PROBLEMS

Section 4.4

•P21 If in a certain material whose atoms are in a cubic array the interatomic distance is 1.7×10^{-10} m and the mass of one atom is 8.2×10^{-26} kg, what would be the density of this material?

•P22 A block of one mole of a certain material whose atoms are in a cubic array has dimensions of 5 cm by 4 cm by 0.5 cm. What is the interatomic distance?

•P23 The diameter of a copper atom is approximately 2.28×10^{-10} m. The mass of one mole of copper is 64 g. Assume that the atoms are arranged in a simple cubic array. Remember to convert to SI units. **(a)** What is the mass of one copper atom, in kg? **(b)** How many copper atoms are there in a cubical block of copper that is 4.6 cm on each side? **(c)** What is the mass of the cubical block of copper, in kg?

•P24 One mole of tungsten (6.02×10^{23} atoms) has a mass of 184 g, as shown in the periodic table on the inside front cover of the textbook. The density of tungsten is 19.3 g/cm^3. What is the approximate diameter of a tungsten atom (length of a bond) in a solid block of the material? Make the simplifying assumption that the atoms are arranged in a cubic array.

Section 4.5

•P25 If a chain of 50 identical short springs linked end to end has a stiffness of 270 N/m, what is the stiffness of one short spring?

•P26 A certain spring has stiffness 190 N/m. The spring is then cut into two equal lengths. What is the stiffness of one of these half-length springs?

•P27 Forty-five identical springs are placed side by side (in parallel) and connected to a large massive block. The stiffness of the 45-spring combination is 20,250 N/m. What is the stiffness of one of the individual springs?

•P28 A certain spring has stiffness 140 N/m. The spring is then cut into two equal lengths. What is the stiffness of one of these half-length springs?

•P29 Five identical springs, each with stiffness 390 N/m, are attached in parallel (that is, side by side) to hold up a heavy weight. If these springs were replaced by an equivalent single spring, what should be the stiffness of this single spring?

••P30 A hanging titanium wire with diameter 2 mm (2×10^{-3} m) is initially 3 m long. When a 5 kg mass is hung from it, the wire stretches an amount 0.4035 mm, and when a 10 kg mass is hung from it, the wire stretches an amount 0.807 mm. A mole of titanium has a mass of 48 grams, and its density is 4.51 grams/cm^3. Find the approximate value of the effective spring stiffness of the interatomic force, and explain your analysis.

••P31 A hanging copper wire with diameter 1.4 mm (1.4×10^{-3} m) is initially 0.95 m long. When a 36 kg mass is hung from it, the wire stretches an amount 1.83 mm, and when a 72 kg mass is hung from it, the wire stretches an amount 3.66 mm. A mole of copper has a mass of 63 g, and its density is 9 g/cm^3. Find the approximate value of the effective spring stiffness of the interatomic force.

••P32 One mole of tungsten (6.02×10^{23} atoms) has a mass of 184 g, and its density is 19.3 g/cm^3, so the center-to-center distance between atoms is 2.51×10^{-10} m. You have a long thin bar of tungsten, 2.5 m long, with a square cross section, 0.15 cm on a side. You hang the rod vertically and attach a 415 kg mass to the bottom, and you observe that the bar becomes 1.26 cm longer. From these measurements, it is possible to determine the stiffness of one interatomic bond in tungsten. **(a)** What is the spring stiffness of the entire wire, considered as a single macroscopic (large scale), very stiff spring? **(b)** How many side-by-side atomic chains (long springs) are there in this wire (Figure 4.55)? This is the same as the number of atoms on the bottom surface of the tungsten wire. Note that the cross-sectional area of one tungsten atom is $(2.51 \times 10^{-10})^2$ m^2.

