required to maintain a constant velocity? Explain this seeming contradiction.

Q5 In a lab experiment you observe that a pendulum swings with a "period" (time for one round trip) of 2 s. In an iterative calculation of the motion, which of the following would NOT be a reasonable choice for Δt, for either hand or computer iterative calculations? **(a)** 1 s **(b)** 0.1 s **(c)** 0.05 s **(d)** 0.01 s

Q6 A comet passes near the Sun. When the comet is closest to the Sun, it is 9×10^{10} m from the Sun. You need to choose a time step to use in predicting the comet's motion. Which of the following would be a reasonable distance for the comet to move in one time step, doing an iterative calculation *by hand*? **(a)** 1×10^2 m **(b)** 1×10^{10} m **(c)** 1×10^{11} m **(d)** 1×10^9 m

Q7 A ball moves in the direction of the arrow labeled c in Figure 2.53. The ball is struck by a stick that briefly exerts a force on the ball in the direction of the arrow labeled e. Which arrow best describes the direction of $\Delta\vec{p}$, the change in the ball's momentum?

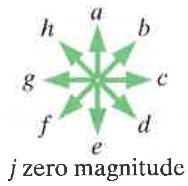

Figure 2.53

PROBLEMS

Section 2.1

•P8 A system is acted upon by two forces, $\langle 18,47,-23\rangle$ N, and $\langle -20,-13,41\rangle$ N. What is the net force acting on the system?

•P9 A truck driver slams on the brakes and the momentum of the truck changes from $\langle 65{,}000,0,0\rangle$ kg·m/s to $\langle 26{,}000,0,0\rangle$ kg·m/s in 4.1 s due to a constant force of the road on the wheels of the truck. As a vector, write the net force exerted on the truck by the surroundings.

•P10 At a certain instant a particle is moving in the $+x$ direction with momentum $+8$ kg·m/s. During the next 0.13 s a constant force acts on the particle, with $F_x = -7$ N and $F_y = +5$ N. What is the *magnitude* of the momentum of the particle at the end of this 0.13 s interval?

•P11 At $t = 16.0$ s an object with mass 4 kg was observed to have a velocity of $\langle 9,29,-10\rangle$ m/s. At $t = 16.2$ s its velocity was $\langle 18,20,25\rangle$ m/s. What was the average net force acting on the object?

••P12 A proton (mass 1.7×10^{-27} kg) interacts electrically with a neutral HCl molecule located at the origin. At a certain time t, the proton's position is $\langle 1.6 \times 10^{-9},0,0\rangle$ m and the proton's velocity is $\langle 3600,600,0\rangle$ m/s. The force exerted on the proton by the HCl molecule is $\langle -1.12 \times 10^{-11},0,0\rangle$ N. At a time $t + 3.4 \times 10^{-14}$ s, what is the approximate velocity of the proton? (You may assume that the force was approximately constant during this interval.)

••P13 A Ping-Pong ball is acted upon by the Earth, air resistance, and a strong wind. Here are the positions of the ball at several times.

Early time interval:

- At $t = 12.35$ s, the position was $\langle 3.17,2.54,-9.38\rangle$ m.
- At $t = 12.37$ s, the position was $\langle 3.25,2.50,-9.40\rangle$ m.

Late time interval:

- At $t = 14.35$ s, the position was $\langle 11.25,-1.50,-11.40\rangle$ m.
- At $t = 14.37$ s, the position was $\langle 11.27,-1.86,-11.42\rangle$ m.

(a) In the early time interval, from $t = 12.35$ s to $t = 12.37$ s, what was the average momentum of the ball? The mass of the Ping-Pong ball is 2.7 grams (2.7×10^{-3} kg). Express your result as a vector. **(b)** In the late time interval, from $t = 14.35$ s to $t = 14.37$ s, what was the average momentum of the ball? Express your result as a vector. **(c)** In the time interval from $t = 12.35$ s (the start of the early time interval) to $t = 14.35$ s (the start of the late time interval), what was the average net force acting on the ball? Express your result as a vector.

