3. Divide $\Delta\vec{p}$ by Δt to get $\Delta\vec{p}/\Delta t$ (Figure 5.37). The smaller Δt, the closer the result is to $d\vec{p}/dt$.

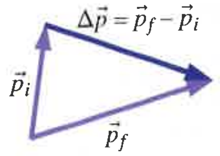

Figure 5.62

Momentarily at rest vs. uniform motion and equilibrium

For a system in equilibrium $d\vec{p}/dt = \vec{0}$.

For a system in uniform motion $d\vec{p}/dt = \vec{0}$.

For a system that is momentarily at rest, $d\vec{p}/dt \neq \vec{0}$.

The Momentum Principle: Parallel and perpendicular components

$$\frac{dp}{dt}\hat{p} = \vec{F}_{\parallel} = (\vec{F}_{\text{net}} \bullet \hat{p})\hat{p} \quad \text{and} \quad p\frac{d\hat{p}}{dt} = \vec{F}_{\perp} = \vec{F}_{\text{net}} - \vec{F}_{\parallel}$$

Valid only for a moving object, because $\hat{p}$ and $d\hat{p}/dt$ are undefined when $\vec{p} = \vec{0}$.

Component of net force parallel to momentum changes magnitude of momentum.

Component of net force perpendicular to momentum changes direction of momentum.

Effect of perpendicular component of net force on a particle moving along a curving path

$$p\left|\frac{d\hat{p}}{dt}\right| = p\frac{v}{R} = |\vec{F}_{\text{net}\perp}|$$

R is the radius of the kissing circle. The component of net force perpendicular to the particle's momentum changes its direction but not the magnitude of its momentum (speed). Must be a smoothly curving motion; not valid if there is an abrupt change of direction.

The dot product

$$\vec{A} \bullet \vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

QUESTIONS

Q1 You are riding in the passenger seat of an American car, sitting on the right side of the front seat. The car makes a sharp left turn. You feel yourself thrown to the right and your right side hits the right door. Is there a force that pushes you to the right? What object exerts that force? What really happens? Draw a diagram to illustrate and clarify your analysis.

Q2 A student said, "When the Moon goes around the Earth, there is an inward force due to the Moon and an outward force due to centrifugal force, so the net force on the Moon is zero." Give two or more physics reasons why this is wrong.

Q3 A space shuttle is in a circular orbit near the Earth. An astronaut floats in the middle of the shuttle, not touching the walls. On a diagram, draw and label **(a)** the momentum $\vec{p}_1$ of the astronaut at this instant, **(b)** all of the forces (if any) acting on the astronaut at this instant, **(c)** the momentum $\vec{p}_2$ of the astronaut a short time Δt later, and **(d)** the momentum change (if any) $\Delta\vec{p}$ in this time interval. **(e)** Why does the astronaut seem to "float" in the shuttle?

It is ironic that we say the astronaut is "weightless" despite the fact that the only force acting on the astronaut is the astronaut's weight (that is, the gravitational force of the Earth on the astronaut).

Q4 Tarzan swings back and forth on a vine. At the microscopic level, why is the tension force on Tarzan by the vine greater than it would be if he were hanging motionless?

Q5 Tarzan swings from a vine. When he is at the bottom of his swing, as shown in Figure 5.63, which is larger in magnitude: the force by the Earth on Tarzan, the force by the vine (a tension force) on Tarzan, or neither (same magnitude)? Explain how you know this.

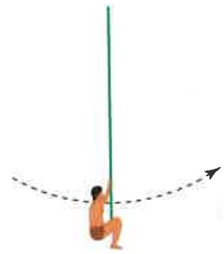

Figure 5.63

PROBLEMS

Section 5.4

•P6 A rope is attached to a block, as shown in Figure 5.64. The rope pulls on the block with a force of 210 N, at an angle of $\theta = 23°$ to the horizontal (this force is equal to the tension in the rope).

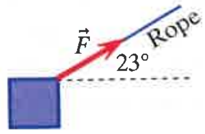

Figure 5.64

(a) What is the x component of the force on the block due to the rope? **(b)** What is the y component of the force on the block due to the rope?

••**P7** A box of mass 40 kg hangs motionless from two ropes, as shown in Figure 5.65. The angle is 38 degrees. Choose the box as the system. The x axis runs to the right, the y axis runs up, and the z axis is out of the page.

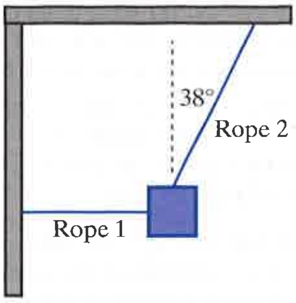

Figure 5.65

(a) Draw a free-body diagram for the box. **(b)** Is $d\vec{p}/dt$ of the box zero or nonzero? **(c)** What is the y component of the gravitational force acting on the block? (A component can be positive or negative). **(d)** What is the y component of the force on the block due to rope 2? **(e)** What is the magnitude of $\vec{F}_2$? **(f)** What is the x component of the force on the block due to rope 2? **(g)** What is the x component of the force on the block due to rope 1?

