser emit?

Section 8.2

•P9 How much energy in electron volts is required to ionize a hydrogen atom (that is, remove the electron from the proton), if initially the atom is in the state $N = 2$? (Remember that $N = 1$ if the atom is in the lowest energy level.)

•P10 The mean lifetime of a certain excited atomic state is 5 ns. What is the probability of the atom staying in this state for 10 ns or more?

•P11 At $t = 0$ all of the atoms in a collection of 10000 atoms are in an excited state whose lifetime is 25 ns. Approximately how many atoms will still be in this excited state at $t = 12$ ns?

•P12 $N = 1$ is the lowest electronic energy state for a hydrogen atom. **(a)** If a hydrogen atom is in state $N = 4$, what is $K + U$ for this atom (in eV)? **(b)** The hydrogen atom makes a transition to state $N = 2$. Now what is $K + U$ in electron volts for this atom? **(c)** What is the energy (in eV) of the photon emitted in the transition from level $N = 4$ to $N = 2$? **(d)** Which of the arrows in Figure 8.40 represents this transition?

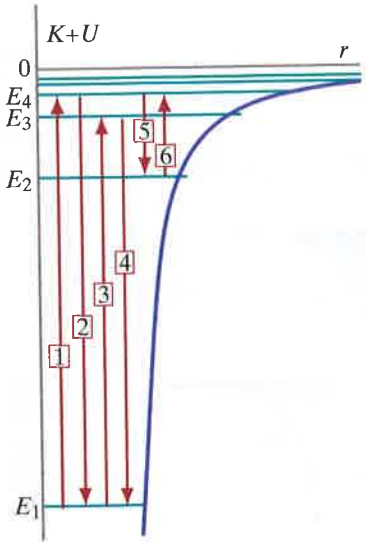

Figure 8.40

•P13 The Franck–Hertz experiment involved shooting electrons into a low-density gas of mercury atoms and observing discrete amounts of kinetic energy loss by the electrons. Suppose that instead a similar experiment is done with a very cold gas of atomic hydrogen, so that all of the hydrogen atoms are initially in the ground state. If the kinetic energy of an electron is 11.6 eV just before it collides with a hydrogen atom, how much kinetic energy will the electron have just after it collides with and excites the hydrogen atom?

••P14 Hydrogen atoms: **(a)** What is the minimum kinetic energy in electron volts that an electron must have to be able to ionize a hydrogen atom that is in its ground state (that is, remove the electron from being bound to the proton)? **(b)** If electrons of energy 12.8 eV are incident on a gas of hydrogen atoms in their ground state, what are the energies of the photons that can be emitted by the excited gas? **(c)** If instead of electrons, photons of all energies between 0 and 12.8 eV are incident on a gas of hydrogen atoms in the ground state, what are the energies at which the photons are absorbed?

••P15 Suppose we have reason to suspect that a certain quantum object has only three quantum states. When we excite such an object we observe that it emits electromagnetic radiation of three different energies: 2.48 eV (green), 1.91 eV (orange), and 0.57 eV (infrared). **(a)** Propose two possible energy-level schemes for this system. **(b)** Explain how to use an absorption measurement to distinguish between the two proposed schemes.

••P16 Predict how many emission lines will be seen by a human in the visible spectrum of atomic hydrogen. Give the energies of the emitted photons, and specify the energy levels involved in the transitions that are responsible for these lines.

••P17 Assume that a hypothetical object has just four quantum states, with the following energies:

−1.0 eV (third excited state)
−1.8 eV (second excited state)
−2.9 eV (first excited state)
−4.8 eV (ground state)

(a) Suppose that material containing many such objects is hit with a beam of energetic electrons, which ensures that there are always some objects in all of these states. What are the six energies of photons that could be strongly emitted by the material? (In actual quantum objects there are often "selection rules" that forbid certain emissions even though there is enough energy; assume that there are no such restrictions here.) List the photon emission energies. **(b)** Next, suppose that the beam of electrons is shut off so that all of the objects are in the ground state almost all the time. If electromagnetic radiation with a wide range of energies is passed through the material, what will be the three energies of photons corresponding to missing ("dark") lines in the spectrum? Remember that there is hardly any absorption from excited states, because emission from an excited state happens very quickly, so there is never a significant number of objects in an excited state. Assume that the detector is sensitive to a wide range of photon energies, not just energies in the visible region. List the dark-line energies.

