system at each location, largest first. **(d)** Rank-order the locations on the path in terms of the sum of the kinetic energy and the potential energy of the system at each location, largest first.

Q6 Figure 6.77 is a graph of the energy of a system of a planet interacting with a star. The gravitational potential energy U_g is shown as the thick curve, and plotted along the vertical axis are various values of $K + U_g$.

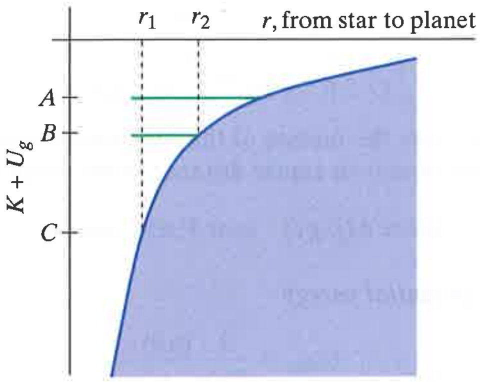

Figure 6.77

Suppose that $K + U_g$ of the system is A. Which of the following statements are true? **(a)** The potential energy of the system decreases as the planet moves from r_1 to r_2. **(b)** When the separation between the two bodies is r_2, the kinetic energy of the system is $(A - B)$. **(c)** The system is a bound system; the planet can never escape. **(d)** The planet will escape. **(e)** When the separation between the two bodies is r_2, the kinetic energy of the system is $(B - C)$. **(f)** The kinetic energy of the system is greater when the distance between the star and planet is r_1 than when the distance between the two bodies is r_2.

Suppose instead that $K + U_g$ of the system is B. Which of the following statements are true? **(a)** When the separation between the planet and star is r_2, the kinetic energy of the system is zero. **(b)** The planet and star cannot get farther apart than r_2. **(c)** This is not a bound system; the planet can escape. **(d)** When the separation between the planet and star is r_2, the potential energy of the system is zero.

Q7 A particle moves inside a circular glass tube under the influence of a tangential force of constant magnitude F (Figure 6.78). Explain why we cannot associate a potential energy with this force. How is this situation different from the case of a block on the end of a string, which is swung in a circle?

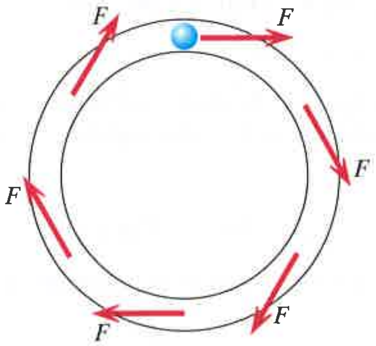

Figure 6.78

Q8 Show the validity of the relation $E^2_{\text{particle}} - (pc)^2 = (mc^2)^2$ when $m \neq 0$, by making these substitutions:

$$E_{\text{particle}} = \frac{mc^2}{\sqrt{1-(v/c)^2}} \quad \text{and} \quad p = \frac{mv}{\sqrt{1-(v/c)^2}}$$

PROBLEMS

Section 6.2

•P9 It is not very difficult to accelerate an electron to a speed that is 99% of the speed of light, because the electron has such a very small mass. What is the ratio of the kinetic energy K to the rest energy mc^2 in this case? In the definition of what we mean by kinetic energy K, $E = mc^2 + K$, you must use the full relativistic expression for E, because v/c is not small compared to 1.

•P10 A pitcher can throw a baseball at about 100 mi/h (about 44 m/s). What is the ratio of the kinetic energy to the rest energy mc^2? (Can you use $K \approx \frac{1}{2}mv^2$?)

•P11 What is the speed of an electron whose total energy is equal to the total energy of a proton that is at rest? What is the kinetic energy of this electron?

•P12 The point of this question is to compare rest energy and kinetic energy at low speeds. A baseball is moving at a speed of 17 m/s. Its mass is 145 g (0.145 kg). **(a)** What is its rest energy? **(b)** Is it okay to calculate its kinetic energy using the expression $\frac{1}{2}mv^2$? **(c)** What is its kinetic energy? **(d)** Which is true? A. The kinetic energy is approximately equal to the rest energy. B. The kinetic energy is much bigger than the rest energy. C. The kinetic energy is much smaller than the rest energy.

•P13 The point of this question is to compare rest energy and kinetic energy at high speeds. An alpha particle (a helium nucleus) is moving at a speed of 0.9993 times the speed of light. Its mass is 6.40×10^{-27} kg. **(a)** What is its rest energy? **(b)** Is it okay to calculate its kinetic energy using the expression $\frac{1}{2}mv^2$? **(c)** What is its kinetic energy? **(d)** Which is true? A. The kinetic energy is approximately equal to the rest energy. B. The kinetic energy is much bigger than the rest energy. C. The kinetic energy is much smaller than the rest energy.

•P14 A runner whose mass is 60 kg runs in the $+x$ direction at a speed of 7 m/s. **(a)** What is the kinetic energy of the runner? **(b)** The runner turns around and runs in the $-x$ direction at the same speed. Now what is the kinetic energy of the runner?

•P15 A baseball of mass 144 g has a velocity of $\langle 22, 23, -11 \rangle$ m/s. What is the kinetic energy of the baseball?

•P16 One mole of helium atoms has a mass of 4 grams. If a helium atom in a balloon has a kinetic energy of 1.437×10^{-21} J, what is the speed of the helium atom? (The speed is much lower than the speed of light.)

•P17 You throw a ball of mass 160 g upward (Figure 6.79). When the ball is 2 m above the ground, headed upward (the initial state), its speed is 19 m/s. Later, when the ball is again 2 m above the ground, this time headed downward (the final state), its speed is 19 m/s. What is the change in the kinetic energy of the ball from initial to final state?

