

**Q11** Describe a situation in which it would be appropriate to neglect the effects of air resistance.

**Q12** Describe a situation in which neglecting the effects of air resistance would lead to significantly wrong predictions.

**Q13** Throw a ball straight up and catch it on the way down, at the same height. Taking into account air resistance, does the ball take longer to go up or to come down? Why?

**Q14** You drag a block with constant speed  $v$  across a table with friction. Explain in detail what you have to do in order to change to a constant speed of  $2v$  on the same surface. (That is, the puzzle is to explain how it is possible to drag a block with sliding friction at different constant speeds.)

**Q15** Figure 7.48 is a portion of a graph of energy terms vs. time for a mass on a spring, subject to air resistance. Identify and

label the three curves as to what kind of energy each represents. Explain briefly how you determined which curve represented which kind of energy.

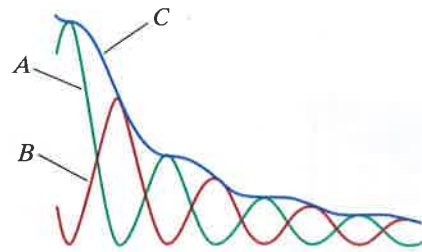


Figure 7.48

## PROBLEMS

### Section 7.2

**•P16** A spring has a relaxed length of 6 cm and a stiffness of 100 N/m. How much work must you do to change its length from 5 cm to 9 cm?

**•P17** A horizontal spring with stiffness 0.5 N/m has a relaxed length of 15 cm. A mass of 20 g is attached and you stretch the spring to a total length of 25 cm. The mass is then released from rest and moves with little friction. What is the speed of the mass at the moment when the spring returns to its relaxed length of 15 cm?

**•P18** A spring whose stiffness is 800 N/m has a relaxed length of 0.66 m. If the length of the spring changes from 0.55 m to 0.96 m, what is the change in the potential energy of the spring?

**••P19** A spring with stiffness  $k_s$  and relaxed length  $L$  stands vertically on a table. You hold a mass  $M$  just barely touching the top of the spring. **(a)** You *very slowly* let the mass down onto the spring a certain distance, and when you let go, the mass doesn't move. How much did the spring compress? How much work did *you* do? **(b)** Now you again hold the mass just barely touching the top of the spring, and then let go. What is the maximum compression of the spring? State what approximations and simplifying assumptions you made. **(c)** Next you push the mass down on the spring so that the spring is compressed an amount  $s$ , then let go, and the mass starts moving upward and goes quite high. When the mass is a height of  $2L$  above the table, what is its speed?

**••P20** A horizontal spring-mass system has low friction, spring stiffness 200 N/m, and mass 0.4 kg. The system is released with an initial compression of the spring of 10 cm and an initial speed of the mass of 3 m/s. **(a)** What is the maximum stretch during the motion? **(b)** What is the maximum speed during the motion? **(c)** Now suppose that there is energy dissipation of 0.01 J per cycle of the spring-mass system. What is the average power input in watts required to maintain a steady oscillation?

**••P21** A package of mass 9 kg sits on an airless asteroid of mass  $8.0 \times 10^{20}$  kg and radius  $8.7 \times 10^5$  m. We want to launch the package in such a way that it will never come back, and when it is very far from the asteroid it will be traveling with speed 226 m/s. We have a large and powerful spring whose

stiffness is  $2.8 \times 10^5$  N/m. How much must we compress the spring?

**••P22** A mass of 0.3 kg hangs motionless from a vertical spring whose length is 0.8 m and whose unstretched length is 0.65 m. Next the mass is pulled down so the spring has a length of 0.9 m and is given an initial speed upward of 1.2 m/s. What is the maximum length of the spring during the following motion? What approximations or simplifying assumptions did you make?

**•••P23** A relaxed spring of length 0.15 m stands vertically on the floor; its stiffness is 1000 N/m. You release a block of mass 0.4 kg from rest, with the bottom of the block 0.8 m above the floor and straight above the spring. How long is the spring when the block comes momentarily to rest on the compressed spring?