••**P8** A helicopter flies to the right (in the $+x$ direction) at a constant speed of 12 m/s, parallel to the surface of the ocean. A 900 kg package of supplies is suspended below the helicopter by a cable as shown in Figure 5.66: the package is also traveling to the right in a straight line, at a constant speed of 12 m/s. The pilot is concerned about whether or not the cable, whose breaking strength is listed at 9300 N, is strong enough to support this package under these circumstances. **(a)** Choose the package as the system, and draw a free-body diagram. **(b)** What is the magnitude of the tension in the cable supporting the package? **(c)** Write the force exerted on the package by the cable as a vector. **(d)** What is the magnitude of the force exerted by the air on the package? **(e)** Write the force on the package by the air as a vector. **(f)** Is the cable in danger of breaking?

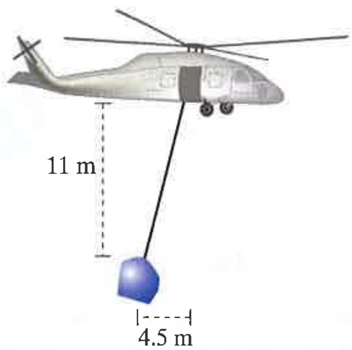

Figure 5.66

••**P9** You pull with a force of 255 N on a rope that is attached to a block of mass 30 kg, and the block slides across the floor at a constant speed of 1.1 m/s. The rope makes an angle $\theta = 40°$ with the horizontal. Both the force and the velocity of the block are in the xy plane. See Figure 5.67.

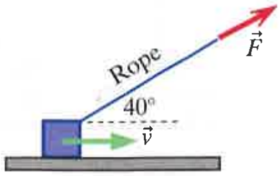

Figure 5.67

(a) Express the tension force exerted by the rope on the block as a vector. **(b)** Express the force exerted by the floor on the block as a vector.

••**P10** A ball of mass 450 g hangs from a spring whose stiffness is 110 N/m. A string is attached to the ball and you are pulling the string to the right, so that the ball hangs motionless, as shown in Figure 5.68. In this situation the spring is stretched, and its length is 15 cm. What would be the relaxed length of the spring, if it were detached from the ball and laid on a table?

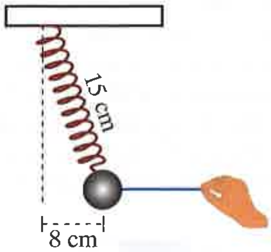

Figure 5.68

••**P11** An 800 kg load is suspended as shown in Figure 5.69. **(a)** Calculate the tension in all three wires (that is, the magnitude of the tension force exerted by each of these wires). **(b)** These wires are made of a material whose value for Young's modulus is 1.3×10^{11} N/m^2. The diameter of the wires is 1.1 mm. What is the strain (fractional stretch) in each wire?

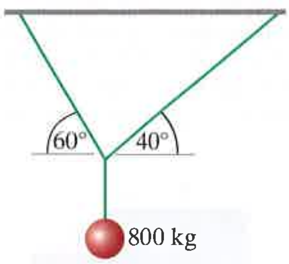

Figure 5.69

•••**P12** A cylindrical steel rod with diameter 0.7 mm is 2 m long when lying horizontally (Figure 5.70). The density of this particular steel is 7.85 g/cm^3, and Young's modulus is 2×10^{11} N/m^2. **(a)** You now hang the rod vertically from a support and attach a 60 kg mass to the bottom end. How much does the rod stretch? **(b)** Next you remove the 60 kg mass so the rod hangs under its own weight. Describe qualitatively how the stretch of the interatomic bonds depends on position along the rod. **(c)** Calculate how much the rod stretches under its own weight. (This requires the use of integral calculus, because the tension and strain vary along the rod.) **(d)** If the rod were twice as thick, how much would it stretch?

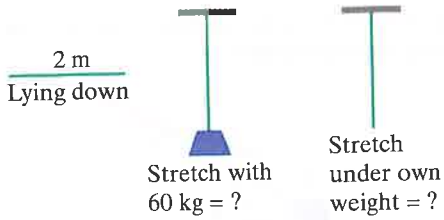

Figure 5.70

Section 5.5

••P13 An elevator is accelerating upward at a rate of 3.2 m/s^2. A block of mass 30 kg hangs by a low-mass rope from the ceiling, and another block of mass 80 kg hangs by a low-mass rope from the upper block. **(a)** What are the tensions in the upper and lower ropes? **(b)** What are the tensions in the upper and lower ropes when the elevator accelerates downward at a rate of 3.2 m/s^2?

••P14 A 10 kg block is placed on top of a 35 kg block (Figure 5.71). A force of 350 N is applied to the right on the lower block, and the upper block slips on the lower block (accelerating less than the lower block). The coefficient of kinetic friction between the upper block and the lower block is 0.2, and the coefficient of kinetic friction between the lower block and the floor is 0.7. **(a)** What is the acceleration of the upper block? **(b)** What is the acceleration of the lower block? **(c)** How big would the coefficient of static friction between the upper and lower block have to be so that the upper block would not slip on the lower block?

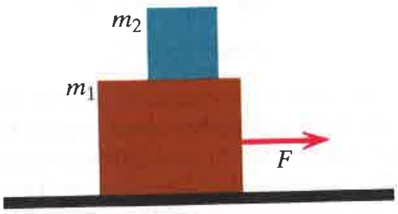

Figure 5.71

••P15 In Figure 5.72 $m_1 = 12$ kg and $m_2 = 5$ kg. The kinetic coefficient of friction between m_1 and the floor is 0.3 and that between m_2 and the floor is 0.5. You push with a force of magnitude $F = 110$ N. **(a)** What is the acceleration of the center of mass? **(b)** What is the magnitude of the force that m_1 exerts on m_2?