Figure 6.79

•**P18** A fan cart of mass 0.8 kg initially has a velocity of $\langle 0.9,0,0\rangle$ m/s. Then the fan is turned on, and the air exerts a constant force of $\langle -0.3,0,0\rangle$ N on the cart for 1.5 s. **(a)** What is the change in momentum of the fan cart over this 1.5 s interval? **(b)** What is the change in kinetic energy of the fan cart over this 1.5 s interval?

Section 6.3

•**P19** You push a crate 3 m across the floor with a 40 N force whose direction is 30° below the horizontal. How much work do you do?

•**P20** You pull your little sister across a flat snowy field on a sled. Your sister plus the sled have a mass of 20 kg. The rope is at an angle of 35 degrees to the ground. You pull a distance of 50 m with a force of 30 N. How much work do you do?

•**P21** A 2-kg ball rolls off a 30-m-high cliff, and lands 25 m from the base of the cliff. Express the displacement and the gravitational force in terms of vectors and calculate the work done by the gravitational force. Note that the gravitational force is $\langle 0,-mg,0\rangle$, where g is a *positive* number (+9.8 N/kg).

•**P22** A boat is coasting toward a dock you're standing on, and as it comes toward you, you push back on it with a force of 300 N. As you do this, you back up a distance of 2 m. How much work do you do on the boat?

•**P23** A jar of honey with a mass of 0.5 kg is nudged off the kitchen counter and falls 1 m to the floor. What force acts on the jar during its fall? How much work is done by this force?

•**P24** An object that is originally at location $\langle -17,0,0\rangle$ m moves to location $\langle -25,0,0\rangle$ m as shown in Figure 6.80. While it is moving it is acted on by a constant force of $\langle 22,0,0\rangle$ N.

Figure 6.80

(a) How much work is done on the object by this force? **(b)** Does the kinetic energy of the object increase or decrease?

Figure 6.81

(c) A different object moves from location $\langle -25,0,0\rangle$ m to location $\langle -17,0,0\rangle$ m, as shown in Figure 6.81. While it is moving it is acted on by a constant force of $\langle 22,0,0\rangle$ N. How much work is done on the second object by this force? **(d)** Does the kinetic energy of the object increase or decrease?

•**P25** A constant force $\langle 23,-12,32\rangle$ N acts through a displacement $\langle 0.12,0.31,-0.24\rangle$ m. How much work does this force do?

•**P26** One end of a spring whose spring constant is 20 N/m is attached to the wall, and you pull on the other end, stretching it from its equilibrium length of 0.2 m to a length of 0.3 m. Estimate the work done by dividing the stretching process into two stages and using the average force you exert to calculate work done during each stage.

•**P27** An electron traveling through a curving wire in an electric circuit experiences a constant force of 5×10^{-19} N, always in the direction of its motion through the wire. How much work is done on the electron by this force as it travels through 0.5 m of the wire?

•**P28** You bring a boat toward the dock by pulling on a rope with a force of 130 N through a distance of 6 m. **(a)** How much work do you do? (Include the appropriate sign.) **(b)** Then you slow the boat down by pushing against it with a force of 40 N, opposite to the boat's movement of 5 m. How much work do you do? (Include the appropriate sign.) **(c)** What is the total amount of work that you do?

••**P29** You push a box out of a carpeted room and along a hallway with a waxed linoleum floor. While pushing the crate 2 m out of the room you exert a force of 34 N; while pushing it 6 m along the hallway you exert a force of 13 N. To slow it down you exert a force of 40 N through a distance of 2 m, opposite to the motion. How much work do you do in all?

Section 6.4

•**P30** A ball of mass 0.7 kg falls downward, as shown in Figure 6.82. Initially you observe it to be 4.5 m above the ground. After a short time it is just about to hit the ground.

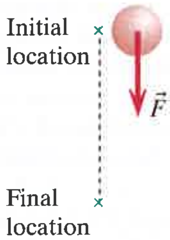

Figure 6.82

(a) During this interval how much work was done on the ball by the gravitational force? **(b)** Does the kinetic energy of the ball increase or decrease?

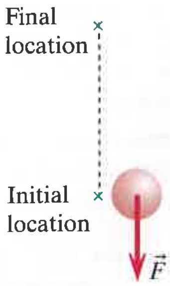

Figure 6.83

(c) The ball hits the ground and bounces back upward, as shown in Figure 6.83. After a short time it is 4.5 m above the ground again. During this second interval (between leaving the ground and reaching a height of 4.5 m) how much work was done on the ball by the gravitational force? **(d)** Does the kinetic energy of the ball increase or decrease?

•P31 A lithium nucleus has mass 5.1×10^{-27} kg. If its speed is 0.984c (that is, $|\vec{v}|/c = 0.984$), what are the values of the particle energy, the rest energy, and the kinetic energy?

Next an electric force acts on the lithium nucleus and does 4.7×10^{-9} J of work on the particle. Now what are the values of the particle energy, the rest energy, and the kinetic energy?

•P32 A space probe in outer space has a mass of 111 kg, and it is traveling at a speed of 29 m/s. When it is at location $\langle 445, 535, -350 \rangle$ m it begins firing two booster rockets. The rockets exert constant forces of $\langle 90, 150, 195 \rangle$ N and $\langle 90, -90, -585 \rangle$ N, respectively. The rockets fire until the spacecraft reaches location $\langle 449, 541, -354 \rangle$ m. Now what is its speed? There is negligible mass loss due to the rocket exhaust.

••P33 An object with mass 100 kg moved in outer space. When it was at location $\langle 9, -24, -4 \rangle$ its speed was 3.5 m/s. A single constant force $\langle 250, 400, -170 \rangle$ N acted on the object while the object moved from location $\langle 9, -24, -4 \rangle$ m to location $\langle 15, -17, -8 \rangle$ m. Then a different single constant force $\langle 140, 250, 150 \rangle$ N acted on the object while the object moved from location $\langle 15, -17, -8 \rangle$ m to location $\langle 19, -24, -3 \rangle$ m. What is the speed of the object at this final location?