••P14 A 0.7 kg block of ice is sliding by you on a very slippery floor at 2.5 m/s. As it goes by, you give it a kick perpendicular to its path. Your foot is in contact with the ice block for 0.003 s. The block eventually slides at an angle of 22 degrees from its original direction. The overhead view shown in Figure 2.54 is approximately to scale. The arrow represents the average force your toe applies briefly to the block of ice.
(a) Which of the possible paths shown in the diagram corresponds to the correct overhead view of the block's path? **(b)** Which components of the block's momentum are changed by the impulse applied by your foot? (Check all that apply. The diagram shows a top view, looking down on the xz plane.) **(c)** What is the unit vector $\hat{p}$ in the direction of the block's momentum after the kick? **(d)** What is the x component of the block's momentum after the kick? **(e)** Remember that $\vec{p} = |\vec{p}|\hat{p}$. What is the magnitude of the block's momentum after the kick? **(f)** Use your answers to the preceding questions to find the z component of the block's momentum after the kick (drawing a diagram is helpful). **(g)** What was the magnitude of the average force you applied to the block?

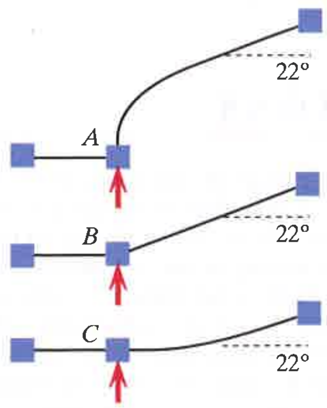

Figure 2.54

••P15 In outer space a rock of mass 5 kg is acted on by a constant net force $\langle 29,-15,40\rangle$ N during a 4 s time interval. At the end of this time interval the rock has a velocity of

$\langle 114, 94, 112\rangle$ m/s. What was the rock's velocity at the beginning of the time interval?

Section 2.2

••P16 A steel safe with mass 2200 kg falls onto concrete. Just before hitting the concrete its speed is 40 m/s, and it smashes without rebounding and ends up being 0.06 m shorter than before. What is the approximate magnitude of the force exerted on the safe by the concrete? How does this compare with the gravitational force of the Earth on the safe? Explain your analysis carefully, and justify your estimates on physical grounds.

••P17 In a crash test, a truck with mass 2500 kg traveling at 24 m/s smashes head-on into a concrete wall without rebounding. The front end crumples so much that the truck is 0.72 m shorter than before. **(a)** What is the average speed of the truck during the collision (that is, during the interval between first contact with the wall and coming to a stop)? **(b)** About how long does the collision last? (That is, how long is the interval between first contact with the wall and coming to a stop?) **(c)** What is the magnitude of the average force exerted by the wall on the truck during the collision? **(d)** It is interesting to compare this force to the weight of the truck. Calculate the ratio of the force of the wall to the gravitational force mg on the truck. This large ratio shows why a collision is so damaging. **(e)** What approximations did you make in your analysis?

••P18 A tennis ball has a mass of 0.057 kg. A professional tennis player hits the ball hard enough to give it a speed of 50 m/s (about 120 mi/h). The ball hits a wall and bounces back with almost the same speed (50 m/s). As indicated in Figure 2.55, high-speed photography shows that the ball is crushed 2 cm (0.02 m) at the instant when its speed is momentarily zero, before rebounding.

Making the very rough approximation that the large force that the wall exerts on the ball is approximately constant during contact, determine the approximate magnitude of this force. *Hint:* Think about the approximate amount of time it takes for the ball to come momentarily to rest. (For comparison note that the gravitational force on the ball is quite small, only about (0.057 kg)(9.8 N/ kg) $\approx$ 0.6 N. A force of 5 N is approximately the same as a force of one pound.)

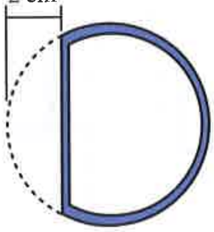

Figure 2.55

••P19 An object is on a collision course with the Earth and is predicted to hit in the center of the target, coming in vertically. The object is roughly spherical, with an approximate diameter of 100 m (the meteor that damaged Chelyabinsk, Russia, in February 2013 had a diameter of about 20 m; the object that killed off the dinosaurs 65 million years ago is thought to have had a diameter of about 10 km). If we start immediately, we can rendezvous with the object when it is 2.5×10^{11} m from the Sun, traveling toward the Sun at a speed of 30 km/s. There is a proposal to implant a rocket engine on the surface of the object to deflect the object enough to miss the Earth. Perform an approximate feasibility analysis of this critical mission. Give details of your design, including the estimates, assumptions, and idealizations you make. Note that the first stage of the most powerful rocket ever used, the Saturn V, had a thrust of 3.4×10^7 N and burned for 170 s. Can the Earth be saved?