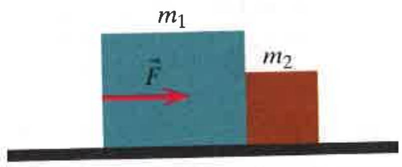

Figure 5.72

Section 5.6

•P16 The radius of a merry-go-round is 11 m, and it takes 12 s to go around once. What is the speed of an atom in the outer rim?

•P17 If the radius of a merry-go-round is 5 m, and it takes 14 s to go around once, what is the speed of an atom at the outer rim? What is the direction of the velocity of this atom: toward the center, away from the center, or tangential?

••P18 At a particular instant the magnitude of the momentum of a planet is 2.3×10^{29} kg · m/s, and the force exerted on it by the star it is orbiting is 8.9×10^{22} N. The angle between the planet's momentum and the gravitational force exerted by the star is 123°. **(a)** What is the parallel component of the force on the planet by the star? **(b)** What will the magnitude of the planet's momentum be after 9 h?

••P19 The angle between the gravitational force on a planet by a star and the momentum of the planet is 61° at a particular instant. At this instant the magnitude of the planet's momentum is 3.1×10^{29} kg · m/s, and the magnitude of the gravitational force on the planet is 1.8×10^{23} N. **(a)** What is the parallel component of the force on the planet by the star? **(b)** What will be the magnitude of the planet's momentum after 8 h?

••P20 A planet orbits a star in an elliptical orbit. At a particular instant the momentum of the planet is $\langle -2.6 \times 10^{29}, -1.0 \times 10^{29}, 0\rangle$ kg · m/s, and the force on the planet by the star is $\langle -2.5 \times 10^{22}, -1.4 \times 10^{23}, 0\rangle$ N. Find $\vec{F}_{\parallel}$ and $\vec{F}_{\perp}$.

••P21 A planet of mass 6×10^{24} kg orbits a star in a highly elliptical orbit. At a particular instant the velocity of the planet is $\langle 4.5 \times 10^4, -1.7 \times 10^4, 0\rangle$ m/s, and the force on the planet by the star is $\langle 1.5 \times 10^{22}, 1.9 \times 10^{23}, 0\rangle$ N. Find $\vec{F}_{\parallel}$ and $\vec{F}_{\perp}$.

Section 5.7

•P22 An object moving at a constant speed of 23 m/s is making a turn with a radius of curvature of 4 m (this is the radius of the kissing circle). The object's momentum has a magnitude of 78 kg · m/s. What is the magnitude of the rate of change of the momentum? What is the magnitude of the net force?

•P23 The radius of a merry-go-round is 7 m, and it takes 12 s to make a complete revolution. **(a)** What is the speed of an atom on the outer rim? **(b)** What is the direction of the momentum of this atom? **(c)** What is the direction of the rate of change of the momentum of this atom?

•P24 A proton moving in a magnetic field follows the curving path shown in Figure 5.73. The dashed circle is the kissing circle tangent to the path when the proton is at location A. The proton is traveling at a constant speed of 7.0×10^5 m/s, and the radius of the kissing circle is 0.08 m. The mass of a proton is 1.7×10^{-27} kg. Refer to the directional arrows shown at the right in Figure 5.73 when answering the questions below.

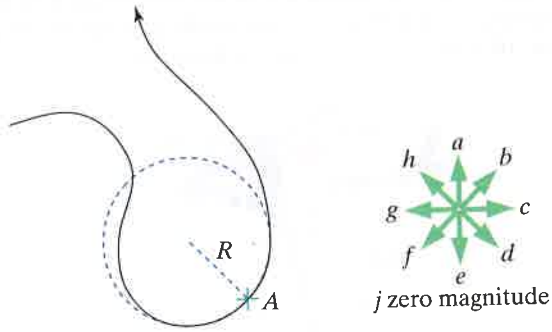

Figure 5.73

(a) When the proton is at location A, what are the magnitude and direction of $(d|\vec{p}|/dt)\hat{p}$, the parallel component of $d\vec{p}/dt$? **(b)** When the proton is at location A, what are the magnitude and direction of $|\vec{p}|\hat{p}/dt$, the perpendicular component of $d\vec{p}/dt$?

•P25 A proton moving in a magnetic field follows the curving path shown in Figure 5.74, traveling at constant speed in the direction shown. The dashed circle is the kissing circle tangent to the path when the proton is at location A. Refer to the directional arrows shown at the right in Figure 5.74 when answering the questions below.

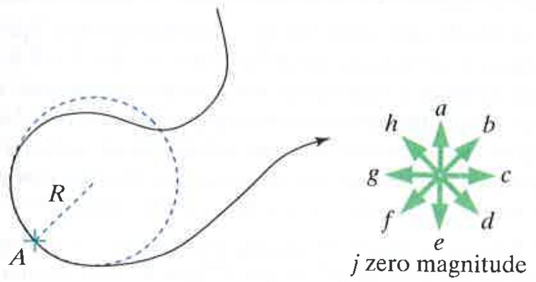

Figure 5.74

(a) When the proton is at location A, what is the direction of the proton's momentum? **(b)** When the proton is at location A, what is the direction of $d\vec{p}/dt$? **(c)** The mass of a proton is 1.7×10^{-27} kg. The proton is traveling at a constant speed of 6.0×10^5 m/s, and the radius of the kissing circle is 0.07 m. What is the magnitude of $d\vec{p}/dt$ of the proton?