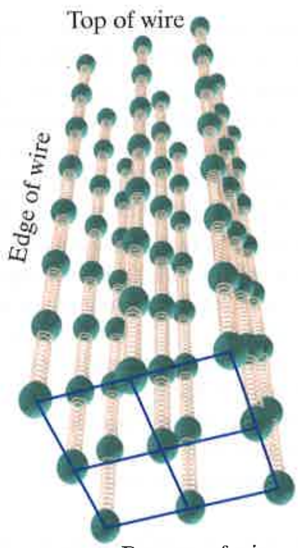

Figure 4.55

(c) How many interatomic bonds are there in one atomic chain running the length of the wire? **(d)** What is the stiffness of a single interatomic "spring"?

••P33 A hanging iron wire with diameter 0.08 cm is initially 2.5 m long. When a 52 kg mass is hung from it, the wire stretches an amount 1.27 cm. A mole of iron has a mass of 56 g, and its density is 7.87 g/cm^3. **(a)** What is the length of an interatomic bond in iron (diameter of one atom)? **(b)** Find the approximate value of the effective spring stiffness of one interatomic bond in iron.

Section 4.6

•P34 Steel is very stiff, and Young's modulus for steel is unusually large, 2×10^{11} N/m^2. A cube of steel 28 cm on a side supports a load of 85 kg that has the same horizontal cross section as the steel cube. **(a)** What is the magnitude of the normal force that the steel cube exerts on the load? **(b)** What is the compression of the steel cube? That is, what is the small change in height of the steel cube due to the load it supports? Give your answer as a positive number. The compression of a wide, stiff support can be extremely small.

•P35 Suppose that you are going to measure Young's modulus for three rods by measuring their stretch when they are suspended vertically and weights are hung from them (Figure 4.56).

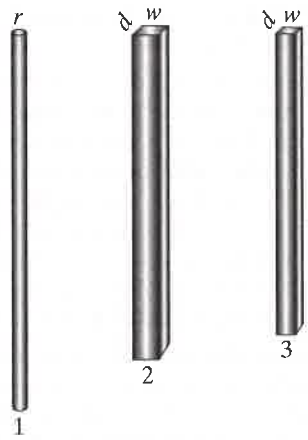

Figure 4.56

Rod 1 is 2.7 m long and cylindrical with radius 4 mm (1 mm is 0.001 m). Rod 2 is 3.2 m long by 12 mm wide by 6 mm deep. Rod 3 is 3 m long by 6 mm wide by 6 mm deep. The definition of Young's modulus, $Y = (F/A)/(\Delta L/L)$, includes the quantity A, the cross-sectional area. **(a)** What is the cross-sectional area of rod 1? **(b)** What is the cross-sectional area of rod 2? **(c)** What is the cross-sectional area of rod 3?

••P36 Young's modulus for aluminum is $6.2 \times 10^{10}\ \text{N/m}^2$. The density of aluminum is $2.7\ \text{g/cm}^3$, and the mass of one mole (6.02×10^{23} atoms) is 27 g. If we model the interactions of neighboring aluminum atoms as though they were connected by springs, determine the approximate spring constant of such a spring. Repeat this analysis for lead: Young's modulus for lead is $1.6 \times 10^{10}\ \text{N/m}^2$, the density of lead is $11.4\ \text{g/cm}^3$, and the mass of one mole is 207 g. Make a note of these results, which we will use for various purposes later on. Note that aluminum is a rather stiff material, whereas lead is quite soft.

••P37 Suppose that we hang a heavy ball with a mass of 10 kg (about 22 lb) from a steel wire 3 m long that is 3 mm in diameter. Steel is very stiff, and Young's modulus for steel is unusually large, $2 \times 10^{11}\ \text{N/m}^2$. Calculate the stretch ΔL of the steel wire. This calculation shows why in many cases it is a very good approximation to pretend that the wire doesn't stretch at all ("ideal nonextensible wire").

••P38 You hang a heavy ball with a mass of 14 kg from a gold wire 2.5 m long that is 2 mm in diameter. You measure the stretch of the wire, and find that the wire stretched 0.00139 m. **(a)** Calculate Young's modulus for the wire. **(b)** The atomic mass of gold is 197 g/mole, and the density of gold is $19.3\ \text{g/cm}^3$. Calculate the interatomic spring stiffness for gold.

