The center of mass

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{M_{total}}$$

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots}{M_{total}}$$

$$\vec{p}_{sys} = M_{total}\vec{v}_{cm}$$

Complex systems; determinism; chaos

QUESTIONS

Q1 Which fundamental interaction (gravitational, electromagnetic, strong, or weak) is responsible for each of these processes? How do you know? **(a)** A neutron outside a nucleus decays into a proton, electron, and antineutrino. **(b)** Protons and neutrons attract each other in a nucleus. **(c)** The Earth pulls on the Moon. **(d)** Protons in a nucleus repel each other.

Q2 Why is the value of the constant g different on Earth and on the Moon? Explain in detail.

Q3 You hold a tennis ball above your head, then open your hand and release the ball, which begins to fall. At this instant (ball starting to fall, no longer in contact with your hand), what can you conclude about the relative magnitudes of the force on the ball by the Earth and the force on the Earth by the ball? Explain.

Q4 Suppose that you are going to program a computer to carry out an iterative calculation of motion involving electric forces. Assume that as usual we use the final velocity in each time interval as the approximate average velocity during that interval. Which of the following calculations should be done *before* starting the repetitive calculation loop? Which of the calculations listed above should go *inside* the repetitive loop, and in what order?

(a) Define constants such as $\frac{1}{4\pi\varepsilon_0}$.
(b) Update the (vector) position of each object.
(c) Calculate the (vector) forces acting on the objects.
(d) Specify the initial (vector) momentum of each object.
(e) Specify an appropriate value for the time step.
(f) Specify the mass of each object.
(g) Update the (vector) momentum of each object.
(h) Specify the initial (vector) position of each object.

Q5 A bullet traveling horizontally at a very high speed embeds itself in a wooden block that is sitting at rest on a very slippery sheet of ice. You want to find the speed of the block just after the bullet embeds itself in the block. **(a)** What should you choose as the system to analyze? **(b)** Which of the following statements is true? (1) After the collision, the speed of the block with the bullet stuck in it is the same as the speed of the bullet before the collision. (2) The momentum of the block with the bullet stuck in it is the same as the momentum of the bullet before the collision. (3) The initial momentum of the bullet is greater than the momentum of the block with the bullet stuck in it.

Q6 You hang from a tree branch, then let go and fall toward the Earth. As you fall, the y component of your momentum, which was originally zero, becomes large and negative. **(a)** Choose yourself as the system. There must be an object in the surroundings whose y momentum must become equally large, and positive. What object is this? **(b)** Choose yourself and the Earth as the system. The y component of your momentum is changing. Does the total momentum of the system change? Why or why not?

Q7 One kind of radioactivity is called "alpha decay." For example, the nucleus of a radium-220 atom can spontaneously split into a radon-216 nucleus plus an alpha particle (a helium nucleus containing two protons and two neutrons). Consider a radium-220 nucleus that is initially at rest. It spontaneously decays, and the alpha particle travels off in the $+z$ direction. What can you conclude about the motion of the new radon-216 nucleus? Be as precise as you can, and explain your reasoning.

Q8 A bowling ball is initially at rest. A Ping-Pong ball moving in the $+z$ direction hits the bowling ball and bounces off it, traveling back in the $-z$ direction. Consider a time interval Δt extending from slightly before to slightly after the collision. **(a)** In this time interval, what is the sign of Δp_z for the system consisting of both balls? **(b)** In this time interval, what is the sign of Δp_z for the system consisting of the bowling ball alone?

Q9 The windshield of a speeding car hits a hovering insect. Consider the time interval from just before the car hits the insect to just after the impact. **(a)** For which choice of system is the change of momentum zero? **(b)** Is the magnitude of the change of momentum of the bug bigger than, the same as, or smaller than that of the car? **(c)** Is the magnitude of the change of velocity of the bug bigger than, the same as, or smaller than that of the car?

PROBLEMS

Section 3.2

•P10 At a particular instant the magnitude of the gravitational force exerted by a planet on one of its moons is 3×10^{23} N. If the mass of the moon were three times as large, what would be the magnitude of the force? If instead the distance between the moon and the planet were three times as large (no change in mass), what would be the magnitude of the force?

•P11 Masses M and m attract each other with a gravitational force of magnitude F. Mass m is replaced with a mass $3m$, and it is moved four times farther away. Now what is the magnitude of the force?

•P12 A 3 kg ball and a 5 kg ball are 2 m apart, center to center. What is the magnitude of the gravitational force that the 3 kg ball exerts on the 5 kg ball? What is the magnitude of the gravitational force that the 5 kg ball exerts on the 3 kg ball?

•P13 The mass of the Earth is 6×10^{24} kg, and the mass of the Moon is 7×10^{22} kg. At a particular instant the Moon is at location $\langle 2.8 \times 10^8, 0, -2.8 \times 10^8 \rangle$ m, in a coordinate system whose origin is at the center of the Earth. **(a)** What is $\vec{r}_{M-E}$, the relative position vector from the Earth to the Moon? **(b)** What is $|\vec{r}_{M-E}|$? **(c)** What is the unit vector $\hat{r}_{M-E}$? **(d)** What is the gravitational force exerted by the Earth on the Moon? Your answer should be a vector.