••P34 Outside the space shuttle, you and a friend pull on two ropes to dock a satellite whose mass is 700 kg. The satellite is initially at position $\langle 3.5, -1, 2.4 \rangle$ m and has a speed of 4 m/s. You exert a force $\langle -400, 310, -250 \rangle$ N. When the satellite reaches the position $\langle 7.1, 3.2, 1.2 \rangle$ m, its speed is 4.01 m/s. How much work did your friend do?

••P35 A crate with a mass of 100 kg glides through a space station with a speed of 3.5 m/s. An astronaut speeds it up by pushing on it from behind with a force of 180 N, continually pushing with this force through a distance of 6 m. The astronaut moves around to the front of the crate and slows the crate down by pushing backward with a force of 170 N, backing up through a distance of 4 m. After these two maneuvers, what is the speed of the crate?

••P36 An object with mass 120 kg moved in outer space from location $\langle 11, -24, -5 \rangle$ to location $\langle 17, -28, -10 \rangle$ m. A single constant force $\langle 250, 440, -220 \rangle$ N acted on the object while the object moved. Its final speed was 7.9 m/s. What was the speed of the object at the initial location?

••P37 An object with mass 120 kg moved in outer space. When it was at location $\langle 7, -20, -8 \rangle$ its speed was 12 m/s. A single constant force $\langle 250, 490, -160 \rangle$ N acted on the object while the object moved to location $\langle 10, -29, -13 \rangle$ m. What is the speed of the object at this final location?

••P38 Jack and Jill are maneuvering a 3000 kg boat near a dock. Initially the boat's position is $\langle 2, 0, 3 \rangle$m and its speed is 1.3 m/s. As the boat moves to position $\langle 4, 0, 2 \rangle$m, Jack exerts a force $\langle -400, 0, 200 \rangle$ N and Jill exerts a force $\langle 150, 0, 300 \rangle$ N. **(a)** How much work does Jack do? **(b)** How much work does Jill do? **(c)** Without doing any calculations, say what is the angle between the (vector) force that Jill exerts and the (vector) velocity of the boat. Explain briefly how you know this. **(d)** Assuming that we can neglect the work done by the water on the boat, what is the final speed of the boat?

••P39 An electron traveling at a speed 0.99c encounters a region where there is a constant electric force directed opposite to its momentum. After traveling 3 m in this region, the electron's speed was observed to decrease to 0.93c. What was the magnitude of the electric force acting on the electron?

••P40 A mass of 0.12 kg hangs from a vertical spring in the lab room. You grab hold of the mass and throw it vertically downward. The speed of the mass just after leaving your hand is 3.40 m/s. **(a)** While the mass moved downward a distance of 0.07 m, how much work was done on the mass by the Earth? **(b)** The speed of the mass has decreased to 2.85 m/s. How much work was done on the mass by the spring?

••P41 An electron (mass 9×10^{-31} kg) is traveling at a speed of 0.91c in an electron accelerator. An electric force of 1.6×10^{-13} N is applied in the direction of motion while the electron travels a distance of 2 m. You need to find the new speed of the electron. Which of the following steps must be included in your solution to this problem? **(a)** Calculate the initial particle energy $\gamma_i mc^2$ of the electron. **(b)** Calculate the final particle energy $\gamma_f mc^2$ of the electron. **(c)** Determine how much time it takes to move this distance. **(d)** Use the expression $\frac{1}{2}m|\vec{v}|^2$ to find the kinetic energy of the electron. **(e)** Calculate the net work done on the electron. **(f)** Use the final energy of the electron to find its final speed.

What is the new speed of the electron as a fraction of c?

••P42 SLAC, the Stanford Linear Accelerator Center, located at Stanford University in Palo Alto, California, accelerates electrons through a vacuum tube 2 mi long (it can be seen from an overpass of the Junipero Serra freeway that goes right over the accelerator). Electrons that are initially at rest are subjected to a continuous force of 2×10^{-12} N along the entire length of two mi (1 mi is 1.6 km) and reach speeds very near the speed of light. A similar analysis in a previous chapter required numerical integration, but with the new techniques of this chapter you can analyze the motion analytically. **(a)** Calculate the final energy, momentum, and speed of the electron. **(b)** Calculate the approximate time required to go the 2 mi distance.

Section 6.5

•P43 In positron-emission tomography (PET) used in medical research and diagnosis, compounds containing unstable nuclei that emit positrons are introduced into the brain, destined for a site of interest in the brain. When a positron is emitted, it goes only a short distance before coming nearly to rest. It forms a bound state with an electron, called "positronium," which is rather similar to a hydrogen atom. The binding energy of positronium is very small compared to the rest energy of an electron. After a short time the positron and electron annihilate. In the annihilation, the positron and the electron disappear, and all of their rest energy goes into two photons (particles of light) that have zero mass; all their energy is kinetic energy. These high-energy photons, called "gamma rays," are emitted at nearly 180° to each other. What energy of gamma ray (in MeV, million electron volts) should each of the detectors be made sensitive to? (The mass of an electron or positron is 9×10^{-31} kg.)

••P44 A nucleus whose mass is 3.499612×10^{-25} kg undergoes spontaneous alpha decay. The original nucleus disappears and there appear two new particles: a He-4 nucleus of mass 6.640678×10^{-27} kg (an "alpha particle" consisting of two protons and two neutrons) and a new nucleus of mass

3.433132×10^{-25} kg (note that the new nucleus has less mass than the original nucleus, and it has two fewer protons and two fewer neutrons). **(a)** When the alpha particle has moved far away from the new nucleus (so the electric interactions are negligible), what is the combined kinetic energy of the alpha particle and new nucleus? **(b)** How many electron volts is this? In contrast to this nuclear reaction, chemical reactions typically involve only a few eV.