••P39 A hanging wire made of an alloy of iron with diameter 0.09 cm is initially 2.2 m long. When a 66 kg mass is hung from it, the wire stretches an amount 1.12 cm. A mole of iron has a mass of 56 g, and its density is $7.87\ \text{g/cm}^3$. Based on these experimental measurements, what is Young's modulus for this alloy of iron?

•••P40 A certain coiled wire with uneven windings has the property that to stretch it an amount s from its relaxed length requires a force that is given by $F = bs^3$, so its behavior is different from a normal spring. You suspend this device vertically, and its unstretched length is 25 cm. **(a)** You hang a mass of 18 g from the device, and you observe that the length is now 29 cm. What is b, including units? **(b)** Which of the following were needed in your analysis in part (a)? (1) The Momentum Principle, (2) The fact that the gravitational force acting on an object near the Earth's surface is approximately mg, (3) The force law for an ordinary spring ($F = k_s s$), (4) The rate of change of momentum being zero **(c)** Next you take hold of the hanging 18 g mass and throw it straight downward, releasing it when the length of the device is 33 cm and the speed of the mass is 5 m/s. After a very short time, 0.0001 s later, what is the stretch of the device, and what was the change in the speed of the mass (including the correct sign of the change) during this short time interval? It helps enormously to draw a diagram showing the forces that act on the mass after it leaves your hand.

Section 4.7

••P41 Two blocks of mass m_1 and m_3, connected by a rod of mass m_2, are sitting on a low-friction surface, and you push to the left on the right block (mass m_1) with a constant force of magnitude F (Figure 4.57).

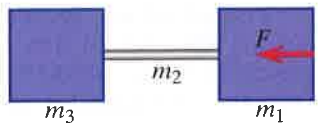

Figure 4.57

(a) What is the acceleration dv_x/dt of the blocks? **(b)** What is the vector force $\vec{F}_{\text{net}\,3}$ exerted by the rod on the block of mass m_3? ($|\vec{F}_{\text{net}\,3}|$ is approximately equal to the compression force in the rod near its left end.) What is the vector force $\vec{F}_{\text{net}\,1}$ exerted by the rod on the block of mass m_1? ($|\vec{F}_{\text{net}\,1}|$ is approximately equal to the compression force in the rod near its right end.) **(c)** Suppose that instead of pushing on the right block (mass m_1), you pull to the left on the left block (mass m_3) with a constant force of magnitude F. Draw a diagram illustrating this situation. Now what is the vector force $\vec{F}_{\text{net}\,3}$ exerted by the rod on the block of mass m_3?

Section 4.8

•P42 A 5 kg box with initial speed 4 m/s slides across the floor and comes to a stop after 0.7 s. **(a)** What is the coefficient of kinetic friction? **(b)** How far does the box move? **(c)** You put a 3 kg block in the box, so the total mass is now 8 kg, and you launch this heavier box with an initial speed of 4 m/s. How long does it take to stop?

•P43 A 3 kg block measures 5 cm by 10 cm by 20 cm. When it slides on a 10 cm by 20 cm face, it moves with constant speed when pulled horizontally by a force whose magnitude is 3 N. How big a horizontal force must be applied to pull it with constant speed if it slides on a 5 cm by 20 cm face?

•P44 A 15 kg box sits on a table. The coefficient of static friction μ_s between table and box is 0.3, and the coefficient of kinetic friction μ_k is 0.2. **(a)** What is the force required to start the box moving? **(b)** What is the force required to keep it moving at constant speed? **(c)** What is the force required to maintain an acceleration of 2 m/s/s?

•P45 A 20 kg box is being pushed across the floor by a constant force $\langle 90, 0, 0 \rangle$ N. The coefficient of kinetic friction for the table and box is 0.25. At $t = 5$ s the box is at location $\langle 8, 2, -1 \rangle$ m, traveling with velocity $\langle 3, 0, 0 \rangle$ m/s. What is its position and velocity at $t = 5.6$ s?