•P14 A star exerts a gravitational force of magnitude $|\vec{F}|$ on a planet. The distance between the star and the planet is r. If the planet were three times farther away (that is, if the distance between the bodies were $3r$), by what factor would the force on the planet due to the star change?

•P15 A planet exerts a gravitational force of magnitude 7×10^{22} N on a star. If the planet were four times closer to the star (that is, if the distance between the star and the planet were 1/4 what it is now), what would be the magnitude of the force on the star due to the planet?

•P16 A moon orbits a planet in the xy plane, as shown in Figure 3.58. You want to calculate the force on the moon by the planet at each location labeled by a letter (A, B, C, D). At each of these locations, what are: **1.** the unit vector $\hat{r}$, **2.** the unit vector $\hat{F}$ in the direction of the force?

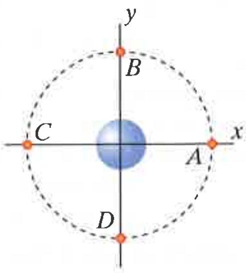

Figure 3.58

•P17 The mass of the Sun is 2×10^{30} kg, and the mass of Mercury is 3.3×10^{23} kg. The distance from the Sun to Mercury is 4.8×10^{10} m. **(a)** Calculate the magnitude of the gravitational force exerted by the Sun on Mercury. **(b)** Calculate the magnitude of the gravitational force exerted by Mercury on the Sun.

•P18 Measurements show that Jupiter's gravitational force on a mass of 1 kg near Jupiter's surface would be 24.9 N. The radius of Jupiter is 71500 km (71.5×10^6 m). From these data determine the mass of Jupiter.

•P19 A planet of mass 4×10^{24} kg is at location $\langle 5e11, -2e11, 0 \rangle$ m. A star of mass 5×10^{30} kg is at location $\langle -2e11, 3e11, 0 \rangle$ m. It will be useful to draw a diagram of the situation, including the relevant vectors. **(a)** What is the relative position vector $\vec{r}$ pointing from the planet to the star? **(b)** What is the distance between the planet and the star? **(c)** What is the unit vector $\hat{r}$ in the direction of $\vec{r}$? **(d)** What is the magnitude of the force exerted on the planet by the star? **(e)** What is the magnitude of the force exerted on the star by the planet? **(f)** What is the (vector) force exerted on the planet by the star? **(g)** What is the (vector) force exerted on the star by the planet?

•P20 A planet of mass 4×10^{24} kg is at location $\langle -6e11, 3e11, 0 \rangle$ m. A star of mass 6×10^{30} kg is at location $\langle 5e11, -3e11, 0 \rangle$ m. What is the force exerted on the planet by the star? (It will probably be helpful to draw a diagram, including the relevant vectors.)

••P21 Two copies of this textbook are standing right next to each other on a bookshelf. Make a rough estimate of the magnitude of the gravitational force that the books exert on each other. Explicitly list all quantities that you had to estimate, and all simplifications and approximations you had to make to do this calculation. Compare your result to the gravitational force on the book by the Earth.

Section 3.3

•P22 The mass of Mars is 6.4×10^{23} kg and its radius is 3.4×10^6 m. What is the value of the constant g on Mars?

•P23 At what height above the surface of the Earth is there a 1% difference between the approximate magnitude of the gravitational field (9.8 N/kg) and the actual magnitude of the gravitational field at that location? That is, at what height y above the Earth's surface is $GM_E/(R_E + y)^2 = 0.99 GM_E/R_E^2$?

•P24 Calculate the approximate gravitational force exerted by the Earth on a human standing on the Earth's surface. Compare with the approximate gravitational force of a human on another human at a distance of 3 m. What approximations or simplifying assumptions must you make?

•••P25 A steel ball of mass m falls from a height h onto a scale calibrated in newtons. The ball rebounds repeatedly to nearly the same height h. The scale is sluggish in its response to the intermittent hits and displays an *average* force F_{avg}, such that $F_{avg}T = F\Delta t$, where $F\Delta t$ is the brief impulse that the ball imparts to the scale on every hit, and T is the time between hits.

Calculate this average force in terms of m, h, and physical constants. Compare your result with the scale reading if the ball merely rests on the scale. Explain your analysis carefully (but briefly).

Section 3.5

•P26 The mass of the Sun is 2×10^{30} kg, the mass of the Earth is 6×10^{24} kg, and their center-to-center distance is 1.5×10^{11} m. Suppose that at some instant the Sun's momentum is zero (it's at rest). Ignoring all effects but that of the Earth, what will the Sun's speed be after one day? (Very small changes in the velocity of a star can be detected using the "Doppler" effect, a change in the frequency of the starlight, which has made it possible to identify the presence of planets in orbit around a star.)