Section 2.4

For all problems in this section, use the approximation that $\vec{v}_{avg} \approx \vec{p}_f/m$ for each time step.

••P20 Suppose that you are navigating a spacecraft far from other objects. The mass of the spacecraft is 1.5×10^5 kg (about 150 tons). The rocket engines are shut off, and you're coasting along with a constant velocity of $\langle 0, 20, 0\rangle$ km/s. As you pass the location $\langle 12, 15, 0\rangle$ km you briefly fire side-thruster rockets, so that your spacecraft experiences a net force of $\langle 6 \times 10^4, 0, 0\rangle$ N for 20 s. After turning off the thrusters, you then continue coasting with the rocket engines turned off. (The ejected gases have a mass that is very small compared to the mass of the spacecraft.) **(a)** Using a step size of 20 seconds, predict where you will be 40 s after you began firing the thrusters. (Note that for the second step the thrusters are off.) **(b)** Where would you have been if you had not fired the thrusters?

••P21 You throw a metal block of mass 0.25 kg into the air, and it leaves your hand at time $t = 0$ at location $\langle 0, 2, 0\rangle$ m with velocity $\langle 3, 4, 0\rangle$ m/s. At this low velocity air resistance is negligible. Using the iterative method shown in Section 2.4 with a time step of 0.05 s, calculate step by step the position and velocity of the block at $t = 0.05$ s, $t = 0.10$ s, and $t = 0.15$ s.

••P22 A small space probe, of mass 240 kg, is launched from a spacecraft near Mars. It travels toward the surface of Mars, where it will land. At a time 20.7 s after it is launched, the probe is at the location $\langle 4.30 \times 10^3, 8.70 \times 10^2, 0\rangle$ m, and at this same time its momentum is $\langle 4.40 \times 10^4, -7.60 \times 10^3, 0\rangle$ kg·m/s. At this instant, the net force on the probe due to the gravitational pull of Mars plus the air resistance acting on the probe is $\langle -7 \times 10^3, -9.2 \times 10^2, 0\rangle$ N. Assuming that the net force on the probe is approximately constant over this time interval, what are the momentum and position of the probe 20.9 s after it is launched? Divide the interval into two time steps, and use the approximation $\vec{v}_{avg} \approx \vec{p}_f/m$.

••P23 A soccer ball of mass 0.43 kg is rolling with velocity $\langle 0, 0, 2.2\rangle$ m/s, when you kick it. Your kick delivers an impulse of magnitude 1.3 N·s in the $-x$ direction. The net force on the rolling ball, due to the air and the grass, is 0.25 N in the direction opposite to the direction of the ball's momentum. Using a time step of 0.5 s, find the position of the ball at a time 1.5 s after you kick it, assuming that the ball is at the origin at the moment it is kicked. Use the approximation $\vec{v}_{avg} \approx \vec{p}_f/m$.

••P24 As your spaceship coasts toward Mars, you need to move a heavy load of 1200 kg along a hallway of the spacecraft that has a 90° right turn, without touching the walls, floor, or ceiling, by working remotely, using devices attached to the load that can be programmed to fire blasts of compressed air for up to 1.0 s in any desired direction. During a blast the load is subjected to a force of 20 N. The center of the load must move 3 m along the first section of the hallway, starting from rest, then 4 m along the second section, ending at rest. Let the starting point be $\langle 0, 0, 0\rangle$ m, with the first section ending at $\langle 0, 3, 0\rangle$ m and the second section ending at $\langle 4, 3, 0\rangle$ m. Using just three blasts of compressed air, choose the times when these blasts should be scheduled, their durations, and their directions. How long does it take to complete the entire move?

Section 2.5

•**P25** A runner starts from rest and in 3 s reaches a speed of 8 m/s. Assume that her speed changed at a constant rate (constant net force). **(a)** What was her average speed during this 3 s interval? **(b)** How far did she go in this 3 s interval?

•**P26** The driver of a car traveling at a speed of 18 m/s slams on the brakes and comes to a stop in 4 s. If we assume that the car's speed changed at a constant rate (constant net force): **(a)** What was the car's average speed during this 4 s interval? **(b)** How far did the car go in this 4 s interval?