•P46 You drag a block across a table while a friend presses down on the block. The coefficient of friction between the table and the block is 0.6. The vertical component of the force exerted by the table on the block is 190 N. How big is the horizontal component of the force exerted by the table on the block?

••P47 For this problem you will need measurements of the position vs. time of a block sliding on a table, starting with some initial velocity, slowing down, and coming to rest. If you do not have an appropriate laboratory setup for making these measurements, your instructor will provide you with such data. Analyze these data to determine the coefficient of friction and to see how well they support the assertion that the force of sliding friction is essentially independent of the speed of sliding.

•••P48 It is sometimes claimed that friction forces always slow an object down, but this is not true. If you place a box of mass 8 kg on a moving horizontal conveyor belt, the friction force of the belt acting on the bottom of the box speeds up the box. At first there is some slipping, until the speed of the box catches up to the speed of the belt, which is 5 m/s. The coefficient of kinetic friction between box and belt is 0.6. **(a)** How much time does it take for the box to reach this final speed? **(b)** What is the distance (relative to the floor) that the box moves before reaching the final speed of 5 m/s?

Section 4.10

•••P49 A chain of length L and mass M is suspended vertically by one end with the bottom end just above a table. The chain is released and falls, and the links do not rebound off the table, but they spread out so that the top link falls very nearly the full distance L. Just before the instant when the entire chain has fallen onto the table, how much force does the table exert on the chain? Assume that the chain links have negligible interaction with each other as the chain drops, and make the approximation that there is a very large number of links. *Hint*: Consider the instantaneous rate of change of momentum of the chain as the last link hits the table.

Section 4.11

•P50 A ball whose mass is 1.4 kg is suspended from a spring whose stiffness is 4 N/m. The ball oscillates up and down with an amplitude of 14 cm. **(a)** What is the angular frequency ω? **(b)** What is the frequency? **(c)** What is the period? **(d)** Suppose this apparatus were taken to the Moon, where the strength of the gravitational field is only 1/6 of that on Earth. What would be the period on the Moon? (Consider carefully how the period depends on properties of the system; look at the equation.)

•P51 A mass of 2.2 kg is connected to a horizontal spring whose stiffness is 8 N/m. When the spring is relaxed, $x = 0$. The spring is stretched so that the initial value of $x = +0.18$ m. The mass is released from rest at time $t = 0$. Remember that when the argument of a trigonometric function is in radians, on a calculator you have to switch the calculator to radians or convert the radians to degrees. Predict the position x when $t = 1.15$ s.

•P52 A bouncing ball is an example of an anharmonic oscillator. If you quadruple the maximum height, what happens to the period? (Assume that the ball keeps returning almost to the same height.)

••P53 Here on Earth you hang a mass from a vertical spring and start it oscillating with amplitude 1.7 cm. You observe that it takes 2.1 s to make one round-trip. You construct another vertical oscillator with a mass 6 times as heavy and a spring 10 times as stiff. You take it to a planet where $g_{\text{planet}} = 6.8$ N/kg. You start it oscillating with amplitude 3.3 cm. How long does it take for the mass to make one round-trip?

••P54 In the approximation that the Earth is a sphere of uniform density, it can be shown that the gravitational force it exerts on a mass m inside the Earth at a distance r from the center is $mg(r/R)$, where R is the radius of the Earth. (Note that at the surface, the force is indeed mg, and at the center it is zero). Suppose that there were a hole drilled along a diameter straight through the Earth, and the air were pumped out of the hole. If an object is released from one end of the hole, how long will it take to reach the other side of the Earth? Include a numerical result.

••P55 A spring suspended vertically is 18 cm long. When you suspend a 30 g weight from the spring, at rest, the spring is 22 cm long. Next you pull down on the weight so the spring is 23 cm long and you release the weight from rest. What is the period of oscillation?

••P56 It was found that a 20 g mass hanging from a particular spring had an oscillation period of 1.2 s. **(a)** When two 20 g masses are hung from this spring, what would you predict for the period in seconds? Explain briefly.