••P27 At $t = 532.0$ s after midnight, a spacecraft of mass 1400 kg is located at position $\langle 3 \times 10^5, 7 \times 10^5, -4 \times 10^5 \rangle$ m, and at that time an asteroid whose mass is 7×10^{15} kg is located at position $\langle 9 \times 10^5, -3 \times 10^5, -12 \times 10^5 \rangle$ m. There are no other objects nearby. **(a)** Calculate the (vector) force acting on the spacecraft. **(b)** At $t = 532.0$ s the spacecraft's momentum was $\vec{p}_i$, and at the later time $t = 538.0$ s its momentum was $\vec{p}_f$. Calculate the (vector) change of momentum $\vec{p}_f - \vec{p}_i$.

••P28 **(a)** In outer space, far from other objects, block 1 of mass 40 kg is at position $\langle 7, 11, 0 \rangle$ m, and block 2 of mass 1000 kg is at position $\langle 18, 11, 0 \rangle$ m. What is the (vector) gravitational force acting on block 2 due to block 1? It helps to make a sketch of the situation. **(b)** At 4.6 s after noon both blocks were at rest at the positions given above. At 4.7 s after noon, what is the (vector) momentum of block 2? **(c)** At 4.7 s after noon, what is the (vector) momentum of block 1? **(d)** At 4.7 s after noon, which one of the following statements is true? A. Block 1 and block 2 have the same speed. B. Block 2 is moving faster than block 1. C. Block 1 is moving faster than block 2.

••P29 A star of mass 7×10^{30} kg is located at $\langle 5 \times 10^{12}, 2 \times 10^{12}, 0 \rangle$ m. A planet of mass 3×10^{24} kg is located at $\langle 3 \times 10^{12}, 4 \times 10^{12}, 0 \rangle$ m and is moving with a velocity of $\langle 0.3 \times 10^4, 1.5 \times 10^4, 0 \rangle$ m/s. **(a)** At a time 1×10^6 s later, what is the new velocity of the planet? **(b)** Where is the planet at this later time? **(c)** Explain briefly why the procedures you followed in parts (a) and (b) were able to produce usable results but wouldn't work if the later time had been 1×10^9 s instead of 1×10^6 s after the initial time. Explain briefly how you could use a computer to get around this difficulty.

••P30 At $t = 0$ a star of mass 4×10^{30} kg has velocity $\langle 7 \times 10^4, 6 \times 10^4, -8 \times 10^4 \rangle$ m/s and is located at $\langle 2.00 \times 10^{12}, -5.00 \times 10^{12}, 4.00 \times 10^{12} \rangle$ m relative to the center of a cluster of stars. There is only one nearby star that exerts a significant force on the first star. The mass of the second star is 3×10^{30} kg, its velocity is $\langle 2 \times 10^4, -1 \times 10^4, 9 \times 10^4 \rangle$ m/s, and this second star is located at $\langle 2.03 \times 10^{12}, -4.94 \times 10^{12}, 3.95 \times 10^{12} \rangle$ m relative to the center of the cluster of stars. **(a)** At $t = 1 \times 10^5$ s, what is the approximate momentum of the first star? **(b)** Discuss briefly some ways in which your result for (a) is approximate, not exact. **(c)** At $t = 1 \times 10^5$ s, what is the approximate position of the first star? **(d)** Discuss briefly some ways in which your result for (b) is approximate, not exact.

•••P31 In June 1997 the NEAR spacecraft ("Near Earth Asteroid Rendezvous"; see http://near.jhuapl.edu/), on its way to photograph the asteroid Eros, passed within 1200 km of asteroid Mathilde at a speed of 10 km/s relative to the asteroid. (See Figure 3.59.) From photos transmitted by the 805 kg spacecraft, Mathilde's size was known to be about 70 km by 50 km by 50 km. The asteroid is presumably made of rock. Rocks on Earth have a density of about 3000 kg/m^3 (3 grams/cm^3). **(a)** Make a rough diagram to show qualitatively the effect on the spacecraft of this encounter with Mathilde. Explain your reasoning. **(b)** Make a very rough estimate of the change in momentum of the spacecraft that would result from encountering Mathilde. Explain how you made your estimate. **(c)** Using your result from part (b), make a rough estimate of how far off course the spacecraft would be, one day after the encounter. **(d)** From actual observations of the location of the spacecraft one day after encountering Mathilde, scientists concluded that Mathilde is a loose arrangement of rocks, with lots of empty space inside. What was it about the observations that must have led them to this conclusion?

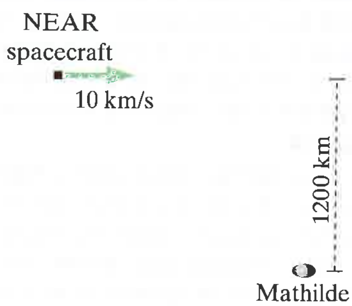

Figure 3.59

Experimental background: The position was tracked by very accurate measurements of the time that it takes for a radio signal to go from Earth to the spacecraft followed immediately by a radio response from the spacecraft being sent back to Earth. Radio signals, like light, travel at a speed of 3×10^8 m/s, so the time measurements had to be accurate to a few nanoseconds ($1 \text{ ns} = 1 \times 10^{-9}$ s).

Section 3.7

•P32 Figure 3.60 shows two positively charged objects (with the same charge) and one negatively charged object. What is the direction of the net electric force on the negatively charged object? If the net force is zero, state this explicitly.