••P45 In a location in outer space far from all other objects, a nucleus whose mass is 3.894028×10^{-25} kg and that is initially at rest undergoes spontaneous alpha decay. The original nucleus disappears, and two new particles appear: a He-4 nucleus of mass 6.640678×10^{-27} kg (an alpha particle consisting of two protons and two neutrons) and a new nucleus of mass 3.827555×10^{-25} kg. These new particles move far away from each other, because they repel each other electrically (both are positively charged).

Because the calculations involve the small difference of (comparatively) large numbers, you need to keep seven significant figures in your calculations, and you need to use the more accurate value for the speed of light, 2.99792e8 m/s.

Choose all particles as the system. Initial state: Original nucleus, at rest. Final state: Alpha particle + new nucleus, far from each other. **(a)** What is the rest energy of the original nucleus? *Give seven significant figures.* **(b)** What is the sum of the rest energies of the alpha particle and the new nucleus? *Give seven significant figures.* **(c)** Did the portion of the total energy of the system contributed by rest energy increase or decrease? **(d)** What is the sum of the kinetic energies of the alpha particle and the new nucleus?

••P46 A proton (1.6726×10^{-27} kg) and a neutron (1.6749×10^{-27} kg) at rest combine to form a deuteron, the nucleus of deuterium or "heavy hydrogen." In this process, a gamma ray (high-energy photon) is emitted, and its energy is measured to be 2.2 MeV (2.2×10^6 eV). **(a)** Keeping all five significant figures, what is the mass of the deuteron? Assume that you can neglect the small kinetic energy of the recoiling deuteron. **(b)** Momentum must be conserved, so the deuteron must recoil with momentum equal and opposite to the momentum of the gamma ray. Calculate approximately the kinetic energy of the recoiling deuteron and show that it is indeed small compared to the energy of the gamma ray.

••P47 Many heavy nuclei undergo spontaneous "alpha decay," in which the original nucleus emits an alpha particle (a helium nucleus containing two protons and two neutrons), leaving behind a "daughter" nucleus that has two fewer protons and two fewer neutrons than the original nucleus. Consider a radium-220 nucleus that is at rest before it decays to radon-216 by alpha decay.

The mass of the radium-220 nucleus is 219.96274 u (unified atomic mass units) where 1 u = 1.6603×10^{-27} kg (approximately the mass of one nucleon).

The mass of a radon-216 nucleus is 215.95308 u, and the mass of an alpha particle is 4.00151 u. Radium has 88 protons, radon has 86 protons, and an alpha particle has 2 protons. **(a)** Make a diagram of the final state of the radon-216 nucleus and the alpha particle when they are far apart, showing the momenta of each particle to the same relative scale. Explain why you drew the lengths of the momentum vectors the way you did. **(b)** Calculate the final kinetic energy of the alpha particle. For the moment, assume that its speed is small compared to the speed of light. **(c)** Calculate the final kinetic energy of the radon-216 nucleus. **(d)** Show that the nonrelativistic approximation was reasonable.

•••P48 A nucleus whose mass is 3.917268×10^{-25} kg undergoes spontaneous alpha decay. The original nucleus disappears and there appear two new particles: a He-4 nucleus of mass 6.640678×10^{-27} kg (an alpha particle consisting of two protons and two neutrons) and a new nucleus of mass 3.850768×10^{-25} kg. (Note that the new nucleus has less mass than the original nucleus, and it has two fewer protons and two fewer neutrons.) **(a)** What is the total kinetic energy of the alpha particle and the new nucleus? **(b)** Use the Conservation of Momentum in order to determine the kinetic energy of the alpha particle and the kinetic energy of the new nucleus.

Section 6.8

•P49 You throw a ball straight up, and it reaches a height of 20 m above your hand before falling back down. What was the speed of the ball just after it left your hand?

•P50 A 1 kg block rests on the Earth's surface. How much energy is required to move the block very far from the Earth, ending up at rest again?

•P51 An object with mass 7 kg moves from a location $\langle 22, 43, -41 \rangle$ m near the Earth's surface to location $\langle -27, 11, 46 \rangle$ m. What is the change in the potential energy of the system consisting of the object plus the Earth?

•P52 The radius of the Moon is 1750 km, and its mass is 7×10^{22} kg. What would be the escape speed from an isolated Moon? Why was a small rocket adequate to lift the lunar astronauts back up from the surface of the Moon?

•P53 Use energy conservation to find the approximate final speed of a basketball dropped from a height of 2 m (roughly the height of a professional basketball player). Why don't you need to know the mass of the basketball?

•P54 Under certain conditions the interaction between a "polar" molecule such as HCl located at the origin and an ion located along the x axis can be described by a potential energy $U = -b/x^2$, where b is a constant. What is F_x, the x component of the force on the ion? What is F_y, the y component of the force on the ion?

•P55 (a) A 0.5 kg teddy bear is nudged off a window sill and falls 2 m to the ground. What is its kinetic energy at the instant it hits the ground? What is its speed? What assumptions or approximations did you make in this calculation? **(b)** A 1.0 kg flowerpot is nudged off a window sill and falls 2 m to the ground. What is its kinetic energy at the instant it hits the ground? What is its speed? How do the speed and kinetic energy compare to that of the teddy bear in part (a)?

•P56 You throw a ball of mass 1.2 kg straight up. You observe that it takes 3.1 s to go up and down, returning to your hand. Assuming we can neglect air resistance, the time it takes to go up to the top is half the total time, 1.55 s. Note that at the top the momentum is momentarily zero, as it changes from heading upward to heading downward. **(a)** Use the Momentum Principle to determine the speed that the ball had just after it left your hand. **(b)** Use the Energy Principle to determine the maximum height above your hand reached by the ball.