**•••P24** Design a “bungee jump” apparatus for adults. A bungee jumper falls from a high platform with two elastic cords tied to the ankles. The jumper falls freely for a while, with the cords slack. Then the jumper falls an additional distance with the cords increasingly tense. You have cords that are 10 m long, and these cords stretch in the jump an additional 24 m for a jumper whose mass is 80 kg, the heaviest adult you will allow to use your bungee jump (heavier customers would hit the ground). You can neglect air resistance. **(a)** Make a series of five simple diagrams, like a comic strip, showing the platform, the jumper, and the two cords at various times in the fall and the rebound. On each diagram, draw and label vectors representing the forces acting on the jumper, and the jumper's velocity. Make the relative lengths of the vectors reflect their relative magnitudes. **(b)** At what instant is there the greatest tension in the cords? How do you know? **(c)** What is the jumper's speed at this instant? **(d)** Is the jumper's momentum changing at this instant or not? (That is, is  $d\vec{p}/dt$  nonzero or zero?) Explain briefly. **(e)** Focus on this instant, and use the principles of this chapter to determine the spring stiffness  $k_s$  for each cord. Explain your analysis. **(f)** What is the maximum tension that each cord must support without breaking? **(g)** What is the maximum acceleration (in g's) that the jumper experiences? What is the direction of this maximum acceleration? **(h)** State clearly what approximations and estimates you have made in your design.

## Section 7.3

••P25 Figure 7.49 is a potential energy curve for the interaction of two neutral atoms. The two-atom system is in a vibrational state indicated by the green horizontal line.

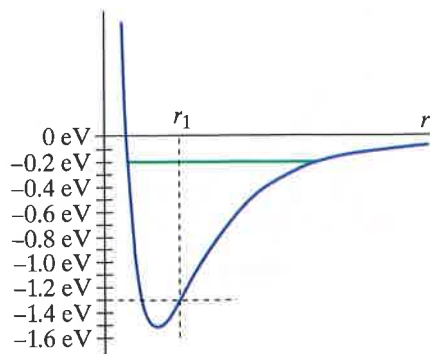


Figure 7.49

(a) At  $r = r_1$ , what are the approximate values of the kinetic energy  $K$ , the potential energy  $U$ , and the quantity  $K + U$ ? (b) What minimum energy must be supplied to cause these two atoms to separate? (c) In some cases, when  $r$  is large, the interatomic potential energy can be expressed approximately as  $U = -a/r^6$ . For large  $r$ , what is the algebraic form of the magnitude of the force the two atoms exert on each other in this case?

## Section 7.4

••P26 Niagara Falls is about 50 m high. What is the temperature rise in kelvins of the water from just before to just after it hits the rocks at the bottom of the falls, assuming negligible air resistance during the fall and that the water doesn't rebound but just splats onto the rock? It is helpful (but not essential) to consider a 1 g drop of water.

••P27 400 g of boiling water (temperature  $100^\circ\text{C}$ , specific heat  $4.2 \text{ J/K/g}$ ) are poured into an aluminum pan whose mass is 600 g and initial temperature  $20^\circ\text{C}$  (the specific heat of aluminum is  $0.9 \text{ J/K/g}$ ). After a short time, what is the temperature of the water? Explain. What simplifying assumptions did you have to make?

••P28 Here are questions about human diet. (a) A typical candy bar provides 280 calories (one "food" or "large" calorie is equal to  $4.2 \times 10^3 \text{ J}$ ). How many candy bars would you have to eat to replace the chemical energy you expend doing 100 sit-ups? Explain your work, including any approximations or assumptions you make. (In a sit-up, you go from lying on your back to sitting up.) (b) How many days of a diet of 2000 large calories are equivalent to the gravitational energy difference for you between sea level and the top of Mount Everest, 8848 m above sea level? (However, the body is not anywhere near 100% efficient in converting chemical energy into change in altitude. Also note that this is in addition to your basal metabolism.)

••P29 You observe someone pulling a block of mass 43 kg across a low-friction surface. While they pull a distance of 3 m in the direction of motion, the speed of the block changes from 5 m/s to 7 m/s. Calculate the magnitude of the force exerted by the person on the block. What was the change in internal energy (chemical energy plus thermal energy) of the person pulling the block?

## Section 7.5

••P30 You place into an insulated container a 1.5 kg block of aluminum at a temperature of  $45^\circ\text{C}$  in contact with a 2.1 kg block of copper at a temperature of  $18^\circ\text{C}$ . The specific heat of aluminum is  $0.91 \text{ J/g}$  and the specific heat of copper is  $0.39 \text{ J/g}$ . What is the final temperature of the two blocks?