•**P27** On a straight road with the $+x$ axis chosen to point in the direction of motion, you drive for 3 h at a constant 30 mi/h, then in a few seconds you speed up to 60 mi/h and drive at this speed for 1 h. **(a)** What was the x component of average velocity for the 4 h period, using the fundamental definition of average velocity, which is the displacement divided by the time interval? **(b)** Suppose that instead you use the equation $v_{avg,x} = (v_{ix} + v_{fx})/2$. What do you calculate for the x component of average velocity? **(c)** Why does the equation used in part (b) give the wrong answer?

•**P28** A ball of mass 0.4 kg flies through the air at low speed, so that air resistance is negligible. **(a)** What is the net force acting on the ball while it is in motion? **(b)** Which components of the ball's momentum will be changed by this force? **(c)** What happens to the x component of the ball's momentum during its flight? **(d)** What happens to the y component of the ball's momentum during its flight? **(e)** What happens to the z component of the ball's momentum during its flight? **(f)** In this situation, why is it legitimate to use the expression for average y component of velocity, $v_{avg,y} = (v_{iy} + v_{fy})/2$, to update the y component of position?

•**P29** For each graph of v_x vs. t numbered 1–6 in Figure 2.56, choose the letter (a–i) corresponding to the appropriate description of motion of a fan cart moving along a track. Not all descriptions will be used. Assume the usual coordinate system ($+x$ to the right, $+y$ up, $+z$ out of the page).

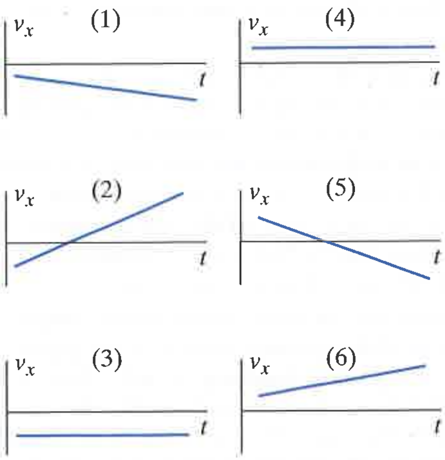

Figure 2.56

(a) A cart moves to the left, gradually slowing down. **(b)** A cart moves to the right, gradually speeding up. **(c)** A cart moves to the left at constant speed. **(d)** A cart moves to the left, gradually slowing down, stops, and moves to the right, speeding up. **(e)** A cart remains stationary and does not move. **(f)** A cart moves to the right, gradually slowing down. **(g)** A cart moves to the right, gradually slowing down, stops, and moves to the left, speeding up. **(h)** A cart moves to the left, gradually speeding up. **(i)** A cart moves to the right at constant speed.

•**P30** A cart rolls with low friction on a track. A fan is mounted on the cart, and when the fan is turned on, there is a constant force acting on the cart. Three different experiments are performed: **(a)** Fan off: The cart is originally at rest. You give it a brief push, and it coasts a long distance along the track in the $+x$ direction, slowly coming to a stop. **(b)** Fan forward: The fan is turned on, and you hold the cart stationary. You then take your hand away, and the cart moves forward, in the $+x$ direction. After traveling a long distance along the track, you quickly stop and hold the cart. **(c)** Fan backward: The fan is turned on facing the "wrong" way, and you hold the cart stationary. You give it a brief push, and the cart moves forward, in the $+x$ direction, slowing down and then turning around, returning to the starting position, where you quickly stop and hold the cart. Figure 2.57 displays four graphs of p_x (numbered 1–4), the x component of momentum, vs. time. The graphs start when the cart is at rest, and end when the cart is again at rest. Match the experiment with the correct graph.

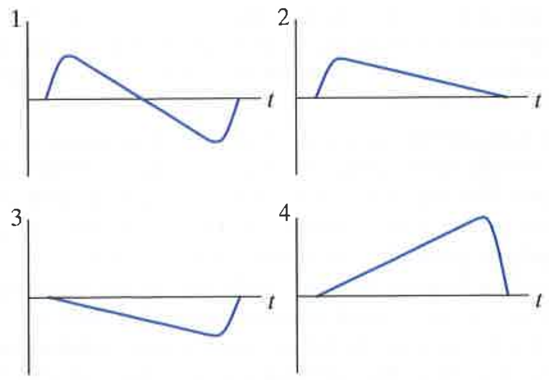

Figure 2.57

•**P31** Consider the three experiments described in Problem 30. Figure 2.58 displays four graphs of $F_{net,x}$, the x component of the net force acting on the cart, vs. time. The graphs start when the cart is at rest, and end when the cart is again at rest. Match the experiment with the graph.