•P57 Suppose that a pitcher can throw a ball straight up at 100 mi/h (about 45 m/s). Use energy conservation to calculate how high the baseball goes. Explain your work. Actually, a pitcher can't attain this high a speed when throwing straight up, so your result will be an overestimate of what a human can do; air resistance also reduces the achievable height.

••P58 The radius of Mars (from the center to just above the atmosphere) is 3400 km (3400×10^3 m), and its mass is 0.6×10^{24} kg. An object is launched straight up from just above the atmosphere of Mars. **(a)** What initial speed is needed so that when the object is far from Mars its final speed is 1000 m/s? **(b)** What initial speed is needed so that when the object is far from Mars its final speed is 0 m/s? (This is called the escape speed.)

••P59 The radius of an airless planet is 2000 km (2×10^6 m), and its mass is 1.2×10^{23} kg. An object is launched straight up from the surface. **(a)** What initial speed is needed so that when the object is far from the planet its final speed is 900 m/s? **(b)** What initial speed is needed so that when the object is far from the planet its final speed is 0 m/s? (This is called the escape speed.)

••P60 The escape speed from an asteroid whose radius is 10 km is only 10 m/s. If you throw a rock away from the asteroid at a speed of 20 m/s, what will be its final speed?

••P61 The escape speed from a very small asteroid is only 24 m/s. If you throw a rock away from the asteroid at a speed of 35 m/s, what will be its final speed?

••P62 Calculate the speed of a satellite in a circular orbit near the Earth (just above the atmosphere). If the mass of the satellite is 200 kg, what is the minimum energy required to move the satellite from this near-Earth orbit to very far away from the Earth?

••P63 A spacecraft is coasting toward Mars. The mass of Mars is 6.4×10^{23} kg and its radius is 3400 km (3.4×10^6 m). When the spacecraft is 7000 km (7×10^6 m) from the center of Mars, the spacecraft's speed is 3000 m/s. Later, when the spacecraft is 4000 km (4×10^6 m) from the center of Mars, what is its speed? Assume that the effects of Mars's two tiny moons, the other planets, and the Sun are negligible. Precision is required to land on Mars, so make an accurate calculation, not a rough, approximate calculation.

••P64 A comet is in an elliptical orbit around the Sun. Its closest approach to the Sun is a distance of 4×10^{10} m (inside the orbit of Mercury), at which point its speed is 8.17×10^4 m/s. Its farthest distance from the Sun is far beyond the orbit of Pluto. What is its speed when it is 6×10^{12} m from the Sun? (This is the approximate distance of Pluto from the Sun.)

••P65 An electron is traveling at a speed of $0.95c$ in an electron accelerator. An electric force of 1.6×10^{-13} N is applied in the direction of motion while the electron travels a distance of 2 m. What is the new speed of the electron?

••P66 You stand on a spherical asteroid of uniform density whose mass is 2×10^{16} kg and whose radius is 10 km (10^4 m). These are typical values for small asteroids, although some asteroids have been found to have much lower average density and are thought to be loose agglomerations of shattered rocks. **(a)** How fast do you have to throw the rock so that it never comes back to the asteroid and ends up traveling at a speed of 3 m/s when it is very far away? **(b)** Sketch graphs of the kinetic energy of the rock, the gravitational potential energy of the rock plus asteroid, and their sum, as a function of separation (distance from center of asteroid to rock). Label the graphs clearly.

•••P67 In the rough approximation that the density of the Earth is uniform throughout its interior, the gravitational field strength (force per unit mass) inside the Earth at a distance r from the center is gr/R, where R is the radius of the Earth. (In actual fact, the outer layers of rock have lower density than the inner core of molten iron.) Using the uniform-density approximation, calculate the amount of energy required to move a mass m from the center of the Earth to the surface. Compare with the amount of energy required to move the mass from the surface of the Earth to a great distance away.

••P68 This problem is closely related to the spectacular impact of the comet Shoemaker-Levy with Jupiter in July 1994:

http://www.jpl.nasa.gov/sl9/sl9.html

A rock far outside our Solar System is initially moving very slowly relative to the Sun, in the plane of Jupiter's orbit around the Sun. The rock falls toward the Sun, but on its way to the Sun it collides with Jupiter. Calculate the rock's speed just before colliding with Jupiter. Explain your calculation and any approximations that you make.

$M_{\text{Sun}} = 2 \times 10^{30}$ kg, $M_{\text{Jupiter}} = 2 \times 10^{27}$ kg
Distance, Sun to Jupiter $= 8 \times 10^{11}$ m
Radius of Jupiter $= 1.4 \times 10^8$ m

•••P69 A pendulum (see Figure 6.84) consists of a very light but stiff rod of length L hanging from a nearly frictionless axle, with a mass m at the end of the rod. **(a)** Calculate the gravitational potential energy as a function of the angle θ, measured from the vertical. **(b)** Sketch the potential energy as a function of the angle θ, for angles from $-210°$ to $+210°$. **(c)** Let $s = L\theta$ = the arc length away from the bottom of the arc. Calculate the tangential component of the force on the mass by taking the (negative) gradient of the energy with respect to s. Does your result make sense? **(d)** Suppose that you hit the stationary hanging mass so it has an initial speed v_i. What is the minimum initial speed needed for the pendulum to go over the top ($\theta = 180°$)? On your sketch of the potential energy (part b), draw and label energy levels for the case in which the initial speed is less than, equal to, or greater than this critical initial speed.