••P31 180 g of boiling water (temperature  $100^\circ\text{C}$ , heat capacity  $4.2 \text{ J/K/g}$ ) are poured into an aluminum pan whose mass is 1050 g and initial temperature  $26^\circ\text{C}$  (the heat capacity of aluminum is  $0.9 \text{ J/K/g}$ ). (a) After a short time, what is the temperature of the water? (b) What simplifying assumptions did you have to make? (1) The thermal energy of the water doesn't change. (2) The thermal energy of the aluminum doesn't change. (3) Energy transfer between the system (water plus pan) and the surroundings was negligible during this time. (4) The heat capacities for both water and aluminum hardly change with temperature in this temperature range. (c) Next you place the pan on a hot electric stove. While the stove is heating the pan, you use a beater to stir the water, doing 29,541 J of work, and the temperature of the water and pan increases to  $86.9^\circ\text{C}$ . How much energy transfer due to a temperature difference was there from the stove into the system consisting of the water plus the pan?

## Section 7.6

••P32 A certain motor is capable of doing 3000 J of work in 11 s. What is the power output of this motor?

••P33 In the Niagara Falls hydroelectric generating plant, the energy of falling water is converted into electricity. The height of the falls is about 50 m. Assuming that the energy conversion is highly efficient, approximately how much energy is obtained from one kilogram of falling water? Therefore, approximately how many kilograms of water must go through the generators every second to produce a megawatt of power ( $1 \times 10^6 \text{ W}$ )?

••P34 Here are questions dealing with human power. (a) If you follow a diet of 2000 food calories per day ( $2000 \text{ kcal}$ ), what is your average rate of energy consumption in watts (power input)? (A food or "large" calorie is a unit of energy equal to  $4.2 \times 10^3 \text{ J}$ ; a regular or "small" calorie is equal to 4.2 J.) Compare with the power input to a table lamp. (b) You can produce much higher power for short periods. Make appropriate measurements as you run up some stairs, and report your measurements. Use these measurements to estimate your power output (this is in addition to your basal metabolism—the power needed when resting). Compare with a horsepower (which is about 750 W) or a toaster (which is about 1000 W).

•••P35 Humans have about 60 ml ( $60 \text{ cm}^3$ ) of blood per kilogram of body mass, and blood makes a complete circuit in about 20 s, to keep tissues supplied with oxygen. Make a crude estimate of the additional power output of your heart (in watts) when you are standing compared with when you are lying down. Note that we're not asking you to estimate the power output of your heart when you are lying down, just the change in power when you are standing up. You will have to estimate the values of some of the relevant parameters. Because we're only looking for a crude estimate, try to construct a model that is as simple as possible. Describe the approximations and estimates you made.

## Section 7.7

••P36 During three hours one winter afternoon, when the outside temperature was  $0^\circ\text{C}$  ( $32^\circ\text{F}$ ), a house heated by electricity was kept at  $20^\circ\text{C}$  ( $68^\circ\text{F}$ ) with the expenditure of



45 kWh (kilowatt-hours) of electric energy. What was the average energy leakage in joules per second through the walls of the house to the environment (the outside air and ground)?

The rate at which energy is transferred between two systems is often proportional to their temperature difference. Assuming this to hold in this case, if the house temperature had been kept at  $25^\circ\text{C}$  ( $77^\circ\text{F}$ ), how many kWh of electricity would have been consumed?

### Section 7.9

•••P37 A man sits with his back against the back of a chair, and he pushes a block of mass  $m = 2$  kg straight forward on a table in front of him, with a constant force  $F = 30$  N, moving the block a distance  $d = 0.3$  m. The block starts from rest and slides on a low-friction surface. (a) How much work does the man do on the block? (b) What is the final kinetic energy  $K$  of the block? (c) What is the final speed  $v$  of the block? (d) How much time  $\Delta t$  does this process take? (e) Consider the system of the man plus the block: how much work does the chair do on the man? (f) What is the internal energy change of the man?