Figure 4.58

(b) When one 20 g mass is supported by two of these vertical, parallel springs (Figure 4.58), what would you predict for the period in seconds? Explain briefly. **(c)** Suppose that you cut one spring into two equal lengths, and you hang one 20 g mass from this half spring. What would you predict for the period in seconds? Explain briefly. **(d)** Suppose that you take a single (full-length) spring and a single 20 g mass to the Moon and watch the system oscillate vertically there. Will the period you observe on the Moon be longer, shorter, or the same as the period you measured on Earth? (The gravitational field strength on the Moon is about one-sixth that on the Earth.) Explain briefly.

••P57 A vertical mass–spring oscillator has an amplitude of 0.06 m and a period of 0.4 s. **(a)** What is the maximum speed of the mass? **(b)** What is the maximum acceleration of the mass?

••P58 In Problem P36 you can find the effective spring stiffness corresponding to the interatomic force for aluminum and lead. Let's assume for the moment that, *very* roughly, other atoms have similar values. **(a)** What is the (very) approximate frequency f for the vibration of H_2, a hydrogen molecule? **(b)** What is the (very) approximate frequency f for the vibration of O_2, an oxygen molecule? **(c)** What is the approximate vibration frequency f of the molecule D_2, both of whose atoms are deuterium atoms (that is, each nucleus has one proton and one neutron)? **(d)** Explain why the *ratio* of the deuterium frequency to the hydrogen frequency is quite accurate, even though you have estimated each of these quantities very approximately, and the effective spring stiffness is normally expected to be significantly different for different atoms. (*Hint:* What interaction is modeled by the effective "spring"?)

••**P59** Find a spring (or a rubber band) and one or more masses with which to study the motion of a mass hanging from a spring (Figure 4.59). **(a)** Measure the value of the spring stiffness k_s of the spring in N/m. The magnitude of a spring force is $k_s s$, where s is the stretch of the spring (change from the unstretched length). Explain briefly how you measured k_s. Include in your report the unstretched length of the spring. **(b)** Measure the period (the round-trip time) of a mass hanging from the vertical spring. Report the mass that you use, and the amplitude of the oscillation. Amplitude is the maximum displacement, plus or minus, from the equilibrium position (the position where the mass can hang motionless). **(c)** With twice the amplitude that you reported in part (b), measure the period again. Since the mass has to move twice as far, one might expect the period to lengthen. Does it?

Figure 4.59

There are unavoidable fluctuations in starting and stopping the timing, but you can minimize the error this contributes by timing many complete cycles so that the starting and stopping fluctuations are a small fraction of the total time measured.

It is good practice to count out loud *starting from zero, not one.* To count five cycles, say out loud "Zero, one, two, three, four, five." If on the other hand you say "One, two, three, four, five," you have actually counted only four cycles, not five.

Since the motion is periodic, you can start (say "zero") at any point in the motion. It is best to start and stop when the mass is moving fast past some marker near the equilibrium point, because it is difficult to estimate the exact time when the mass reaches the very top or the very bottom, because it is moving slowly at those turnaround points. Be sure to measure full round-trip cycles, not the half-cycles between returns to the equilibrium point (but going in the opposite direction).

•••**P60** An object of mass m is attached by two stretched springs (stiffness k_s and relaxed length L_0) to rigid walls, as shown in Figure 4.60. The springs are initially stretched by an amount $(L - L_0)$. When the object is displaced to the right and released, it oscillates horizontally. Starting from the Momentum Principle, find a function of the displacement x of the object and the time t that describes the oscillatory motion. **(a)** What is the period of the motion? **(b)** If L_0 were shorter (so that the springs are initially stretched more), would the period be larger, smaller, or the same?