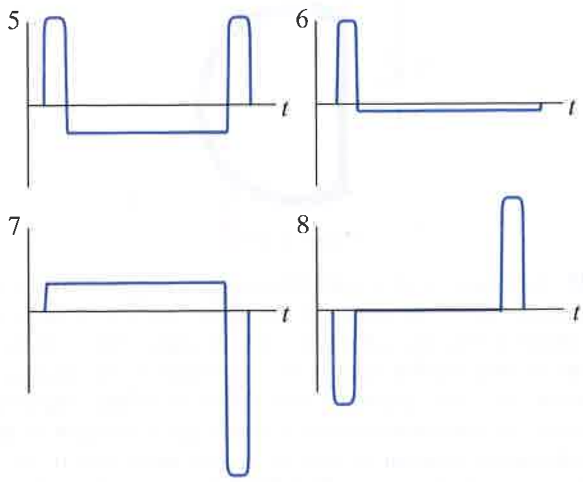

Figure 2.58

•**P32** Consider the three experiments described in Problem 30. Figure 2.59 displays four graphs of x, positioned along the track, vs. time. The graphs start when the cart is at rest, and end when the cart is again at rest. Match the experiment with the graph.

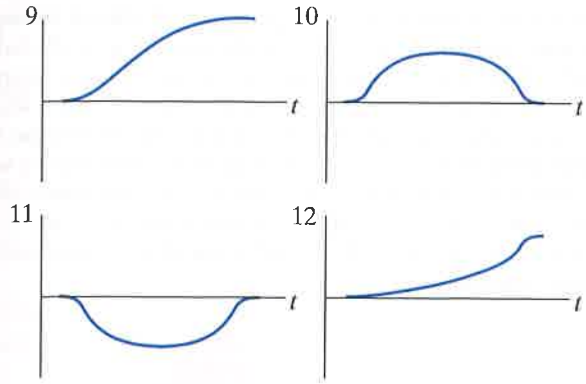

Figure 2.59

••P33 You are a detective investigating why someone was hit on the head by a falling flowerpot. One piece of evidence is a home video taken in a 4th-floor apartment, which happens to show the flowerpot falling past a tall window. Inspection of individual frames of the video shows that in a span of 6 frames the flowerpot falls a distance that corresponds to 0.85 of the window height seen in the video. (*Note:* Standard video runs at a rate of 30 frames per second.) You visit the apartment and measure the window to be 2.2 m high. What can you conclude? Under what assumptions? Give as much detail as you can.

••P34 A soccer ball is kicked at an angle of 59° to the horizontal with an initial speed of 20 m/s. Assume for the moment that we can neglect air resistance. **(a)** For how much time is the ball in the air? **(b)** How far does it go (horizontal distance along the field)? **(c)** How high does it go?

••P35 A ball is kicked from a location $\langle 9,0,-6\rangle$ (on the ground) with initial velocity $\langle -11,16,-6\rangle$ m/s. The ball's speed is low enough that air resistance is negligible. **(a)** What is the velocity of the ball 0.5 s after being kicked? (Use the Momentum Principle!) **(b)** In this situation (constant force), which velocity will give the most accurate value for the location of the ball 0.5 s after it is kicked: the arithmetic average of the initial and final velocities, the final velocity of the ball, or the initial velocity of the ball? **(c)** What is the average velocity of the ball over this time interval (a vector)? **(d)** Use the average velocity to find the location of the ball 0.5 s after being kicked.

Now consider a different time interval: the interval between the initial kick and the moment when the ball reaches its highest point. We want to find how long it takes for the ball to reach this point, and how high the ball goes. **(e)** What is the y component of the ball's velocity at the instant when the ball reaches its highest point (the end of this time interval)? **(f)** Fill in the known quantities in the update form of the Momentum Principle, $mv_{yf} = mv_{yi} + F_{\text{net},y}\Delta t$, leaving as symbols anything that is unknown. **(g)** How long does it take for the ball to reach its highest point? **(h)** Knowing this time, first find the y component of the average velocity during *this* time interval, then use it to find the maximum height attained by the ball.

Now take a moment to reflect on the reasoning used to solve this problem. You should be able to do a similar problem on your own, without prompting. Note that the only equations needed were the Momentum Principle and the expression for the arithmetic average velocity.