••P26 A particle moving at nearly the speed of light ($v \approx c$) passes through a region where it is subjected to a magnetic force of constant magnitude that is always perpendicular to the momentum and has a magnitude of 2×10^{-10} N. As a result, the particle moves along a circular arc with a radius of 8 m. What is the magnitude of the momentum of this particle?

Section 5.8

•P27 A child of mass 40 kg sits on a wooden horse on a carousel. The wooden horse is 5 m from the center of the carousel, which completes one revolution every 90 s. What is $(d|\vec{p}|/dt)\hat{p}$ for the child, both magnitude and direction? What is $|\vec{p}|(d\hat{p}/dt)$ for the child? What is the net force acting on the child? What objects in the surroundings exert this force?

•P28 A child of mass 35 kg sits on a wooden horse on a carousel. The wooden horse is 3.3 m from the center of the carousel, which rotates at a constant rate and completes one revolution every 5.2 s. **(a)** What are the magnitude and direction (tangential in direction of velocity, tangential in the opposite direction of the velocity, radial outward, radial inward) of $(d|\vec{p}|/dt)\hat{p}$, the parallel component of $d\vec{p}/dt$ for the child? **(b)** What are the magnitude and direction of $|\vec{p}|d\hat{p}/dt$, the perpendicular component of $d\vec{p}/dt$ for the child? **(c)** What are the magnitude and direction of the net force acting on the child? **(d)** What objects in the surroundings contribute to this horizontal net force acting on the child? (There are also vertical forces, but these cancel each other if the horse doesn't move up and down.)

•P29 A 30 kg child rides on a playground merry-go-round, 1.4 m from the center. The merry-go-round makes one complete revolution every 5 s. How large is the net force on the child? In what direction does the net force act?

•P30 The orbit of the Earth around the Sun is approximately circular, and takes one year to complete. The Earth's mass is 6×10^{24} kg, and the distance from the Earth to the Sun is 1.5×10^{11} m. What is $(d|\vec{p}|/dt)\hat{p}$ of the Earth? What is $|\vec{p}|(d\hat{p}/dt)$ of the Earth? What is the magnitude of the gravitational force the Sun (mass 2×10^{30} kg) exerts on the Earth? What is the direction of this force?

••P31 You swing a bucket full of water in a vertical circle at the end of a rope. The mass of the bucket plus the water is 3.5 kg. The center of mass of the bucket plus the water moves in a circle of radius 1.3 m. At the instant that the bucket is at the top of the circle, the speed of the bucket is 4 m/s. What is the tension in the rope at this instant?

••P32 In outer space two identical spheres are connected by a taut steel cable, and the whole apparatus rotates about its center. The mass of each sphere is 60 kg. The distance between centers of the spheres is 3.2 m. At a particular instant the velocity of one of the spheres is $\langle 0,5,0\rangle$ m/s and the velocity of the other sphere is $\langle 0,-5,0\rangle$ m/s. What is the tension in the cable?

••P33 In the dark in outer space, you observe a glowing ball of known mass 2 kg moving in the xy plane at constant speed in a circle of radius 6.5 m, with the center of the circle at the origin ($\langle 0,0,0\rangle$ m). You can't see what's making it move in a circle. At time $t = 0$ the ball is at location $\langle -6.5,0,0\rangle$ m and has velocity $\langle 0,40,0\rangle$ m/s.

On your own paper draw a diagram of the situation showing the circle and showing the position and velocity of the ball at time $t = 0$. The diagram will help you analyze the situation. Use letters a–j (Figure 5.75) to answer questions about directions ($+x$ to the right, $+y$ up).

Figure 5.75

At time $t = 0$: **(a)** What is the direction of the vector $\vec{p}$? **(b)** What are the magnitude and direction of $(d|\vec{p}|/dt)\hat{p}$, the parallel component of $d\vec{p}/dt$? **(c)** What are the magnitude and direction of $|\vec{p}|d\hat{p}/dt$, the perpendicular component of $d\vec{p}/dt$? **(d)** Even though you can't see what's causing the motion, what can you conclude must be the direction of the vector $\vec{F}_{net}$? **(e)** Even though you can't see what's causing the motion, what can you conclude must be the vector $\vec{F}_{net}$? **(f)** You learn that at time $t = 0$, two forces act on the ball, and that at this instant one of these forces is $\vec{F}_1 = \langle 196,-369,0\rangle$ N. What is the other force?

••P34 You're driving a vehicle of mass 1350 kg and you need to make a turn on a flat road. The radius of curvature of the turn is 76 m. The coefficient of static friction and the coefficient of kinetic friction are both 0.25. **(a)** What is the fastest speed you can drive and still make it around the turn? Invent symbols for the various quantities and solve algebraically before plugging in numbers. **(b)** Which of the following statements are true about this situation? (1) The net force is nonzero and points away from the center of the kissing circle. (2) The rate of change of the momentum is nonzero and points away from the center of the kissing circle. (3) The rate of change of the momentum is nonzero and points toward the center of the kissing circle. (4) The momentum points toward the center of the kissing circle. (5) The centrifugal force balances the force of the road, so the net force is zero. (6) The net force is nonzero and points toward the center of the kissing circle. **(c)** Look at your algebraic analysis and answer the following question. Suppose that your vehicle had a mass five times as big (6750 kg). Now what is the fastest speed you can drive and still make it around the turn? **(d)** Look at your algebraic analysis and answer the following question. Suppose that you have the original 1350 kg vehicle but the turn has a radius twice as large (152 m). What is the fastest speed you can drive and still make it around the turn? This problem shows why high-speed curves on freeways have very large radii of curvature, but low-speed entrance and exit ramps can have smaller radii of curvature.