••P18 Energy graphs: **(a)** Figure 8.41 shows a graph of potential energy vs. interatomic distance for a particular molecule. What is the direction of the associated force at location A? At location B? At location C? Rank the magnitude of the force at locations A, B, and C. (That is, which is greatest, which is smallest, and are any of these equal to each other?) For the energy level shown on the graph, draw a line whose height is the kinetic energy when the system is at location D.

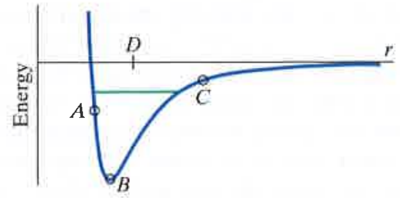

Figure 8.41

(b) Figure 8.42 shows all of the quantized energies (bound states) for one of these molecules. The energy for each state is given on the graph, in electron volts ($1\text{ eV} = 1.6 \times 10^{-19}\text{ J}$). How much energy is required to break a molecule apart, if it is initially in the ground state? (Note that the final state must be an unbound state; the unbound states are not quantized.)

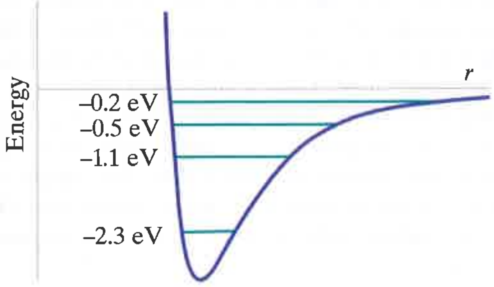

Figure 8.42

(c) At high enough temperatures, in a collection of these molecules there will be at all times some molecules in each of these states, and light will be emitted. What are the energies in electron volts of the emitted light? **(d)** The "inertial" mass of the molecule is the mass that appears in Newton's second law, and it determines how much acceleration will result from applying a given force. Compare the inertial mass of a molecule in the ground state and the inertial mass of a molecule in an excited state 10 eV above the ground state. If there is a difference, briefly explain why and calculate the difference. If there isn't a difference, briefly explain why not.

••P19 A bottle contains a gas with atoms whose lowest four energy levels are -12 eV, -6 eV, -3 eV, and -2 eV. Electrons run through the bottle and excite the atoms so that at all times there are large numbers of atoms in each of these four energy levels, but there are no atoms in higher energy levels. List the energies of the photons that will be emitted by the gas.

Next, the electron beam is turned off, and all the atoms are in the ground state. Light containing a continuous spectrum of photon energies from 0.5 eV to 15 eV shines through the bottle. A photon detector on the other side of the bottle shows that some photon energies are depleted in the spectrum ("dark lines"). What are the energies of the missing photons?

••P20 Suppose we have reason to suspect that a certain quantum object has only three quantum states. When we excite a collection of such objects we observe that they emit electromagnetic radiation of three different energies: 0.3 eV (infrared), 2.0 eV (visible), and 2.3 eV (visible). **(a)** Draw a possible energy-level diagram for one of the quantum objects, which has three bound states. On the diagram, indicate the transitions corresponding to the emitted photons, and check that the possible transitions produce the observed photons and no others. The energy $K + U$ of the ground state is -4 eV. Label the energies of each level ($K + U$, which is *negative*). **(b)** The material is now cooled down to a very low temperature, and the photon detector stops detecting photon emissions. Next, a beam of light with a continuous range of energies from infrared through ultraviolet shines on the material, and the photon detector observes the beam of light after it passes through the material. What photon energies in this beam of light are observed to be significantly reduced in intensity ("dark absorption lines")? Remember that there is hardly any absorption from excited states, because emission from an excited state happens very quickly, so there is never a significant number of objects in an excited state. **(c)** There exists another possible set of energy levels for these objects that produces the same photon emission spectrum. On an alternative energy-level diagram, *different from the one you drew in part (a),* indicate the transitions corresponding to the emitted photons, and check that the possible transitions produce the observed photons and no others. Label the energies of each level ($K + U$, which is *negative*). **(d)** For your second proposed energy-level scheme, what photon energies would be observed to be significantly reduced in intensity in an absorption experiment ("dark absorption lines")? (Given the differences from part (b), you can see that an absorption measurement can be used to tell which of your two energy-level schemes is correct.)