Figure 3.60

•P33 Figure 3.61 shows two negatively charged objects (with the same charge) and one positively charged object. What is the direction of the net electric force on the positively charged object? If the net force is zero, state this explicitly.

Figure 3.61

•P34 The left side of Figure 3.62 shows a proton and an electron. **(a)** What is the direction of the electric force on the electron by the proton? **(b)** What is the direction of the electric force on the proton by the electron? **(c)** How do the magnitudes of these forces compare? The right side of Figure 3.62 shows two electrons. **(d)** What is the direction of the electric force on electron A due to electron B? **(d)** What is the direction of the electric force on electron B due to electron A? **(d)** How do the magnitudes of these forces compare?

Figure 3.62

•P35 An alpha particle contains two protons and two neutrons, and has a net charge of $+2e$. The alpha particle is 1 mm away from a single proton, which has a charge of $+e$. Which statement about the magnitudes of the electric forces between the particles is correct? **(a)** The force on the proton by the alpha particle is larger than the force on the alpha particle by the proton. **(b)** The force on the alpha particle by the proton is larger than the force on the proton by the alpha particle. **(c)** The forces are equal in magnitude. **(d)** Not enough information is given.

••P36 A proton and an electron are separated by 1×10^{-10} m, the radius of a typical atom. Calculate the magnitude of the electric force that the proton exerts on the electron, and the magnitude of the electric force that the electron exerts on the proton.

••P37 Two thin hollow plastic spheres, about the size of a Ping-Pong ball with masses ($m_1 = m_2 = 2 \times 10^{-3}$ kg), have been rubbed with wool. Sphere 1 has a charge $q_1 = -2 \times 10^{-9}$ C and is at location $\langle 0.50, -0.20, 0\rangle$ m. Sphere 2 has a charge $q_2 = -4 \times 10^{-9}$ C and is at location $\langle -0.40, 0.40, 0\rangle$ m. It will be useful to draw a diagram of the situation, including the relevant vectors. **(a)** What is the relative position vector $\vec{r}$ pointing from q_1 to q_2? **(b)** What is the distance between q_1 and q_2? **(c)** What is the unit vector $\hat{r}$ in the direction of $\vec{r}$? **(d)** What is the magnitude of the gravitational force exerted on q_2 by q_1? **(e)** What is the (vector) gravitational force exerted on q_2 by q_1? **(f)** What is the magnitude of the electric force exerted on q_2 by q_1? **(g)** What is the (vector) electric force exerted on q_2 by q_1? **(h)** What is the ratio of the magnitude of the electric force to the magnitude of the gravitational force? **(i)** If the two masses were four times farther away (that is, if the distance between the masses were $4\vec{r}$), what would be the ratio of the magnitude of the electric force to the magnitude of the gravitational force now?

••**P38** A proton is located at $\langle 0, 0, -2 \times 10^{-9}\rangle$ m, and an alpha particle (consisting of two protons and two neutrons) is located at $\langle 1.5 \times 10^{-9}, 0, 2 \times 10^{-9}\rangle$ m. **(a)** Calculate the force the proton exerts on the alpha particle. **(b)** Calculate the force the alpha particle exerts on the proton.

••**P39** Use data from the inside back cover to calculate the gravitational and electric forces two electrons exert on each other when they are 1×10^{-10} m apart (about one atomic radius). Which interaction between two electrons is stronger, the gravitational attraction or the electric repulsion? If the two electrons are at rest, will they begin to move toward each other or away from each other? Note that since both the gravitational and electric forces depend on the inverse square distance, this comparison holds true at all distances, not just at a distance of 1×10^{-10} m.

••**P40** At a particular instant a proton exerts an electric force of $\langle 0, 5.76 \times 10^{-13}, 0\rangle$ N on an electron. How far apart are the proton and the electron?

Section 3.10

•**P41** Two balls of mass 0.3 kg and 0.5 kg are connected by a low-mass spring (Figure 3.63). This device is thrown through the air with low speed, so air resistance is negligible. The motion is complicated: the balls whirl around each other, and at the same time the system vibrates, with continually changing stretch of the spring. At a particular instant, the 0.3 kg ball has a velocity $\langle 4, -3, 2\rangle$ m/s and the 0.5 kg ball has a velocity $\langle 2, 1, 4\rangle$ m/s. **(a)** At this instant, what is the total momentum of the device? **(b)** What is the net gravitational (vector) force exerted by the Earth on the device? **(c)** At a time 0.1 s later, what is the total momentum of the device?

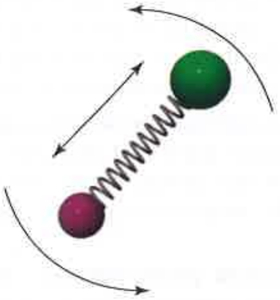

Figure 3.63

Section 3.11

•**P42** At a certain instant object 1 is at location $\langle 10, -8, 6\rangle$ m, moving with velocity $\langle 4, 6, -2\rangle$ m/s. At the same instant object 2 is at location $\langle 3, 0, -2\rangle$ m, moving with velocity $\langle -8, 2, 7\rangle$ m/s. **(a)** What is the location of the center of mass of the two equal-mass objects? **(b)** What is the velocity of the center of mass?