••**P35** What is the minimum speed v that a roller coaster car must have in order to make it around an inside loop and just barely lose contact with the track at the top of the loop (see Figure 5.76)? The center of the car moves along a circular arc of radius R. Include a carefully labeled force diagram. State briefly what approximations you make. Design a plausible roller coaster loop, including numerical values for v and R.

Figure 5.76

••**P36** In outer space a rock of mass 4 kg is attached to a long spring and swung at constant speed in a circle of radius 9 m. The spring exerts a force of constant magnitude 760 N. **(a)** What is the speed of the rock? **(b)** What is the direction of the spring force? **(c)** The relaxed length of the spring is 8.7 m. What is the stiffness of this spring?

••**P37** A child of mass 26 kg swings at the end of an elastic cord. At the bottom of the swing, the child's velocity is horizontal, and the speed is 12 m/s. At this instant the cord is 4.30 m long. **(a)** At this instant, what is the parallel component of the rate of change of the child's momentum? **(b)** At this instant, what is the perpendicular component of the rate of change of the child's momentum? **(c)** At this instant, what is the *net* force acting on the child? **(d)** What is the magnitude of the force that the elastic cord exerts on the child? (It helps to draw a diagram of the forces.) **(e)** The relaxed length of the elastic cord is 4.22 m. What is the stiffness of the cord?

••**P38** An engineer whose mass is 70 kg holds onto the outer rim of a rotating space station whose radius is 14 m and which takes 30 s to make one complete rotation. What is the magnitude of the force the engineer has to exert in order to hold on? What is the magnitude of the net force acting on the engineer?

••**P39** In June 1997 the NEAR spacecraft ("Near Earth Asteroid Rendezvous"; see http://near.jhuapl.edu/), on its way to photograph the asteroid Eros, passed near the asteroid Mathilde. After passing Mathilde, on several occasions rocket propellant was expelled to adjust the spacecraft's momentum in order to follow a path that would approach the asteroid Eros, the final destination for the mission. After getting close to Eros, further small adjustments made the momentum just right to give a circular orbit of radius 45 km (45×10^3 m) around the asteroid. So much propellant had been used that the final mass of the spacecraft while in circular orbit around Eros was only 500 kg. The spacecraft took 1.04 days to make one complete circular orbit around Eros. Calculate what the mass of Eros must be.

••**P40** A Ferris wheel is a vertical, circular amusement ride. Riders sit on seats that swivel to remain horizontal as the wheel turns. The wheel has a radius R and rotates at a constant rate, going around once in a time T. At the bottom of the ride, what are the magnitude and direction of the force exerted by the seat on a rider of mass m? Include a diagram of the forces on the rider.

••**P41** A block with mass 0.4 kg is connected by a spring of relaxed length 0.15 m to a post at the center of a low-friction table. You pull the block straight away from the post and release it, and you observe that the period of oscillation is 0.6 s. Next you stretch the spring to a length of 0.28 m and give the block an initial speed v perpendicular to the spring, choosing v so that the motion is a circle with the post at the center. What is this speed v?

••**P42** When a particle with electric charge q moves with speed v in a plane perpendicular to a magnetic field B, there is a magnetic force at right angles to the motion with magnitude qvB, and the particle moves in a circle of radius r (see Figure 5.77). This equation for the magnetic force is correct even if the speed is comparable to the speed of light. Show that

$$p = \frac{mv}{\sqrt{1-(|\vec{v}|/c)^2}} = qBr$$

even if v is comparable to c.

This result is used to measure relativistic momentum: if the charge q is known, we can determine the momentum of a particle by observing the radius of a circular trajectory in a known magnetic field.

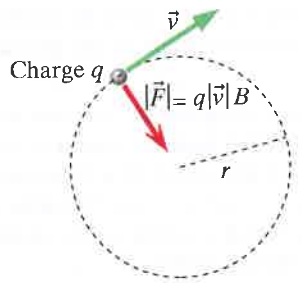

Figure 5.77

••**P43** A ball of unknown mass m is attached to a spring. In outer space, far from other objects, you hold the other end of the spring and swing the ball around in a circle of radius 1.5 m at constant speed. **(a)** You time the motion and observe that going around 10 times takes 6.88 s. What is the speed of the ball? **(b)** Is the momentum of the ball changing or not? How can you tell? **(c)** If the momentum is changing, what interaction is causing it to change? If the momentum is not changing, why isn't it? **(d)** The relaxed length of the spring is 1.2 m, and its stiffness is 1000 N/m. While you are swinging the ball, since the radius of the circle is 1.5 m, the length of the spring is also 1.5 m. What is the magnitude of the force that the spring exerts on the ball? **(e)** What is the mass m of the ball?

••**P44** A sports car (and its occupants) of mass M is moving over the rounded top of a hill of radius R. At the instant when the car is at the very top of the hill, the car has a speed v. You can safely neglect air resistance. **(a)** Taking the sports car as the system of interest, what object(s) exert nonnegligible forces on this system? **(b)** At the instant when the car is at the very top of the hill, draw a diagram showing the system as a dot, with force vectors whose tails are at the location of the dot. Label the force vectors (that is, give them algebraic names). Try to make the lengths of the force vectors be proportional to the magnitudes of the forces. **(c)** Starting from the Momentum Principle, calculate the force exerted by the road on the car. **(d)** Under what conditions will the force exerted by the road on the car be zero? Explain.