••P21 Assume that a hypothetical object has just four quantum states, with the energies shown in Figure 8.43.

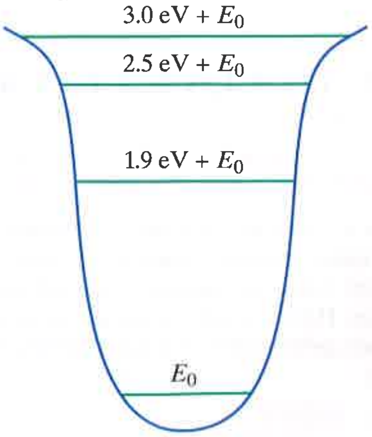

Figure 8.43

(a) Suppose that the temperature is high enough that in a material containing many such objects, at any instant some objects are found in all of these states. What are all the energies of photons that could be strongly emitted by the material?

(In actual quantum objects there are often "selection rules" that forbid certain emissions even though there is enough energy; assume that there are no such restrictions here.) **(b)** If the temperature is very low and electromagnetic radiation with a wide range of energies is passed through the material, what will be the energies of photons corresponding to missing ("dark") lines in the spectrum? (Assume that the detector is sensitive to a wide range of photon energies, not just energies in the visible region.)

Section 8.3

••P22 A certain material is kept at very low temperature. It is observed that when photons with energies between 0.2 and 0.9 eV strike the material, only photons of 0.4 eV and 0.7 eV are absorbed. Next, the material is warmed up so that it starts to emit photons. When it has been warmed up enough that 0.7 eV photons begin to be emitted, what other photon energies are also observed to be emitted by the material? Explain briefly.

••P23 Some material consisting of a collection of microscopic objects is kept at a high temperature. A photon detector capable of detecting photon energies from infrared through ultraviolet observes photons emitted with energies of 0.3 eV, 0.5 eV, 0.8 eV, 2.0 eV, 2.5 eV, and 2.8 eV. These are the only photon energies observed. **(a)** Draw and label a possible energy-level diagram for one of the microscopic objects, which has four bound states. On the diagram, indicate the transitions corresponding to the emitted photons. Explain briefly. **(b)** Would a spring–mass model be a good model for these microscopic objects? Why or why not? **(c)** The material is now cooled down to a very low temperature, and the photon detector stops detecting photon emissions. Next, a beam of light with a continuous range of energies from infrared through ultraviolet shines on the material, and the photon detector observes the beam of light after it passes through the material. What photon energies in this beam of light are observed to be significantly reduced in intensity ("dark absorption lines")? Explain briefly.

Section 8.4

•P24 For a certain diatomic molecule, the lowest-energy photon observed in the vibrational spectrum is 0.17 eV. What is the energy of a photon emitted in a transition from the 5th excited vibrational energy level to the 2nd excited vibrational energy level, assuming no change in the rotational energy?

••P25 Consider a microscopic spring–mass system whose spring stiffness is 50 N/m, and the mass is 4×10^{-26} kg. **(a)** What is the smallest amount of vibrational energy that can be added to this system? **(b)** What is the difference in mass (if any) of the microscopic oscillator between being in the ground state and being in the first excited state? **(c)** In a collection of these microscopic oscillators, the temperature is high enough that the ground state and the first three excited states are occupied. What are possible energies of photons emitted by these oscillators?

••P26 Molecular vibrational energy levels: **(a)** A HCl molecule can be considered to be a quantized harmonic oscillator, with quantized vibrational energy levels that are evenly spaced. Make a rough estimate of this uniform energy spacing in electron volts (where $1\ \text{eV} = 1.6 \times 10^{-19}$ J). You will need to make some rough estimates of atomic properties based on prior work. For comparison with the spacing of these vibrational energy states, note that the spacing between quantized energy levels for "electronic" states such as in atomic hydrogen is of the order of several electron volts. **(b)** List several photon energies that would be emitted if a number of these vibrational energy levels were occupied due to collisional excitation. To what region of the spectrum (x-ray, visible, microwave, etc.) do these photons belong? (See Figure 8.1 at the beginning of the chapter.)