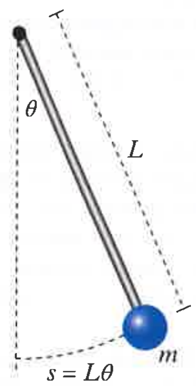

Figure 6.84

Section 6.9

•P70 (a) Two protons are a distance 4×10^{-9} m apart. What is the electric potential energy of the system consisting of the two protons? If the two protons move closer together, will the electric potential energy of the system increase, decrease, or remain the same? **(b)** A proton and an electron are a distance 4×10^{-9} m apart. What is the electric potential energy of the system consisting of the proton and the electron? If the proton and the electron move closer together, will the electric potential energy of the system increase, decrease, or remain the same? **(c)** Which of the following statements are true? A. In some situations charged particles released from rest would move in a direction that increases electric potential energy, but not in other situations. B. If released from rest two protons would move closer together, increasing the potential energy of the system. C. If any two charged particles are released from rest, they will spontaneously move in the direction in which the potential energy of the system will be decreased.

••P71 **(a)** A particle with mass M and charge $+e$ and its antiparticle (same mass M, charge $-e$) are initially at rest, far from each other. They attract each other and move toward each other. Make a graph of energy terms vs. separation distance r between the two particles. Label the various energies involved in this process. Include the rest energy of the particles, assuming that the other energy terms are comparable to the rest energy. **(b)** When the particle and antiparticle collide, they annihilate and produce two new particles, one with mass m (much smaller than M) and charge $+e$, and its antiparticle (same mass m, charge $-e$). When these two particles have moved far away from each other, how fast are they going? Is this speed large or small compared to c? **(c)** Now take the specific case of a proton and antiproton colliding to form a positive and negative pion. Each pion has a mass of 2.5×10^{-28} kg. When the pions have moved far away, how fast are they going? **(d)** How far apart must the two pions be (in meters) for their electric potential energy to be negligible compared to their kinetic energy? Be explicit and quantitative about your criterion and your result.

••P72 Four protons, each with mass M and charge $+e$, are initially held at the corners of a square that is d on a side. They are then released from rest. What is the speed of each proton when the protons are very far apart?

Section 6.11

•P73 There are three different ways to get from location A to location E in Figure 6.85. Along path 1, you take an elevator directly from location A straight up to location E. Along path 2, you walk from A to B, climb a rope from B to C, then walk from C to E. Along path 3, you walk from location A to D, then climb a ramp up to location E. The following questions focus on the work done on a 80 kg person by the Earth while following each of these paths.

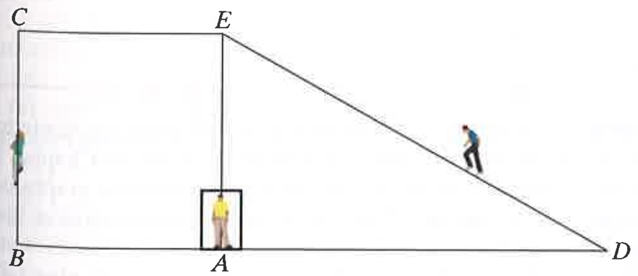

Figure 6.85

Taking the origin at location A, the coordinates of the other locations are: $B\ \langle 0,-3,0\rangle$; $C\ \langle -3,13,0\rangle$; $D\ \langle 33,0,0\rangle$; $E\ \langle 0,13,0\rangle$. **(a)** Path 1 (A to E): What is the displacement vector $\Delta\vec{r}$ for the path from A to E on the elevator? **(b)** What is the gravitational force acting on the person along path 1, as a vector? **(c)** What is the work done by the gravitational force of the Earth on the 80 kg person during the elevator ride from A to E? **(d)** Path 2 (A to B to C to E): What is the work done by the gravitational force of the Earth on the person as he walks from A to B? **(e)** What is the work done by the gravitational force of the Earth on the person as he climbs from B to C? **(f)** What is the work done by the gravitational force of the Earth on the person as he walks from C to E? **(g)** What is the total work done by the gravitational force of the Earth on the person as the person goes from A to E along path 2? **(h)** Path 3 (A to D to E): What is the work done by the gravitational force of the Earth on the person as he walks from A to D? **(i)** What is the work done by the gravitational force of the Earth on the person as he climbs from D to E? **(j)** What is the total work done by the gravitational force of the Earth on the person as the person goes from A to E along path 3? **(k)** You calculated the work done by the gravitational force of the Earth on the person along three different paths. How do these quantities compare with each other?

•P74 Refer to Figure 6.86. Calculate the change in electric energy along two different paths in moving charge q away from charge Q from A to B along a radial path, then to C along a circle centered on Q, then to D along a radial path. Also calculate the change in energy in going directly from A to D along a circle centered on Q. Specifically, what are $U_B - U_A$, $U_C - U_B$, $U_D - U_C$, and their sum? What is $U_D - U_A$? Also, calculate the round-trip difference in the electric energy when moving charge q along the path from A to B to C to D to A.

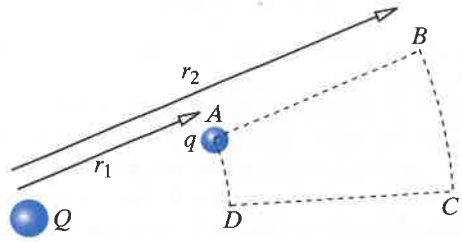

Figure 6.86

Section 6.12

••P75 One of the possible fission modes of Uranium-235 involves nearly equal fragments, palladium nuclei with $Q_1 = Q_2 = 46e$. The masses of the two palladium nuclei add up to less than the mass of the original nucleus. (In addition to the two main fission fragments there are typically one or more free neutrons in the final state; in your analysis make the simplifying assumption that there are no free neutrons, just two palladium nuclei.)