•**P43** The mass of the Earth is 6×10^{24} kg, the mass of the Moon is 7×10^{22} kg, and the center-to-center distance is 4×10^{8} m. How far from the center of the Earth is the center of mass of the Earth–Moon system? Note that the Earth's radius is 6.4×10^{6} m.

•**P44** The mass of the Sun is 2×10^{30} kg, the mass of the Earth is 6×10^{24} kg, and the center-to-center distance is 1.5×10^{11} m. How far from the center of the Sun is the center of mass of the Sun–Earth system? Note that the Sun's radius is 7×10^{8} m.

••**P45** Two rocks are tied together with a string of negligible mass and thrown into the air. At a particular instant, rock 1, which has a mass of 0.1 kg, is headed straight up with a speed of 5 m/s, and rock 2, which has a mass of 0.25 kg, is moving parallel to the ground, in the $+x$ direction, with a speed of 7.5 m/s. **(a)** What is the total momentum of the system consisting of both rocks and the string? **(b)** What is the velocity of the center of mass of the system?

••**P46** A tennis ball of mass 0.06 kg traveling at a velocity of $\langle 9, -2, 13\rangle$ m/s is about to collide with an identical tennis ball whose velocity is $\langle 4, 5, -10\rangle$ m/s. **(a)** What is the total momentum of the system of the two tennis balls? **(b)** What is the velocity of the center of mass of the two tennis balls?

Section 3.12

•**P47** In outer space, far from other objects, two rocks collide and stick together. Before the collision their momenta were $\langle -10, 20, -5\rangle$ kg·m/s and $\langle 8, -6, 12\rangle$ kg·m/s. What was their total momentum before the collision? What must be the momentum of the combined object after the collision?

•**P48** When they are far apart, the momentum of a proton is $\langle 3.4 \times 10^{-21}, 0, 0\rangle$ kg·m/s as it approaches another proton that is initially at rest. The two protons repel each other electrically, without coming close enough to touch. When they are once again far apart, one of the protons now has momentum $\langle 2.4 \times 10^{-21}, 1.55 \times 10^{-21}, 0\rangle$ kg·m/s. At that instant, what is the momentum of the other proton?

••**P49** You and a friend each hold a lump of wet clay. Each lump has a mass of 30 g. You each toss your lump of clay into the air, where the lumps collide and stick together. Just before the impact, the velocity of one lump was $\langle 3, 3, -3\rangle$ m/s, and the velocity of the other lump was $\langle -3, 0, -3\rangle$ m/s. **(a)** What was the total momentum of the lumps just before the impact? **(b)** What is the momentum of the stuck-together lump just after the collision? **(c)** What is the velocity of the stuck-together lump just after the collision?

••**P50** A car of mass 2800 kg collides with a truck of mass 4700 kg, and just after the collision the car and truck slide along, stuck together. The car's velocity just before the collision was $\langle 40, 0, 0\rangle$ m/s, and the truck's velocity just before the collision was $\langle -14, 0, 29\rangle$ m/s. **(a)** What is the velocity of the stuck-together car and truck just after the collision? **(b)** In your analysis in part (a), why can you neglect the effect of the force of the road on the car and truck?

••**P51** A car of mass 2045 kg moving in the x direction at a speed of 29 m/s strikes a hovering mosquito of mass

2.5 mg, and the mosquito is smashed against the windshield. The interaction between the mosquito and the windshield is an electric interaction between the electrons and protons in the mosquito and those in the windshield. **(a)** What is the approximate momentum change of the mosquito? Give magnitude and direction. Explain any approximations you make. **(b)** At a particular instant during the impact, when the force exerted on the mosquito by the car is F, what is the magnitude of the force exerted on the car by the mosquito? **(c)** What is the approximate momentum change of the car? Give magnitude and direction. Explain any approximations you make. **(d)** Qualitatively, why is the collision so much more damaging to the mosquito than to the car?

••**P52** A bullet of mass 0.105 kg traveling horizontally at a speed of 300 m/s embeds itself in a block of mass 2 kg that is sitting at rest on a nearly frictionless surface. What is the speed of the block after the bullet embeds itself in the block?

••**P53** Object A has mass $m_A = 8$ kg and initial momentum $\vec{p}_{A,i} = \langle 20,-5,0 \rangle$ kg·m/s, just before it strikes object B, which has mass $m_B = 11$ kg. Just before the collision object B has initial momentum $\vec{p}_{B,i} = \langle 5,6,0 \rangle$ kg·m/s. **(a)** Consider a system consisting of both objects A and B. What is the total initial momentum of this system just before the collision? **(b)** The forces that A and B exert on each other are very large but last for a very short time. If we choose a time interval from just before to just after the collision, what is the approximate value of the impulse applied to the two-object system due to forces exerted on the system by objects outside the system? **(c)** Therefore, what does the Momentum Principle predict that the total final momentum of the system will be just after the collision? **(d)** Just after the collision, object A is observed to have momentum $\vec{p}_{A,f} = \langle 18,5,0 \rangle$ kg·m/s. What is the momentum of object B just after the collision?