••P27 A hot bar of iron glows a dull red. Using our simple ball-spring model of a solid (Figure 8.23), answer the following questions, explaining in detail the processes involved. You will need to make some rough estimates of atomic properties based on prior work. **(a)** What is the approximate energy of the lowest-energy spectral emission line? Give a numerical value. **(b)** What is the approximate energy of the highest-energy spectral emission line? Give a numerical value. **(c)** What is the quantum number of the highest-energy occupied state? **(d)** Predict the energies of two other lines in the emission spectrum of the glowing iron bar. (*Note*: Our simple model is too simple—the actual spectrum is more complicated. However, this simple analysis gets at some important aspects of the phenomenon.)

COMPUTATIONAL PROBLEMS

The VPython programs used in these computational problems are available on the Wiley student website for this textbook.

•P28 There is a "random" module for Python that contains a function (also named "random") that generates a random number, which we can use to model photon emission at random times. Here is a little program that graphs and prints successive values generated by this function (the graph is shown in Figure 8.44):

```
from visual import *
from visual.graph import *
from random import random
gg = gcurve(color=color.yellow)
n = 0
while n < 20:
    rate(10)
    a = random()
    print(a)
    gg.plot(pos=(n,a))
    n= n + 1
```

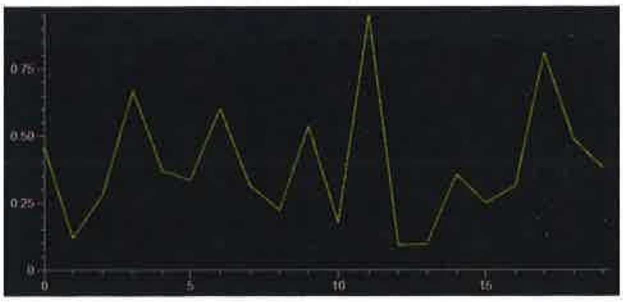

Figure 8.44

(a) Run this program several times, looking at the graph produced each time. What is being plotted in this graph? Is the graph the same each time you run the program? **(b)** What is the maximum random number generated by the `random()` function? **(c)** What is the minimum number generated by the `random()` function?

••P29 Refer to Problem P28 for an explanation of the `random()` function. The VPython program below models random photon emission in a collection of excited atoms. Read through the program, then answer the questions below. **(a)** In which line of code is it decided whether a particular atom will emit or not? How is a random number used in this decision? **(b)** What is the probability that a given atom will emit a photon in one nanosecond? **(c)** In which line of code is the count of excited atoms decreased after an atom emits a photon? **(d)** Start with 5 excited atoms, and run the program 10 times. In your observations, what is the longest time it takes for every atom to emit a photon? What is the shortest time? **(e)** Increase the number of atoms until the results of every run look the same. Approximately how many atoms are required? (You may wish to use a larger value for `rate()`). **(f)** With 10,000 atoms, drag the mouse across the graph and find a vertical bar whose height is $10000/e = 10000 * 0.368$. What is the value of t at this location? This value is called the "mean lifetime" and can be shown to be equal to the reciprocal of the emission rate (emissions per second, which is `P/dt`). How does your mean lifetime compare with `dt/P`)?

```
from visual import *
from visual.graph import *
from random import random

Natoms = 5
# P is the probability for an atom to emit
# during a time interval dt
P = 0.1
dt = 0.2  # ns
t = 0
tmax = 5*dt/P # 5 mean lifetimes
# Create a bar graph
gdisplay(xtitle='t, ns',
         ytitle='Atoms in excited state')
excited      =      gvbars(color=color.yellow,
delta=dt/2)

while t < tmax:
    rate(10)
    # Show number of excited atoms remaining
    excited.plot(pos=(t,Natoms))
    emissions = 0
    atom = 0
    while atom < Natoms:
        if random() < P: # emits?
            # count emissions in this dt
            emissions = emissions + 1
        atom = atom + 1
    Natoms = Natoms - emissions
    t = t + dt
```

Figure 8.45 shows sample output of this program.