Now suppose that the man is sitting on a train that is moving in a straight line with speed  $V = 15$  m/s, and you are standing on the ground as the train goes by, moving to your right. From your perspective (that is, in your reference frame), answer the following questions: (g) What is the initial speed  $v_i$  of the block? (h) What is the final speed  $v_f$  of the block? (i) What is the initial kinetic energy  $K_i$  of the block? (j) What is the final kinetic energy  $K_f$  of the block? (k) What is the change in kinetic energy  $\Delta K = K_f - K_i$ , and how does this compare with the change in kinetic energy in the man's reference frame? (l) How far does the block move ( $\Delta x$ )? (m) How much work does the man do on the block, and how does this compare with the work done by the man in his reference frame, and with  $\Delta K$  in your reference frame? (n) How far does the chair move? (o) Consider the system of the man plus the block: how much work does the chair do on the man, and how does this compare with the work done by the chair in the man's reference frame? (p) What is the internal energy change of the man, and how does this compare with the internal energy change in his reference frame?

### Section 7.10

•P38 A coffee filter of mass 1.8 g dropped from a height of 4 m reaches the ground with a speed of 0.8 m/s. How much kinetic energy  $K_{\text{air}}$  did the air molecules gain from the falling coffee filter? Start from the Energy Principle, and choose as the system the coffee filter, the Earth, and the air.

•P39 You are standing at the top of a 50 m cliff. You throw a rock in the horizontal direction with speed 10 m/s. If you neglect air resistance, where would you predict it would hit on the flat plain below? Is your prediction too large or too small as a result of neglecting air resistance?

••P40 A simple experiment can allow you to determine approximately the dependence of the air resistance force on the speed of a falling object. The basic logic of the experiment is this: if we can determine the air resistance force on two objects of the same shape as they fall at different speeds, we can figure out a mathematical relationship between the air resistance force and speed. For example, if the air resistance force is eight times as large when the speed is only twice as large, then we may conclude that  $F \propto v^3$ . With only a timer and a ruler, it is possible to measure the terminal speed of a falling object. A simple way to increase the terminal speed of an object is to retain its shape but make it more massive. This will increase the gravitational force

on the object, and therefore in order to get to a speed at which  $\vec{F}_{\text{net}} = \vec{0}$  and the object falls at a constant terminal speed, the air resistance force will need to be larger. If you use a paper object (Figure 7.50) as the falling object, you can make it more massive without changing its shape simply by stacking two or more of these objects together.

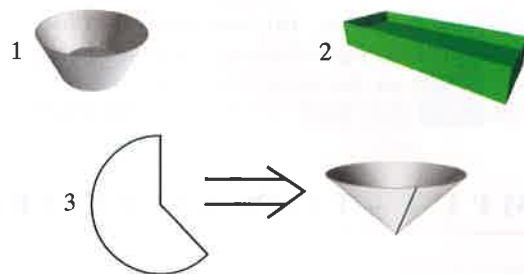


Figure 7.50 A flat-bottomed coffee filter, a folded dollar bill, or a paper cone can be used in Problem P40. The mass of a dollar bill is 1 g.

(a) Measure the terminal speed of a single falling paper object. Drop the object from as high a starting position as possible. Start timing when you think the object has reached terminal speed (think of a way to test this). Do this several times and average your results. (b) At terminal speed, what is the value of the air resistance force on a single falling object? If you don't have the equipment to measure the mass of a filter, leave your answer in terms of  $m$ . (c) Repeat this measurement for stacks of different numbers of objects of the same shape; one of the measurements should be with a stack of four objects. Qualitatively, as the mass of the falling object increases, what happens to the terminal speed? Explain why. (d) Now determine the mathematical form of the dependence of the air resistance force on speed, using your measured terminal speeds. (There are many ways to do this.)

•P41 A box with its contents has a total mass of 36 kg. It is dropped from a very high building. (a) After reaching terminal speed, what is the magnitude of the air resistance force acting upward on the falling box? (b) The box survived the fall and is returned to the top of the building. More objects are put into the box, and the box with its contents now has a total mass of 71 kg. The box is dropped, and it reaches a higher terminal speed than before. After reaching terminal speed, what is the magnitude of the air resistance force acting upward on the falling box? (The fact that the heavier object reaches a higher terminal speed shows that the air resistance force increases with increasing speed.)