••**P45** A small block of mass m is attached to a spring with stiffness k_s and relaxed length L. The other end of the spring is fastened to a fixed point on a low-friction table. The block slides on the table in a circular path of radius $R > L$. How long does it take for the block to go around once?

••**P46** A person of mass 70 kg rides on a Ferris wheel whose radius is 4 m. The person's speed is constant at 0.3 m/s. The person's location is shown by a dot in Figure 5.78. **(a)** What is the magnitude of the rate of change of the momentum of the person at the instant shown?

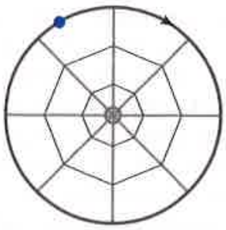

Figure 5.78

(b) What is the direction of the rate of change of momentum of the person at the instant shown? **(c)** What is the magnitude of the *net* force acting on the person at the instant shown? Draw the *net* force vector on the diagram at this instant, with the tail of the vector on the person.

••**P47** The planets in our Solar System have orbits around the Sun that are nearly circular, and $v \ll c$. Calculate the period T (a year—the time required to go around the Sun once) for a planet whose orbit radius is r. This is the relationship discovered by Kepler and explained by Newton. (It can be shown by advanced techniques that this result also applies to elliptical orbits if you replace r by the semimajor axis, which is half the longer, major axis of the ellipse.) Use this analytical solution for circular motion to predict the Earth's orbital speed, using the data for Sun and Earth on the inside back cover of the textbook.

•••**P48** In the 1970s the astronomer Vera Rubin made observations of distant galaxies that she interpreted as indicating that perhaps 90% of the mass in a galaxy is invisible to us ("dark matter"). She measured the speed with which stars orbit the center of a galaxy, as a function of the distance of the stars from the center. The orbital speed was determined by measuring the Doppler shift of the light from the stars, an effect that makes light shift toward the red end of the spectrum ("red shift") if the star has a velocity component away from us, and makes light shift toward the blue end of the spectrum if the star has a velocity component toward us.

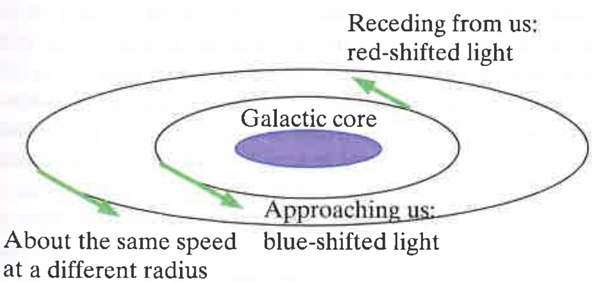

Figure 5.79

She found that for stars farther out from the center of the galaxy, the orbital speed of the star hardly changes with distance from the center of the galaxy, as is indicated in Figure 5.79. The visible components of the galaxy (stars, and illuminated clouds of dust) are most dense at the center of the galaxy and thin out rapidly as you move away from the center, so most of the visible mass is near the center. **(a)** Predict the speed v of a star going around the center of a galaxy in a circular orbit, as a function of the star's distance r from the center of the galaxy, assuming that almost all of the galaxy's mass M is concentrated at the center. **(b)** Construct a logical argument as to why Rubin concluded that much of the mass of a galaxy is not visible to us. Reason from principles discussed in this chapter, and your analysis of part (a). Explain your reasoning. You need to address the following issues: (i) Rubin's observations are not consistent with your prediction in (a). (ii) Most of the *visible* matter is in the center of the galaxy. (iii) Your prediction in (a) assumed that most of the mass is at the center.

This issue has not yet been resolved, and is still a current topic of astrophysics research. Here is a discussion by Rubin of her work: "Dark Matter in Spiral Galaxies" by Vera C. Rubin, *Scientific American*, June 1983 (96–108). You can find several graphs of the rotation curves for spiral galaxies on page 101 of this article.

Section 5.9

••**P49** The Ferris wheel in Figure 5.80 is a vertical, circular amusement ride with radius 10 m. Riders sit on seats that swivel to remain horizontal. The Ferris wheel rotates at a constant rate, going around once in 10.5 s. Consider a rider whose mass is 56 kg.

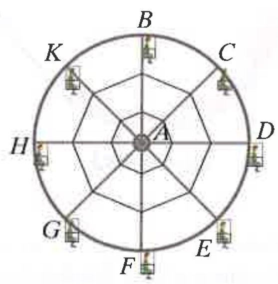

Figure 5.80

(a) At the bottom of the ride, what is the rate of change of the rider's momentum? **(b)** At the bottom of the ride, what is the vector gravitational force exerted by the Earth on the rider? **(c)** At the bottom of the ride, what is the vector force exerted by the seat on the rider? **(d)** Next consider the situation at the top of the ride. At the top of the ride, what is the rate of change of the rider's momentum? **(e)** At the top of the ride, what is the vector gravitational force exerted by the Earth on the rider? **(f)** At the top of the ride, what is the vector force exerted by the seat on the rider?

A rider *feels* heavier if the electric, interatomic contact force of the seat on the rider is larger than the rider's weight mg (and the rider sinks more deeply into the seat cushion). A rider *feels* lighter if the contact force of the seat is smaller than the rider's weight (and the rider does not sink as far into the seat cushion). **(g)** Does a rider *feel* heavier or lighter at the bottom of a Ferris wheel ride? **(h)** Does a rider *feel* heavier or lighter at the top of a Ferris wheel ride?