••**P54** In outer space a small rock with mass 5 kg traveling with velocity $\langle 0,1800,0 \rangle$ m/s strikes a stationary large rock head-on and bounces straight back with velocity $\langle 0,-1500,0 \rangle$ m/s. After the collision, what is the vector momentum of the large rock?

••**P55** Two rocks collide in outer space. Before the collision, one rock had mass 9 kg and velocity $\langle 4100,-2600,2800 \rangle$ m/s. The other rock had mass 6 kg and velocity $\langle -450,1800,3500 \rangle$ m/s. A 2 kg chunk of the first rock breaks off and sticks to the second rock. After the collision the 7 kg rock has velocity $\langle 1300,200,1800 \rangle$ m/s. After the collision, what is the velocity of the other rock, whose mass is 8 kg?

••**P56** Two rocks collide with each other in outer space, far from all other objects. Rock 1 with mass 5 kg has velocity $\langle 30,45,-20 \rangle$ m/s before the collision and $\langle -10,50,-5 \rangle$ m/s after the collision. Rock 2 with mass 8 kg has velocity $\langle -9,5,4 \rangle$ m/s before the collision. Calculate the final velocity of rock 2.

••**P57** In outer space two rocks collide and stick together. Here are the masses and initial velocities of the two rocks:

Rock 1: mass = 15 kg, initial velocity = $\langle 10,-30,0 \rangle$ m/s
Rock 2: mass = 32 kg, initial velocity = $\langle 15,12,0 \rangle$ m/s

What is the velocity of the stuck-together rocks after colliding?

••**P58** A bullet of mass m traveling horizontally at a very high speed v embeds itself in a block of mass M that is sitting at rest on a nearly frictionless surface. What is the speed of the block after the bullet embeds itself in the block?

••**P59** A satellite that is spinning clockwise has four low-mass solar panels sticking out as shown. A tiny meteor traveling at high speed rips through one of the solar panels and continues in the same direction but at reduced speed. Afterward, calculate the v_x and v_y components of the center-of-mass velocity of the satellite. In Figure 3.64 $\vec{v}_1$, and $\vec{v}_2$ are the initial and final velocities of the meteor, and $\vec{v}$ is the initial velocity of the center of mass of the satellite, in the x direction.

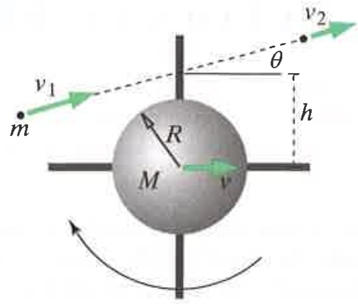

Figure 3.64

••**P60** A tiny piece of space junk of mass m strikes a glancing blow to a spinning satellite. Before the collision the satellite was moving and rotating as shown in Figure 3.65. After the collision the space junk is traveling in a new direction and moving more slowly. The velocities of the space junk before and after the collision are shown in the diagram. The satellite has mass M and radius R. Just after the collision, what are the components of the center-of-mass velocity of the satellite (v_x and v_y)?

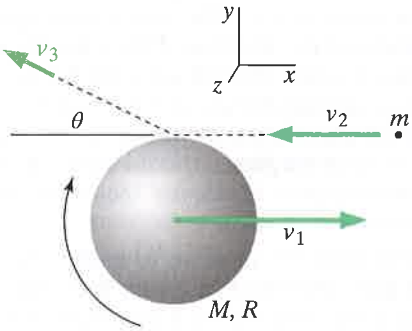

Figure 3.65

••**P61** A space station has the form of a hoop of radius R, with mass M. Initially its center of mass is not moving, but it is spinning. Then a small package of mass m is thrown by a spring-loaded gun toward a nearby spacecraft as shown in Figure 3.66; the package has a speed v after launch. Calculate the center-of-mass velocity (a vector) of the space station after the launch.

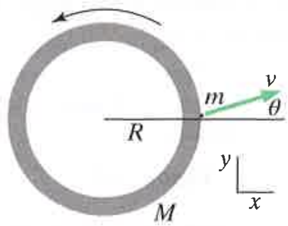

Figure 3.66

••P62 A ball of mass 0.05 kg moves with a velocity $\langle 17,0,0\rangle$ m/s. It strikes a ball of mass 0.1 kg that is initially at rest. After the collision, the heavier ball moves with a velocity of $\langle 3,3,0\rangle$ m/s. **(a)** What is the velocity of the lighter ball after impact? **(b)** What is the impulse delivered to the 0.05 kg ball by the heavier ball? **(c)** If the time of contact between the balls is 0.03 s, what is the force exerted by the heavier ball on the lighter ball?

••P63 Suppose that all the people of the Earth go to the North Pole and, on a signal, all jump straight up. Estimate the recoil speed of the Earth. The mass of the Earth is 6×10^{24} kg, and there are about 6 billion people (6×10^{9}).