••P42 You drop a single coffee filter of mass 1.7 g from a very tall building, and it takes 52 s to reach the ground. In a small fraction of that time the coffee filter reached terminal speed. (a) What was the upward force of the air resistance while the coffee filter was falling at terminal speed? (b) Next you drop a stack of five of these coffee filters. What was the upward force of the air resistance while this stack of coffee filters was falling at terminal speed? (c) Again assuming that the stack reaches terminal speed very quickly, about how long will the stack of coffee filters take to hit the ground? (Hint: Consider the relation between speed and the force of air resistance.)

### Section 7.12

••P43 You can observe the main effects of resonance with very simple experiments. Hold a spring vertically with a mass

suspended at the other end, and observe the frequency of “free” oscillations with your hand kept still. Then stop the oscillations, and move your hand *extremely* slowly up and down in a kind of slow sinusoidal motion. You will see that the mass moves up and down with the same very low frequency. **(a)** How does the amplitude (plus or minus displacement from the center location) of the mass compare with the amplitude of your hand? (Notice that the phase shift of the oscillation is  $0^\circ$ ; the mass moves up when your hand moves up.) **(b)** Next move your hand up and down at a significantly higher frequency than the free-oscillation frequency. How does the amplitude of the mass compare to the amplitude of your hand? (Notice that the phase shift of

the oscillation is  $180^\circ$ ; the mass moves down when your hand moves up.) **(c)** Finally, move your hand up and down at the free-oscillation frequency. How does the amplitude of the mass compare with the amplitude of your hand? (It is hard to observe, but the phase shift of the oscillation is  $90^\circ$ ; the mass is at the midpoint of its travel when your hand is at its maximum height.) **(d)** Change the system in some way so as to increase the air resistance significantly. For example, attach a piece of paper to increase drag. At the free-oscillation frequency, how does this affect the size of the response? A strong dependence of the amplitude and phase shift of the system to the driving frequency is called resonance.

## COMPUTATIONAL PROBLEMS

More detailed and extended versions of some computational modeling problems may be found in the lab activities included in the *Matter & Interactions* resources for instructors.

••P44 Energy graphs with springs: Start with the program you wrote to model the motion of a mass hanging from a spring (Chapter 4, Problem P70). Add the statements necessary to plot graphs of  $K$ ,  $U_{\text{spring}}$ , and  $K + U_{\text{spring}}$  vs. time. **(a)** Use initial conditions that produce 1D vertical oscillations. Is the sum  $K + U_{\text{spring}}$  constant over time? Should it be? Explain. **(b)** Use your graphs to explain the flow of energy within this system. If you need to plot an additional quantity in order to support your explanation, do so. **(c)** Now experiment with different initial conditions that produce 2D or 3D oscillations. What do your graphs show about energy flow within this system?

••P45 Air resistance: Start with the program you wrote to model the motion of a mass hanging from a spring (Chapter 4, Problem P70). Add the statements necessary to plot graphs of  $K$ ,  $U$ , and  $K + U$  vs. time. Use initial conditions that produce 1D vertical oscillations. **(a)** Make sure that the plots generated by your program make sense. Have you included all potential energy terms? **(b)** Add an air resistance force to your model. You may wish to modify the hanging mass to be a flat disk. Choose an approximate value for the drag coefficient  $C$  in your model, and adjust this value until the model behaves appropriately for a system experiencing the effects of air resistance. **(c)** Consider the energy graphs produced by the model. Is the rate of energy dissipation constant? If not, at what point(s) in the oscillation cycle is this rate largest? Explain this on physical grounds.



Figure 7.51

••P46 Sliding and viscous friction: Write a program to model a horizontal mass-spring system, like the one shown in Figure 7.51. Give the spring a relaxed length of 0.9 m and a stiffness of 1.5 N/m. Give the block a mass of 0.02 kg. **(a)** Compare the period of the oscillations of your model system (without any friction) to

the period you would expect, to make sure your model behaves correctly. **(b)** Add graphs of  $K$ ,  $U$ , and  $K + U$ . Again, check to make sure these make physical sense, in the absence of friction. **(c)** Now add a viscous friction force to the model. Adjust the viscous friction coefficient to make the behavior of the system clear. (A reasonable starting value is about 0.03). **(d)** Is the rate of energy loss constant over time? Use the energy graphs produced by your program to explain physically why the rate of energy loss varies as observed. **(e)** Replace the viscous friction force with a sliding friction force. A reasonable value for the coefficient in this model is about 0.15. **(f)** Compare the shape of the graph of  $K + U$  for sliding friction to the shape of the  $K + U$  graph for viscous friction. Explain the difference in the shapes in physical terms (consider the mathematical expressions for these forces).