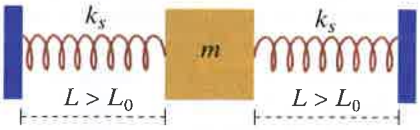

Figure 4.60

•••**P61** A simple pendulum (Figure 4.61) consists of a small mass of mass m swinging at the end of a low-mass string of length L (in contrast to a pendulum whose mass is distributed, such as a rod swinging from one end). **(a)** Show that the tangential momentum p of the small mass along the arc to the right (increasing θ) obeys the following equation, where s is the arc length $L\theta$:

$$\frac{dp}{dt} = -mg\sin\theta = -mg\sin\left(\frac{s}{L}\right)$$

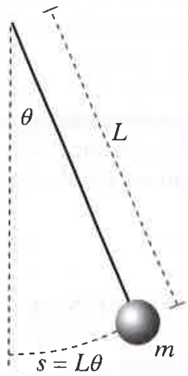

Figure 4.61

(b) What form does this equation take for small-amplitude swings? (Note that if θ is measured in radians, $\sin\theta \approx \theta$ for small angles. For example, even for an angle as large as $\theta = 30°$, $\theta = \pi/6 = 0.524$ radians, which is close to $\sin(30°) = 0.5$, and the approximation gets better for smaller angles.) **(c)** Compare the form of the approximate equation you obtained in part (b) with the form of the Momentum Principle for a spring–mass system. By comparing these equations for different systems, determine the period of a simple pendulum for small-amplitude swings. (Note that any time you can approximate the Momentum Principle for some system by an equation that looks like the equation for a mass on a spring, you know that the system will move approximately sinusoidally.) **(d)** Make a simple pendulum and predict its period, then measure the period. Do this for a long pendulum and for a short pendulum. Report your experimental data and results, with comparisons to your theoretical predictions.

Section 4.13

•**P62** Two metal rods are made of different elements. The interatomic spring stiffness of element A is three times larger than the interatomic spring stiffness for element B. The mass of an atom of element A is three times greater than the mass of an atom of element B. The atomic diameters are approximately the same for A and B. What is the ratio of the speed of sound in rod A to the speed of sound in rod B?

••**P63** You hang a heavy ball with a mass of 41 kg from a silver rod 2.6 m long by 1.5 mm by 3.1 mm. You measure the stretch of the rod, and find that the rod stretched 0.002898 m. Using these experimental data, what value of Young's modulus do you get? The atomic mass of silver is 108 g/mole, and the density of silver is 10.5 g/cm^3. Using this information along with the measured value of Young's modulus, calculate the speed of sound in silver.

••**P64** One mole of nickel (6.02×10^{23} atoms) has a mass of 59 g, and its density is 8.9 g/cm^3. You have a bar of nickel 2.5 m long, with a square cross section, 2 mm on a side. You hang the rod vertically and attach a 40 kg mass to the bottom, and you observe that the bar becomes 1.2 mm longer. Next you remove the 40 kg mass, place the rod horizontally, and strike one end with a hammer. How much time T will elapse before a microphone at the other end of the bar will detect a disturbance?

Section 4.14

•P65 It is hard to imagine that there can be enough air between a book and a table so that there is a net upward (buoyant) force on the book despite the large downward force on the top of the book. About how many air molecules are there between a textbook and a table, if there is an average distance of about 0.01 mm between the uneven surfaces of the book and table?

•P66 Calculate the buoyant force in air on a kilogram of lead (whose density is about $11\ \text{g/cm}^3$). The density of air is $1.3 \times 10^{-3}\ \text{g/cm}^3$. Remember that the buoyant force is equal to the weight of a volume of air that is equal to the volume of the object. (Compare with the weight mg of this much lead.)

•P67 Air pressure at the surface of a fresh water lake near sea level is about $1 \times 10^5\ \text{N/m}^2$. At approximately what depth below the surface does a diver experience a pressure of $3 \times 10^5\ \text{N/m}^2$? How would this be different in sea water, which has higher density than fresh water?

••P68 Here are two examples of floating objects: **(a)** A block of wood 20 cm long by 10 cm wide by 6 cm high has a density of $0.7\ \text{g/cm}^3$ and floats in water (whose density is $1.0\ \text{g/cm}^3$). How far below the surface of the water is the bottom of the block? Explain your reasoning. **(b)** An advertising blimp consists of a gas bag, the supporting structure for the gas bag, and the gondola hung from the bottom, with its cabin, engines, and propellers. The gas bag is about 30 m long with a diameter of about 10 m, and it is filled with helium (density about 4 g per 22.4 l under these conditions). Estimate the total mass of the blimp, including the helium.