The mass of the U-236 nucleus is 235.996 u (unified atomic mass units), and the mass of each Pd-118 nucleus is 117.894 u, where $1\text{ u} = 1.7 \times 10^{-27}$ kg (approximately the mass of one nucleon). Although in most problems you solve in this course it is adequate to use values of constants rounded to two or three significant figures, in this problem you must keep at least six significant figures throughout your calculation. Problems involving mass changes require many significant figures because the changes in mass are small compared to the total mass. **(a)** Calculate the final speed v, when the palladium nuclei have moved far apart (due to their mutual electric repulsion). Is this speed small enough that $p^2/(2m)$ is an adequate approximation for the kinetic energy of one of the palladium nuclei? (It is all right to go ahead and make the nonrelativistic assumption first, but you then must check that the calculated v is indeed small compared to c.) **(b)** Using energy considerations, calculate the distance between centers of the palladium nuclei just after fission, when they are starting from rest. **(c)** A proton or neutron has a radius of roughly 1×10^{-15} m, and a nucleus is a tightly packed collection of nucleons. Experiments show that the radius of a nucleus containing N nucleons is approximately $(1.3 \times 10^{-15}\,\text{m}) \times N^{1/3}$. What is the approximate radius of a palladium nucleus? Draw a sketch of the two palladium nuclei in part (b), and label the distances you calculated in parts (b) and (c). If the two palladium nuclei are nearly touching, this would be consistent with our model of fission, in which the uranium nucleus fissions into two pieces that are nearly at rest. **(d)** The kinetic energy of the fast-moving daughter nuclei is eventually absorbed in the surrounding material and raises the temperature of that material. In a fission power plant, this thermal energy is used to boil water to drive a steam turbine and generate electricity. If a mole of uranium undergoes this fission reaction, how much kinetic energy is generated? For comparison, only around 1×10^6 J are obtained from burning a mole of gasoline, which is why energy from fission is of great interest, if some way can be found to deal with the waste products.

••P76 A proton (^{1}H) and a deuteron (^{2}H, "heavy" hydrogen) start out far apart. An experimental apparatus shoots them toward each other (with equal and opposite momenta). If they get close enough to make actual contact with each other, they can react to form a helium-3 nucleus and a gamma ray (a high-energy photon, which has kinetic energy but zero rest energy):

$$^1\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + \gamma$$

This is one of the thermonuclear or fusion reactions that takes place inside a star such as our Sun.

The mass of the proton is 1.0073 u (unified atomic mass unit, 1.7×10^{-27} kg), the mass of the deuteron is 2.0136 u, the mass of the helium-3 nucleus is 3.0155 u, and the gamma ray is massless. Although in most problems you solve in this course it is adequate to use values of constants rounded to two or three significant figures, in this problem you must keep at least six significant figures throughout your calculation. Problems involving mass changes require many significant figures because the changes in mass are small compared to the total mass. **(a)** The strong interaction has a very short range and is essentially a contact interaction. For this fusion reaction to take place, the proton and deuteron have to come close enough together to touch. The approximate radius of a proton or neutron is about 1×10^{-15} m. What is the approximate initial total kinetic energy of the proton and deuteron required for the fusion reaction to proceed, in joules and electron volts ($1\text{ eV} = 1.6 \times 10^{-19}$ J)? **(b)** Given the initial conditions found in part (a), what is the kinetic energy of the ^{3}He plus the energy of the gamma ray, in joules and in electron volts? **(c)** The net energy released is the kinetic energy of the ^{3}He plus the energy of the gamma ray found in part (b), minus the energy input that you calculated in part (a). What is the net energy release, in joules and in electron volts? Note that you do get back the energy investment made in part (a). **(d)** Kinetic energy can be used to drive motors and do other useful things. If a mole of hydrogen and a mole of deuterium underwent this fusion reaction, how much kinetic energy would be generated? (For comparison, around 1×10^6 J are obtained from burning a mole of gasoline.) **(e)** Which of the following potential energy curves (1–4) in Figure 6.87 is a reasonable representation of the interaction in this fusion reaction? Why?

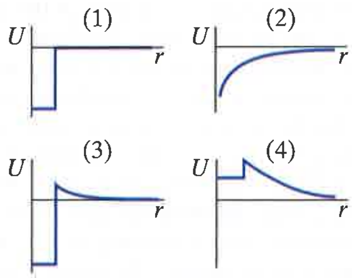

Figure 6.87

As we will study later, the average kinetic energy of a gas molecule is $\frac{3}{2}k_BT$, where k is the "Boltzmann constant," 1.4×10^{-23} J/K, and T is the absolute or Kelvin temperature, measured from absolute zero (so that the freezing point of water is 273 K). The approximate temperature required for the fusion reaction to proceed is very high. This high temperature, required because of the electric repulsion barrier to the reaction, is the main reason why it has been so difficult to make progress toward thermonuclear power generation. Sufficiently high temperatures are found in the interior of the Sun, where fusion reactions take place.

COMPUTATIONAL PROBLEMS

More detailed and extended versions of some computational modeling problems may be found in the lab activities included in the *Matter & Interactions, 4th Edition*, resources for instructors.

••P77 Start with the program you wrote to model the motion of a spacecraft near the Earth, in Chapter 3 (Problem P65). Add the statements necessary to plot graphs of K, U, and $(K + U)$ versus time, like the graphs in Figure 6.71. Use initial conditions that produce an elliptical orbit. **(a)** Refer to the shapes of the plots to explain the flow of energy within the system. **(b)** Use the plot of $K + U$ versus time produced by the program to determine the maximum reasonable value of Δt for this model. **(c)** Predict the shape each plot should have for a perfectly circular orbit. **(d)** Use your energy plots to find (by trial and error) the initial speed of the spacecraft that produces a circular orbit, and explain what information from the plots you used.