COMPUTATIONAL PROBLEMS

More detailed and extended versions of some of these computational modeling problems may be found in the lab activities included in the *Matter & Interactions*, Fourth Edition, resources for instructors.

••P64 To use an `arrow` object to visualize a force, it is usually necessary to scale the length of the arrow in order to make it fit on the screen with the objects exerting and experiencing the force. Watch VPython Instructional Video 5: Scalefactors, at vpython.org/video05.html to learn how to do this.

Now, write a VPython program to do the following: **(a)** Create a sphere representing a planet at the origin, with radius 6.4×10^{6} m. The mass of this planet is 6×10^{24} kg. **(b)** Create five spheres representing 5 spacecraft, at locations $\langle -13\times10^{7}, 6.5\times10^{7}, 0\rangle$ m, $\langle -6.5\times10^{7}, 6.5\times10^{7}, 0\rangle$ m, $\langle 0, 6.5\times10^{7}, 0\rangle$ m, $\langle 6.5\times10^{7}, 6.5\times10^{7}, 0\rangle$ m, and $\langle 13\times10^{7}, 6.5\times10^{7}, 0\rangle$ m. You will have to exaggerate the radius of each spacecraft to make it visible; try 3×10^{6} m. The mass of each spacecraft is 15×10^{3} kg. **(c)** For each spacecraft, have your program calculate the gravitational force exerted on the spacecraft by the planet, and visualize it with an arrow whose tail is at the center of the spacecraft. Use the same scalefactor for all arrows. Check that the display your program produces makes physical sense. **(d)** For each spacecraft, have your program calculate the gravitational force exerted on the planet by the spacecraft, and visualize it with an arrow whose tail is at the center of the planet. Use the same scalefactor as you used in the previous part. Check that the display your program produces makes physical sense.

••P65 Write a computer program to model the motion of a spacecraft of mass 15000 kg that is launched from a location 10 Earth radii from the center of the Earth (Figure 3.67). (Data for the Earth are given on the inside back cover of the textbook. Start with a Δt of 60 s and an initial speed of 2×10^{3} m/s in a direction perpendicular to the line between the spacecraft and the Earth.) **(a)** Vary the initial speed (but not the direction), and have the spacecraft leave a trail. What trajectories can you produce? **(b)** Find an initial speed that produces an elliptical orbit. **(c)** For the elliptical orbit, display arrows indicating the directions of the momentum of the spacecraft and the net force on the spacecraft as it moves. **(d)** Find an initial speed that produces a circular orbit. **(e)** Experiment by increasing and decreasing the time step Δt. What is the largest value of Δt that gives enough accuracy to produce a closed circular orbit?

Figure 3.67

••P66 Extend the program you wrote in the previous problem by including the effect of the Moon, placing the Moon on the opposite side of the Earth from the spacecraft's initial location (Figure 3.68). (Relevant data are given on the inside back cover of the textbook.) To simplify the model, keep both the Earth and the Moon fixed. (This is called a "restricted three-body problem.") **(a)** Find an initial speed for the spacecraft that results in an orbit around both Earth and Moon. **(b)** By adjusting the initial speed of the spacecraft, can you produce a figure-eight trajectory? **(c)** What other interesting trajectories can you produce by varying the initial speed? (Small variations may produce large effects.)

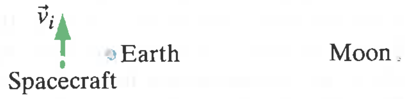

Figure 3.68

•••P67 Once you have completed the previous problem, allow the Earth and Moon to move. Use what you know about the period of their orbits to determine appropriate initial velocities for the Earth and the Moon.

••P68 Model the motion of Mars around the Sun. You can find data for masses and distances online. Use the known period of Mars's orbit to determine an approximate initial velocity. Display a trail, so you can see the shape of the orbit. Determine an appropriate value for Δt. Display arrows representing the momentum of Mars and the net force on Mars. Determine the period of the orbit of the planet in your model. How can you produce noncircular orbits?

••P69 About a half of the visible "stars" are actually systems consisting of two stars orbiting each other, called "binary stars." In your computer model of a planet and Sun (Problem P68), replace the planet with a star whose mass is half the mass of our Sun, and take into account the gravitational effects that the second star has on the Sun. **(a)** Give the second star the speed of the actual Earth, and give the Sun zero initial momentum. What happens? Try a variety of other initial conditions. What kinds of orbits do you find? **(b)** Choose initial conditions so that the total momentum of the two-star system is zero, but the stars are not headed directly at each other. What is special about the motion you observe in this case?

•••P70 Modify your orbit computation to use a different force, such as a force that is proportional to $1/r$ or $1/r^3$, or a constant force, or a force proportional to r (this represents the force of a spring whose relaxed length is nearly zero). How do orbits with these forces differ from the circles and ellipses that result from a $1/r^2$ force? If you want to keep the magnitude of the force roughly the same as before, you will need to adjust the force constant G.