COMPUTATIONAL PROBLEMS

More detailed and extended versions of some of these computational modeling problems may be found in the lab activities included in the *Matter & Interactions, 4th Edition*, resources for instructors.

••P69 Write an iterative computational model to predict and display the motion of a block of mass 60 g that sits on top of a vertical spring of stiffness 8 N/m and relaxed length 20 cm (Figure 4.62). **(a)** Use initial conditions that represent the block at rest on top of the spring, which has been pushed down until its total length is 10 cm. These values are the same as those used in an example in Chapter 2, Section 2.6, so you can use that example to check your work. **(b)** Add a graph of the y component of the block's position vs. time. What is the period of the block's motion? (It will be easiest to determine this if you stop the graph after two or three complete cycles. By holding down the mouse button while dragging over a VPython graph you will get crosshairs and a numerical readout of the coordinates.) **(c)** Double the stiffness of the spring. How does that affect the period of the motion? **(d)** Restore the stiffness of the spring to its original value, but double the mass. How does this affect the period of the motion?

Figure 4.62

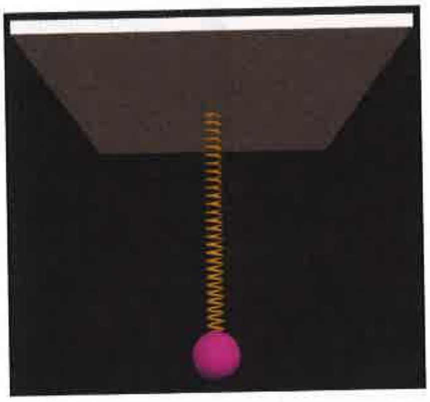

Figure 4.63

••P70 A ball of mass 20 g hangs from a spring whose relaxed length is 20 cm and whose stiffness is 0.9 N/m (Figure 4.63). **(a)** Write a VPython program to model the motion of this spring. **(b)** Add a graph of the y component of the ball's position vs. time. From the graph, determine the period of the ball's motion. (It will be easiest to determine this if you stop the graph after two or three complete cycles. By holding down the mouse button while dragging over a VPython graph you will get crosshairs and a numerical readout of the coordinates.) **(c)** How does doubling the mass affect the period of the oscillations? **(d)** How does keeping the mass unchanged but doubling the spring stiffness affect the period of the oscillations? **(e)** Make the ball leave a trail behind it as it moves. Find initial conditions that result in oscillations in all three dimensions. Rotate the display to make sure the motion is not planar.

••P71 Create a VPython program to model the motion of a 0.03 kg mass connected to two horizontal springs whose stiffness is 4 N/m, and whose relaxed length is 0.5 m, as shown in Figure 4.64. Neglect the effects of gravity (perhaps the system is in outer space). **(a)** Add a graph of x vs. t for the mass, and determine the period of the oscillator from this graph. It will be easiest to do this if you stop the graph after two or three complete cycles. If you hold down the mouse button while dragging over a VPython graph you will get crosshairs and a numerical readout of the coordinates. **(b)** Using the period you determined above, calculate the effective spring stiffness of this two-spring system. Is this the same as the spring stiffness you assigned to each spring in your model? Explain. **(c)** How does changing the amplitude of the oscillations affect the period?