••P78 As in Problem P77, start with the program you wrote to model the motion of a spacecraft near the Earth in Chapter 3 (Problem P64). Add the statements necessary to plot graphs of graphs of K, U, and $(K + U)$ versus r, the separation between the spacecraft and the Earth. Use initial conditions that produce an elliptical orbit. In VPython, when you create a `gcurve` object, it can be helpful to add a moving dot to clarify what is happening in the plot, like this:

```
ugraph = gcurve(color=color.green, dot=True)
```

(a) Refer to the shapes of the plots to explain the flow of energy within the system. **(b)** What effect does changing the initial speed of the spacecraft have on these three plots? **(c)** Predict the shape each plot should have for a perfectly circular orbit. **(d)** Use your energy plots to find (by trial and error) the initial speed of the spacecraft that produces a circular orbit, and explain what information from the plots you used. **(e)** Use your energy plots to help you find the escape speed for this system. **(f)** Describe your plot of $K+U$ when the spacecraft starts with this escape speed.

••P79 Start with the program you wrote to model the motion of a spacecraft near the Earth and a stationary Moon, in Chapter 3 (Problem P66). Add the statements necessary to plot graphs of K, U, and $(K+U)$ versus time, like the graphs in Figure 6.71. Give the spacecraft an initial velocity of $\langle 0, 3.27\times10^3, 0\rangle$ m/s. **(a)** Refer to the shapes of the plots to explain the flow of energy within the system. **(b)** Use the plot of $K+U$ versus time produced by the program to determine the maximum reasonable value of Δt for this model. **(c)** What changes in the energy graphs when you vary the initial speed?

••P80 As in Problem P79, start with the program you wrote to model the motion of a spacecraft near the Earth and a stationary Moon, in Chapter 3 (Problem P66). Add the statements necessary to plot graphs of graphs of K, U, and $(K+U)$ versus r, the separation between the spacecraft and the Earth. Give the spacecraft an initial velocity of $\langle 0, 3.27\times10^3, 0\rangle$ m/s. In VPython, when you create a `gcurve` object, it can be helpful to add a moving dot to clarify what is happening in the plot, like this:

```
ugraph = gcurve(color=color.green, dot=True)
```

(a) Why are the shapes of the graphs of U and K versus separation different for this three-body system than they were for the two-body (spacecraft + Earth) system? **(b)** Refer to the shapes of the plots to explain the flow of energy within the system. **(c)** What effect does changing the initial speed of the spacecraft have on these three plots?

••P81 Start with the program you wrote in Problem P71 in Chapter 3 to model the Ranger 7 mission to the Moon. (If you have not already written this program, do it now.) **(a)** Add a calculation of the work done by the gravitational forces of the Earth and the Moon to your analysis of sending a spacecraft to the Moon. You need to approximate the work by adding up the amount of work done by gravitational forces along each step of the path:

$$W = \Sigma\vec{F}\bullet\Delta\vec{r} = \vec{F}_1\bullet\Delta\vec{r}_1 + \vec{F}_2\bullet\Delta\vec{r}_2 + \vec{F}_3\bullet\Delta\vec{r}_3 + \cdots$$

(b) Compare the numerical value of the work with the change in the kinetic energy (final kinetic energy just before crashing on the Moon, minus initial kinetic energy when released above the Earth's atmosphere). **(c)** Modify your program to make graphs of two quantities: the kinetic energy of the spacecraft, and the work done by the Earth and the Moon on the spacecraft, as a function of time. **(d)** Find a way to determine the maximum value of Δt you can use without introducing significant errors into your computation.

••P82 Energy conservation is a powerful check on the accuracy of a numerical integration. Modify the program for the Chapter 3 problem on the Ranger 7 mission to the Moon to plot graphs of kinetic energy, of gravitational potential energy, and of the sum of the kinetic energy and the gravitational potential energy, vs. position. Does the kinetic plus potential energy remain constant? What if you vary the step size (which varies the accuracy of the numerical integration)? Vary the launch speed, and explain the effect that this has on your graphs.

••P83 Use energy conservation to calculate analytically (that is, without doing a numerical integration) the final speed of the spacecraft just before it hits the Moon. Include the gravitational effect of the Moon. Use a launch speed of 1.3×10^4 m/s. Modify your program for the Chapter 3 problem on the Ranger 7 mission to the Moon to print out the speed of the spacecraft when it hits the surface of the Moon, and compare this value to your analytical result. What questions that could be addressed in the numerical integration are you *not* able to answer by doing this energy calculation?

ANSWERS TO CHECKPOINTS

1 Particle energy = 5.2×10^{-13} J, rest energy = 8.1×10^{-14} J; kinetic energy = 4.4×10^{-13} J
2 12 m/s; 27 miles per hour; 1.2×10^{-12} kg (an extremely small amount of mass!)
3 1.4 J
4 **(a)** decreases, **(b)** increases, **(c)** decreases, **(d)** increases, **(e)** stays the same
5 189 J
6 400 J
7 40.8; 39.8; very large; very small
8 8.5×10^{-4}, approximately 0.1%
9 Six pairs: $U_{12}+U_{13}+U_{14}+U_{23}+U_{24}+U_{34}$
10 Same ΔU
11 $\frac{1}{2}m(v_{xi}^2) - \frac{1}{2}m(v_{xi}^2+v_{yi}^2) + mg(y_f - y_i) = 0$, so $y_f = y_i + v_{yi}^2/(2g)$
12 **(a)** $\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r} = (0.2\times10^6\text{ eV})(1.6\times10^{-19}\text{ J/eV})$, **(b)** the electric potential energy term
13 **(a)** decrease, **(b)** A is bound; C is unbound; B is trapped. In a quantum system in state B there is some probability of "tunneling" through the barrier and getting out of the trap!
14 A: go out in a straight line and stop, then fall back; B: elliptical (motion with varying separation); C: nearly circular (nearly constant separation)
15 1.12×10^4 m/s
16 About 1.7×10^{-10}; accuracy of laboratory scale is far below what would be needed to detect this
17 About 0.1%
18 $-GMm/y^2$; 0