••P71 The first U.S. spacecraft to photograph the Moon close up was the unmanned *Ranger 7* photographic mission in 1964. The spacecraft, shown in Figure 3.69, contained television cameras that transmitted close-up pictures of the Moon back to Earth as the spacecraft approached the Moon. The spacecraft did not have retro-rockets to slow itself down, and it eventually simply crashed onto the Moon's surface, transmitting its last photos immediately before impact.

Figure 3.69 (Image courtesy of NASA)

Figure 3.70 is the first image of the Moon taken by a U.S. spacecraft, *Ranger 7*, on July 31, 1964, about 17 minutes before impact on the lunar surface. To find out more about the actual *Ranger* lunar missions, see http://nssdc.gsfc.nasa.gov/planetary/lunar/ranger.html.

Figure 3.70 (Image courtesy of NASA)

Create a computational model of the *Ranger 7* mission. Start your model when the spacecraft, whose mass is = 173 kg, has been brought to 50 km above the Earth's surface (5×10^4 m) by several stages of large rockets, and has a speed of around 1×10^4 m/s. All fuel has been used up, and the spacecraft now coasts toward the Moon. Data for the Earth and Moon may be found on the inside back cover of the textbook.

For this simple model, keep the Earth and Moon fixed in space during the mission, and ignore the effect of the Sun. **(a)** Compute and display the path of the spacecraft, having it leave a trail. **(b)** Determine experimentally the approximate *minimum* initial speed needed to reach the Moon, to three significant figures (this is the speed that the spacecraft obtained from the multistage rocket, at the time of release above the Earth's atmosphere). **(c)** Check your result by decreasing the time step size until your results do not change significantly. **(d)** Use a launch speed 10% larger than the approximate minimum value found in part (b). How long does it take to go to the Moon, in hours or days? **(e)** What is the "impact speed" of the spacecraft (its speed just before it hits the Moon's surface)? Make sure that your spacecraft crashes on the surface of the Moon, and not at its center!

•••**P72** In the *Ranger 7* model, take into account the motion of the Moon around the Earth and the motion of the Earth around the Sun. In addition, the Sun and other planets exert gravitational forces on the spacecraft.

•••**P73** In the *Ranger 7* analysis (the Moon voyage), you used a simplified model in which you neglected among other things the effect of Venus. An important aspect of physical modeling is making estimates of how large the neglected effects might be. Venus and the Earth have similar size and mass. At its closest approach to the Earth, Venus is about 40 million km away (4×10^{10} m). In the real world, Venus would attract the Earth and the Moon as well as the spacecraft, but to get an idea of the size of the effects, imagine that the Earth, the Moon, and Venus are all fixed in position. (See Figure 3.71.)

Figure 3.71

If we take Venus into account, make a rough estimate of whether the spacecraft will miss the Moon entirely. How large a sideways deflection of the crash site will there be? Explain your reasoning and approximations. If you expect a significant effect, modify your program to include the effects of Venus.

•••**P74** Create a computational model of the motion of a three-body gravitational system, with all three objects free to move, and plot the trajectories, leaving trails behind the objects. Calculate all of the forces before using these forces to update the momenta and positions of the objects. Otherwise the calculations of gravitational forces would mix positions corresponding to different times.

Try different initial positions and initial momenta. Find at least one set of initial conditions that produces a long-lasting orbit, one set of initial conditions that results in a collision with a massive object, and one set of initial conditions that allows one of the objects to wander off without returning. Report the masses and initial conditions that you used.

ANSWERS TO CHECKPOINTS

1 **(a)** 4×10^{25} N, **(b)** 8×10^{25} N, **(c)** 4.4×10^{24} N

2 **(a)** (1) $\langle -1,0,0 \rangle$, (2) $\langle 1,0,0 \rangle$, (3) $\langle 1,0,0 \rangle$, **(b)** (1) $\langle 1,0,0 \rangle$, (2) $\langle -1,0,0 \rangle$, (3) $\langle -1,0,0 \rangle$

3 **(a)** 0.285 N/kg, **(b)** 19.9 N, **(c)** 0.029 as much (the astronaut's weight on the asteroid is about 3% of what it would be on Earth)

4 **(a)** 588 N, **(b)** 588 N

5 The magnitude of $\vec{p}_{future}$ will be less than the magnitude of $\vec{p}_{now}$ (so speed decreases) because part of $\vec{F}_{net,now}$ is opposite to $\vec{p}_{now}$. The direction of $\vec{p}_{future}$ will also be different (it will have a larger $+y$ component).

6 **(a)** 2.02×10^{-9} N, **(b)** 2.02×10^{-9} N

7 **(a)** Si: 14 p and 14 n, **(b)** Sn: 50 p and 69 n, **(c)** Au: 79 p and 118 n, **(d)** Th: 90 p and 142 n, **(e)** The farther you go in the periodic table, the more "excess" neutrons with their strong interactions are needed to offset the proton repulsions.

8 $\langle 0.57, 5.43, 0 \rangle$ kg·m/s

9 **(a)** $\langle 0.04, 0.04, -0.1 \rangle$ kg·m/s, **(b)** $\langle 0.04, 0.04, -0.1 \rangle$ kg·m/s, **(c)** $\langle 1, 1, -2.5 \rangle$ m/s

10 6, 6