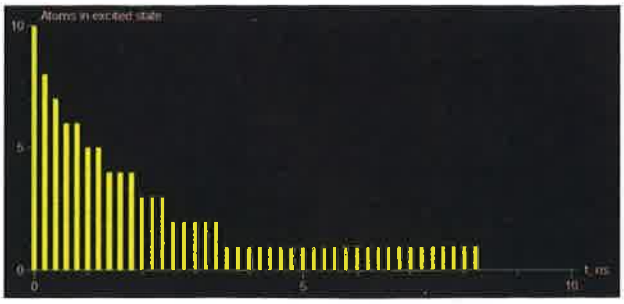

Figure 8.45

••P30 Refer to Problem P28 for an explanation of the `random()` function. The VPython program below models the random emission of red and green photons from a collection of atoms in their second excited state. These atoms can either emit a green photon and drop to the ground state, or they can emit a red photon and drop to the first excited state. Read the program carefully before running it, then answer the questions below. **(a)** What quantity is plotted in each of the graphs generated by the program? **(b)** Which is higher: the probability of emission of a red photon or a green photon? **(c)** Run the program repeatedly. Approximately, how much does the number of green emissions vary in repeated trials? **(d)** It can be shown that statistically one expects that the number of green emissions in repeated trials lies in the range $N \pm \sqrt{N}$, where N is the average number (which we expect to be `Pgreen*Natoms`). Do 30 or more trials and determine the experimental average number N and the fraction of trials in which the number of green emissions is within the range $N \pm \sqrt{N}$.

```
from visual import *
from visual.graph import *
from random import random

# Start with 100 atoms in an excited state
# (try larger or smaller numbers)

Natoms = 100
# P is the probability for an atom to emit
# during a time interval dt
P = 0.1
# Pgreen is the probability that when an
# atom emits, it emits a green photon
Pgreen = 0.3
dt = 0.2  # ns
t = 0
tmax = 5*dt/P # 5 mean lifetimes
# Create bar graphs
gdisplay(xtitle='t, ns', xmax=tmax,
         ytitle='Emissions of green photons')
greeng = gvbars(color=color.green, delta=dt/2)
gdisplay(y=400, xtitle='t, ns', xmax=tmax,
        ytitle='Emissions of red photons')
redg = gvbars(color=color.red, delta=dt/2)
greens = reds = 0
```

```
while t < tmax:
    rate(100)
    atom = 0
    g = r = 0
    while atom < Natoms:
        if random() < P: # emits?
            if random() < Pgreen: # green?
                g = g + 1
            else:                # emits red
                r = r + 1
        atom = atom + 1
    greeng.plot(pos=(t,g))
    redg.plot(pos=(t,r))
    greens = greens + g
    reds = reds + r
    Natoms = Natoms - (g + r)
    t = t + dt

print(greens,'green emissions,',
      reds,'red emissions')
```

Figure 8.46 shows sample output of this program.

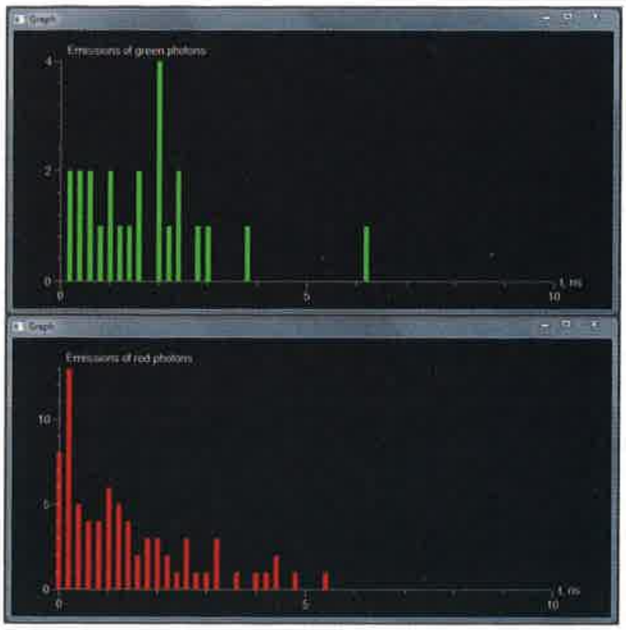

Figure 8.46 The vertical scales in the two graphs are different, due to the different probabilities of green and red emissions.

ANSWERS TO CHECKPOINTS

1 About 2.5 eV; about 3.5×10^{21} per second per square meter
2 5.2 eV
3 Six different photon energies, corresponding to transitions from 4 to 3, 4 to 2, 4 to 1, 3 to 2, 3 to 1, and 2 to 1
4 Just one dark line at 2.4 eV
5 1 eV, 5 eV, and 6 eV
6 0.4 eV (three transitions), 0.8 eV (two transitions), and 1.2 eV (one transition)
7 $N \approx 2 \times 10^{31}$. A system in a very high quantum state behaves like a classical system.
8 2.2 MeV
9 About 294 MeV