••**P50** By weight we usually mean the gravitational force exerted on an object by the Earth. However, when you sit in a chair your own perception of your own weight is based on the contact force the chair exerts upward on your rear end rather than on the gravitational force. The smaller this contact force is, the less weight you perceive, and if the contact force is zero, you feel peculiar and "weightless" (an odd word to describe a situation when the only force acting on you is the gravitational force exerted by the Earth!). Also, in this condition pressure on your inner ear is released, which affects your sense of balance, and your internal organs no longer press on each other, all of which contributes to the odd sensation in your stomach. **(a)** How fast must a roller coaster car go over the top of a circular arc for you to feel "weightless"? The center of the car moves along a circular arc of radius R (see Figure 5.81). Include a carefully labeled force diagram.

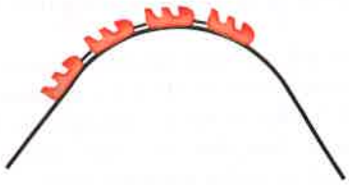

Figure 5.81

(b) How fast must a roller coaster car go through a circular dip for you to feel three times as "heavy" as usual, due to the upward force of the seat on your bottom being three times as large as usual? The center of the car moves along a circular arc of radius R (see Figure 5.82). Include a carefully labeled force diagram.

Figure 5.82

Section 5.10

•P51 A circular pendulum of length 1.1 m goes around at an angle of 28 degrees to the vertical. Predict the speed of the mass at the end of the string. Also predict the period, the time it takes to go around once. Remember that the radius of the circle is the length of the string times the sine of the angle that the string makes to the vertical.

••P52 Use a circular pendulum to determine g. You can increase the accuracy of the time it takes to go around once by timing N revolutions and then dividing by N. This minimizes errors contributed by inaccuracies in starting and stopping the clock. It is wise to start counting from zero (0, 1, 2, 3, 4, 5) rather than starting from 1 (1, 2, 3, 4, 5 represents only four revolutions, not five). It also improves accuracy if you start and stop timing at a well-defined event, such as when the mass crosses in front of an easily visible mark. This was the method used by Newton to get an accurate value of g. Newton was not only a brilliant theorist but also an excellent experimentalist. For a circular pendulum he built a large triangular wooden frame mounted on a vertical shaft, and he pushed this around and around while making sure that the string of the circular pendulum stayed parallel to the slanting side of the triangle.

••P53 **(a)** Many communication satellites are placed in a circular orbit around the Earth at a radius where the period (the time to go around the Earth once) is 24 h. If the satellite is above some point on the equator, it stays above that point as the Earth rotates, so that as viewed from the rotating Earth the satellite appears to be motionless. That is why you see dish antennas pointing at a fixed point in space. Calculate the radius of the orbit of such a "synchronous" satellite. Explain your calculation in detail. **(b)** Electromagnetic radiation including light and radio waves travels at a speed of 3×10^8 m/s. If a phone call is routed through a synchronous satellite to someone not very far from you on the ground, what is the minimum delay between saying something and getting a response? Explain. Include in your explanation a diagram of the situation. **(c)** Some human-made satellites are placed in "near-Earth" orbit, just high enough to be above almost all of the atmosphere. Calculate how long it takes for such a satellite to go around the Earth once, and explain any approximations you make. **(d)** Calculate the orbital speed for a near-Earth orbit, which must be provided by the launch rocket. (The advantages of near-Earth communications satellites include making the signal delay unnoticeable, but with the disadvantage of having to track the satellites actively and having to use many satellites to ensure that at least one is always visible over a particular region.) **(e)** When the first two astronauts landed on the Moon, a third astronaut remained in an orbiter in circular orbit near the Moon's surface. During half of every complete orbit, the orbiter was behind the Moon and out of radio contact with the Earth. On each orbit, how long was the time when radio contact was lost?

••P54 There is no general analytical solution for the motion of a three-body gravitational system. However, there do exist analytical solutions for very special initial conditions. Figure 5.83 shows three stars, each of mass m, which move in the plane of the page along a circle of radius r. Calculate how long this system takes to make one complete revolution. (In many cases three-body orbits are not stable: any slight perturbation leads to a breakup of the orbit.)

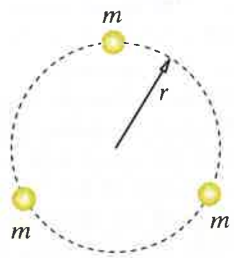

Figure 5.83

••P55 Remarkable data indicate the presence of a massive black hole at the center of our Milky Way galaxy. The W. M. Keck 10-m-diameter telescopes in Hawaii were used by Andrea Ghez and her colleagues to observe infrared light coming directly through the dust surrounding the central region of our galaxy (visible light is multiply-scattered by the dust, blocking a direct view). Stars were observed for several consecutive years to determine their orbits near a motionless center that is completely invisible ("black") in the infrared but whose precise location is known due to its strong output of radio waves, which are observed by radio telescopes. The data were used to show that the object at the center must have a mass that is huge compared to the mass of our own Sun, whose mass is a "mere" 2×10^{30} kg. Figure 5.84 shows positions from 1995 to 2012 of one of the stars, called S0-20, orbiting around the galactic center. The orbit appears nearly circular with the radius shown.