••P47 Effect of air resistance on a baseball: Write a computational model to predict the motion of a baseball that is hit at a speed of 44 m/s (100 mi/h) at an angle of  $45^\circ$  to the horizontal. A baseball has a mass of 155 g and a diameter of 7 cm. The drag coefficient  $C$  for a baseball is about 0.35, and the density of air at sea level is about  $1.3 \times 10^{-3}$  g/cm<sup>3</sup>, or 1.3 kg/m<sup>3</sup>. **(a)** In your initial model, neglect air resistance. How far does the ball go? Is this distance reasonable? (A baseball field is about 400 ft from home plate to the fence in center field. An outfielder cannot throw a baseball in the air from the fence to home plate.) **(b)** Plot a graph of  $K + U_g$  vs. time for the system of baseball plus Earth. **(c)** Now add air resistance to your model. How far does the ball go? **(d)** Compare the graphs of  $K + U_g$  with and without air resistance. **(e)** In Denver, a mile above sea level, the air is about 83% as dense as the air at sea level. Including the effect of air resistance, use your computer model to predict the trajectory and range of a baseball thrown in Denver. How does the predicted range compare with the predicted range at sea level?

••P48 Determining a drag coefficient: Write a VPython program to model the motion of a paper object falling straight down, including the effects of air resistance. If you did the experiments in Problem P40, use your own data. Otherwise, you can use these data from an actual experiment that used coffee filters: filter mass =  $8.52 \times 10^{-3}$  kg, outer radius =  $5.8 \times 10^{-2}$  m, initial height = 2.03 m, drop time for one filter = 1.44 s, drop time for a stack of four coffee filters = 0.88 s. This experiment was performed in Santa Fe, New Mexico (altitude 2100 m), where the air density is only 1.045 kg/m<sup>3</sup>; the air density at an altitude of  $H$  meters is  $e^{-H/8500}$  times the air density at sea level (see Chapter 12).



(a) Add a plot of  $v$  vs.  $t$  to your program. Set the air resistance force to zero and use this plot to check that your program is working correctly. What should this plot look like without air resistance? (b) Now add the air resistance force to your program. What is the initial slope of your graph of speed vs. time? (By dragging the mouse across a VPython graph you can accurately read coordinates off the graph.) What should be the initial slope? Why? (c) Adjust the drag coefficient  $C$  until the drop time predicted by your model matches your experimental data for the case of the smallest mass. (Run your loop until the  $y$  coordinate of the moving object reaches zero, then print the computed time.) (d) What terminal speed does your model predict for this case? (e) Using the value of  $C$  that you determined with the smallest mass, what drop time does your model predict for an object of four times the mass but the same cross section? Does this agree with your measurement? (f) Using the value of  $C$  that you determined with the smallest mass, what terminal speed does your model predict for an object of four times the smallest mass but the same cross section? Is this what you expected? Why or why not?

••P49 Terminal speed for a falling sky diver has been measured to be about 60 m/s. Write a computational model to determine how far (in meters) and how long (in seconds) a sky diver falls before reaching terminal speed. (You will need to think about the meaning of “terminal speed” and how to test for it.) Plot graphs of speed vs. time and position vs. time.

••P50 You can study resonance in a driven oscillator in detail by modifying the program you wrote for Problem P46. Let one end of the spring be moved back and forth sinusoidally by a motor, with a motion given by  $D \sin(\omega_D t)$  (see Figure 7.52). Here  $D$  is the amplitude of the motor motion and  $\omega_D$  is the angular frequency of the motor, which can be varied. (The free-oscillation angular frequency  $\omega_F = \sqrt{k_s/m}$  of the spring-mass system has a fixed value, determined by the spring stiffness  $k_s$  and the mass  $m$ .)