Figure 4.64

••P72 On a space station in outer space a horizontal chain of two identical 3 kg masses and three identical springs of relaxed length 14 m and stiffness 50 N/m is suspended between two walls, as shown in Figure 4.65. Write a VPython program to model the motion of this system. **(a)** Start the motion by displacing the leftmost mass to the left, and observe the motion of the system. **(b)** Add a graph of x vs. t for each mass, making the graphs different colors so you can tell which is which. Is this system a simple harmonic oscillator? **(c)** Experiment with different initial conditions. Make a screen shot of a graph and report the initial conditions that led to this motion. **(d)** Find a set of initial conditions that results in simple sinusoidal motions of both masses. What are the periods of these motions? **(e)** (Optional) The pattern of motion you found in part (d) is called a "normal mode" of the system. This particular system has two such normal modes. Can you find initial conditions that produce the second? **(f)** (Optional) It is interesting to graph p.x for each mass as well as x. To do this you will need to make a separate gdisplay (graph window) for each graph. See the VPython help to learn how to do this. Explain how the plots of p.x and x are related.

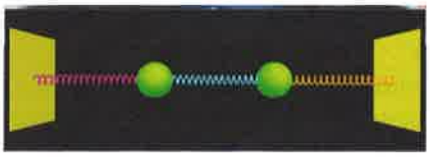

Figure 4.65

••P73 Create a computational model of a system like the one shown in Figure 4.66, in which a ball of mass 30 g is connected by six identical springs of stiffness 30 N/m and relaxed length 50 cm to six fixed walls. **(a)** Find initial conditions that make the system oscillate only along the x axis. **(b)** Find initial conditions that make the system oscillate in such a way that the ball doesn't travel in a straight line. Have the ball leave a trail to check your results. **(c)** Is this system a simple harmonic oscillator? Explain how you determined this.

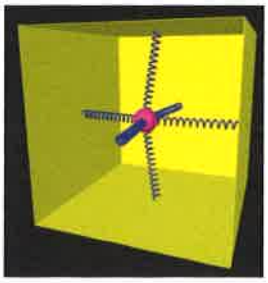

Figure 4.66

••P74 An object of mass m slides with negligible friction on a table. The object is connected to a post by a spring whose stiffness is k_s and whose relaxed length is L_0. Make a computational model of the situation and investigate the various kinds of motions that result from choosing different initial conditions and different values of k_s and L_0. Leave a trail behind the moving object to help in visualizing the motion (Figure 4.67). The program that generated Figure 4.67 used $m = 0.2$ kg, $k_s = 2$ N/m, and $L_0 = 0.16$ m.

Figure 4.67

••P75 A peculiar spring-like device exerts a force whose magnitude is $2 \times 10^9 \times s^7$ N, where s is the stretch in meters. The relaxed length is 0.1 m and the initial stretch is 0.02 m. The device is attached along the x axis to an object of mass 0.1 kg that is free to move with negligible friction on a horizontal surface. Write a program to model this device. A time step Δt of about 0.01 s is a reasonable choice. **(a)** Release the system from rest (the initial speed is zero) and display the motion as a function of time for the first 20 seconds. **(b)** Display on the same graph both the stretch s and the x component of velocity, using different colors. **(c)** How does the motion differ from that of an ordinary spring–mass system? **(d)** How does changing the initial stretch to 0.03 m affect the period of the system? Is this a harmonic oscillator?

•••P76 As explained in Problem P61, a simple pendulum consisting of a small mass of mass m swinging at the end of a low-mass string of length L behaves like a harmonic oscillator when the amplitude of the swings is small. For large-amplitude swings, the small-angle approximation discussed in Problem P61 is not valid, but you can use an iterative model to calculate the motion. For a mass on a string, the largest possible amplitude is 90°, but if the string is replaced by a lightweight rod, the amplitude can be as large as 180° (standing upside-down, with the mass at the top). It turns out that such a pendulum can be modeled by the same equation as the equation derived in part (a) of Problem P61, but the concepts of torque and angular momentum are required to prove it. **(a)** Assume that the equation is valid, and write a computational model to predict the motion of the pendulum. Think of s and p as being like x and p_x so that your program is basically a one-dimensional calculation, with a force $-mg\sin(s/L)$ instead of $-k_s s$. **(b)** Plot the position s and momentum p as a function of time for a pendulum whose amplitude is nearly 180°. **(c)** How does the period of this system depend on the amplitude? Is it a harmonic oscillator?