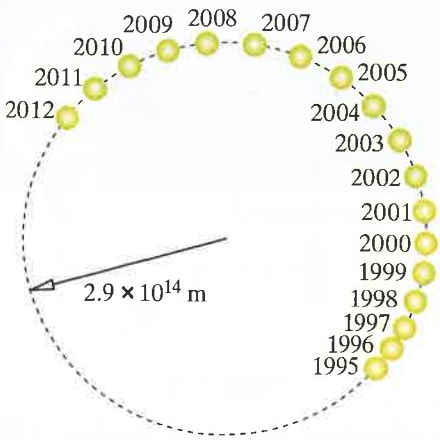

Figure 5.84 Positions on the sky of the star S0-20 near the center of our galaxy.

(a) Using the positions and times shown in Figure 5.84, what is the approximate speed of this star in m/s? Also express the speed as a fraction of the speed of light. **(b)** This is an extraordinarily high speed for a macroscopic object. Is it reasonable to approximate the star's momentum as mv? (Some other stars near the galactic center with highly elliptical orbits move even faster when they are closest to the center.) **(c)** Based on these data, estimate the mass of the massive black hole about which this star is orbiting. (You can see in Figure 5.84 that the speed appears greater in 2007–2008 than in 1995–1996. You're looking at a projection of an elliptical orbit, which only happens to look circular as viewed from Earth, and the black hole is not at the center of this circle. Therefore your result will be approximate and will differ from that obtained by Ghez and colleagues, who carefully made a consistent fit of the position data for seven different stars.) **(d)** How many of our Suns does this represent?

It is now thought that most galaxies have such a black hole at their centers, as a result of long periods of mass accumulation. When many bodies orbit each other, sometimes in an interaction an object happens to acquire enough speed to escape from the group, and the remaining objects are left closer together. Some simulations show that over time, as much as half the mass may be ejected, with agglomeration of the remaining mass. This could be part of the mechanism for forming massive black holes.

For more information, search the web for Andrea Ghez. You may see the term "arc seconds," which is an angular measure of how far apart objects appear in the sky, and "parsecs," which is a distance equal to 3.3 light-years (a light-year is the distance light goes in one year).

•••P56 You put a 10 kg object on a bathroom scale at the North Pole, and the scale reads exactly 10 kg (actually, it measures the force F_N that the scale exerts on the object, but displays a reading in kg). At the North Pole you are 6357 km from the center of the Earth. At the equator, the scale reads a different value due to two effects: (1) The Earth bulges out at the equator (due to its rotation), and you are 6378 km from the center of the Earth. (2) You are moving in a circular path due to the rotation of the Earth (one rotation every 24 hours). Taking into account ***both*** of these effects, what does the scale read at the equator?

COMPUTATIONAL PROBLEMS

More detailed and extended versions of some computational modeling problems may be found in the lab activities included in the *Matter & Interactions, 4th Edition,* resources for instructors.

••P57 Start with the program you wrote to model the motion of a spacecraft near the Earth (Chapter 3). Choose initial conditions that produce an elliptical orbit. **(a)** Create an arrow to represent the net force on the spacecraft, and place the tail of the arrow at the position of the spacecraft. Inside the calculation loop, update the arrow's position every time you update the craft's position. **(b)** Update the axis of the arrow inside the loop, so it always points in the direction of the net force. Find a scale factor that gives the arrow an appropriate length when the craft is near the Earth, and allows the arrow to be visible when the craft is far from the Earth. **(c)** Now create two additional arrows to represent $\vec{F}_{\parallel}$ and $\vec{F}_{\perp}$, the parallel and perpendicular parts of the net force on the spacecraft. In VPython you can take the dot product of two vectors, $\vec{A}$ and $\vec{B}$, like this: `C = dot(A,B)`. Make your program display $\vec{F}_{\text{net}}$, $\vec{F}_{\parallel}$, and $\vec{F}_{\perp}$ as the craft orbits the Earth. **(d)** In what part of the orbit does $\vec{F}_{\parallel}$ point in the same direction as $\vec{p}$? What effect does it have on the craft's momentum? **(e)** In what part of the orbit does $\vec{F}_{\parallel}$ point in the opposite direction to $\vec{p}$? What effect does it have on the craft's momentum? **(f)** Are there any locations in the orbit where $\vec{F}_{\parallel} = 0$? If so, what are they?

••P58 Start with the program you wrote to model the 3D motion of a mass hanging from a spring (Chapter 4). **(a)** Create an arrow to represent the net force on the mass, and place the tail of the arrow at the position of the mass. Update both the position and axis of the arrow inside the calculation loop, using an appropriate scale factor, so the arrow always shows the net force on the mass. **(b)** Now create two additional arrows to represent $\vec{F}_{\parallel}$ and $\vec{F}_{\perp}$, the parallel and perpendicular parts of the net force on the mass. Make your program display $\vec{F}_{\text{net}}$, $\vec{F}_{\parallel}$, and $\vec{F}_{\perp}$ as the mass oscillates. **(c)** Find initial conditions for which $\vec{F}_{\parallel}$ is zero and remains zero. Explain. **(d)** Find initial conditions for which $\vec{F}_{\perp}$ is zero and remains zero. Explain. **(e)** Find initial conditions for which both $\vec{F}_{\parallel}$ and $\vec{F}_{\perp}$ are nonzero most of the time. Explain.

•••P59 Start with the program you wrote to model the motion of a spacecraft near the Earth. Use initial conditions that produce an elliptical orbit. Modify the program so the kissing circle is continuously displayed as the craft orbits the Earth. You may want to use a `ring` object in VPython to display the circle.