We need an expression for the stretch  $s$  of the spring in order to be able to calculate the force  $-k_s s$  of the spring on the mass. We have to do a bit of geometry to figure out the length of the spring when the mass is displaced a distance  $x$  from the equilibrium position, and the motor has moved the other end of the spring by an amount  $D \sin(\omega_D t)$ . In the figure, the spring gets longer when the mass moves to the right ( $+x$ ), but the spring gets shorter when the motor moves to the right ( $-D \sin(\omega_D t)$ ). The new length of the spring is  $L + x - D \sin(\omega_D t)$ , and the net stretch of the spring is this quantity minus the unstretched length  $L$ , yielding ( $s = x - D \sin(\omega_D t)$ ). A check that this is the correct expression for the net stretch of the spring is that if the motor moves to the right the same distance as the mass moves to the right, the spring

will have zero stretch. Replace  $x$  in your computer computation with the quantity  $[x - D \sin(\omega_D t)]$ .

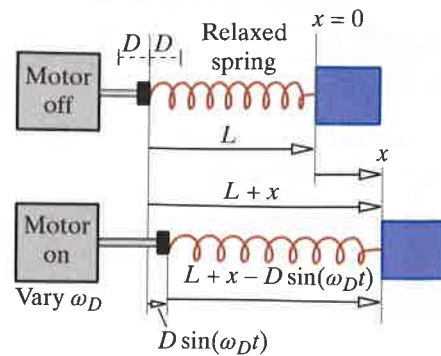


Figure 7.52

Use viscous damping (friction proportional to  $v$ ). The damping should be small. That is, without the motor driving the system ( $D = 0$ ), the mass should oscillate for many cycles. Set  $\omega_D$  to  $0.9\omega_F$  (that is, 0.9 times the free-oscillation angular frequency,  $\omega_F = \sqrt{k_s/m}$ ), and let  $x_0$  be 0. Graph position  $x$  vs. time for enough cycles to show that there is a transient buildup to a “steady state.” In the steady state the energy dissipation per cycle has grown to exactly equal the energy input per cycle.

Vary  $\omega_D$  in the range from 0 to  $2.0\omega_F$ , with closely spaced values in the neighborhood of  $1.0\omega_F$ . Have the computer plot the position of the motor end as well as the position of the mass as a function of time. For each of these driving frequencies, record for later use the steady-state amplitude and the phase shift (the difference in angle between the sinusoids for the motor and for the mass). If a harmonic oscillator is lightly damped, it has a large response only for driving frequencies near its own free-oscillation frequency.

Is the steady-state angular frequency equal to  $\omega_F$  or  $\omega_D$  for these various values of  $\omega_D$ ? (Note that during the transient buildup to the steady state the frequency is not well-defined, because the motion of the mass isn’t a simple sinusoid.)

Sketch graphs of the steady-state amplitude and phase shift vs.  $\omega_D$ . Note that when  $\omega_D = \omega_F$  the amplitude of the mass can be much larger than  $D$ , just as you observed with your hand-driven spring-mass system. Also note the interesting variation of the phase shift as you go from low to high driving frequencies—does this agree with the phase shifts you observed with your hand-driven spring-mass system?

Repeat the analysis with viscous friction twice as large. What happens to the resonance curve (the graph of steady-state amplitude vs. angular frequency  $\omega_D$ )?

## ANSWERS TO CHECKPOINTS

1 No change; same stretch

2 0.012 J

3  $A$  is  $|U|$ ;  $B$  is  $K$ ;  $C$  is  $|K + U|$ ; 1–4 are bound; 5 is unbound.

4 21.36°C; neglect the material of the thin glass.

5  $1.26 \times 10^5$  J;  $7.6 \times 10^4$  J;  $-1.26 \times 10^5$  J

6 9000 J

7 0, since no temperature change;  $-11,000$  J

8 2.5 m/s<sup>2</sup>

9  $\frac{1}{2}k_s A^2$ ;  $\frac{1}{2}k_s A^2$ ; spring plus mass

10 (90 kg)(9.8 N/kg) = 882 N; the net force must be zero.

11  $2.7 \times 10^{-2}$  J

12 Air resistance is proportional to  $v^2$ , so the energy dissipation rate is higher at higher speeds.

13 +4 J; another +4 J; +8 J

14 Denominator gets very small when  $\omega_D = \omega_F$ .