••P36 A small dense ball with mass 1.5 kg is thrown with initial velocity $\langle 5,8,0\rangle$ m/s at time $t=0$ at a location we choose to call the origin ($\langle 0,0,0\rangle$). Air resistance is negligible. **(a)** When the ball reaches its maximum height, what is its velocity (a vector)? It may help to make a simple diagram. **(b)** When the ball reaches its maximum height, what is t? You know how v_y depends on t, and you know the initial and final velocities. **(c)** Between the launch at $t=0$ and the time when the ball reaches its maximum height, what is the average velocity (a vector)? You know how to determine average velocity when velocity changes at a constant rate. **(d)** When the ball reaches its maximum height, what is its location (a vector)? You know how average velocity and displacement are related. **(e)** At a later time the ball's height y has returned to zero, which means that the average value of v_y from $t=0$ to this time is zero. At this instant, what is the time t? **(f)** At the time calculated in part (e), when the ball's height y returns to zero, what is x? (This is called the "range" of the trajectory.) **(g)** At the time calculated in part (e), when the ball's height y returns to zero, what is v_y? **(h)** What was the angle to the x axis of the initial velocity? **(i)** What was the angle to the x axis of the velocity at the time calculated in part (e), when the ball's height y returned to zero?

••P37 Apply the general results obtained in the full analysis of motion under the influence of a constant force in Section 2.5 to answer the following questions. You hold a small metal ball of mass m a height h above the floor. You let go, and the ball falls to the floor. Choose the origin of the coordinate system to be on the floor where the ball hits, with y up as usual. Just after release, what are y_i and v_{iy}? Just before hitting the floor, what is y_f? How much time Δt does it take for the ball to fall? What is v_{fy} just before hitting the floor? Express all results in terms of m, g, and h. How would your results change if the ball had twice the mass?

••P38 In a cathode ray tube (CRT) used in older television sets, a beam of electrons is steered to different places on a phosphor screen, which glows at locations hit by electrons. The CRT is evacuated, so there are few gas molecules present for the electrons to run into. Electric forces are used to accelerate electrons of mass m to a speed $v_0 \ll c$, after which they pass between positively and negatively charged metal plates that deflect the electron in the vertical direction (upward in Figure 2.60, or downward if the sign of the charges on the plates is reversed).

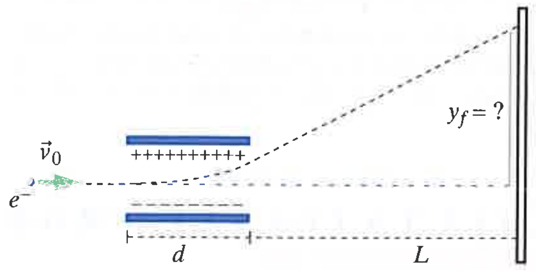

Figure 2.60

While an electron is between the plates, it experiences a uniform vertical force F, but when the electron is outside the plates there is negligible force on it. The gravitational force on the electron is negligibly small compared to the electric force in this situation. The length of the metal plates is d, and the phosphor screen is a distance L from the metal plates. Where does the electron hit the screen? (That is, what is y_f?)

••P39 The performance of two different cars, car 1 and car 2, was measured on a long horizontal test track. Car 1 started from rest and ran with constant acceleration until it was halfway down the track and then stopped accelerating, continuing to run at the attained speed to the end of the track. Car 2 started from rest and ran with a constant acceleration for the entire distance. It was observed that both cars covered the test distance in the same amount of time. **(a)** What was the ratio of the average speed of car 1 to that of car 2? **(b)** What was the ratio of the initial acceleration of car 1 to that of car 2? **(c)** What was the ratio of the final speed of car 1 to that of car 2?

••P40 A driver starts from rest on a straight test track that has markers every 0.1 km. The driver presses on the accelerator and for the entire period of the test holds the car at constant acceleration. The car passes the 0.1 km post at 8 s after starting the test. **(a)** What was the car's acceleration? **(b)** What was the car's speed as it passed the 0.1 km post? **(c)** What does the speedometer read at the 0.2 km post? **(d)** When does the car pass the 0.2 km post?

Section 2.6

For all problems in this section, use the approximation that $\vec{v}_{avg} \approx \vec{p}_f/m$ for each time step.

•P41 The stiffness of a particular spring is 40 N/m. One end of the spring is attached to a wall. When you pull on the other end of the spring with a steady force of 2 N, the spring elongates to a total length of 18 cm. What was the relaxed length of the spring? (Remember to convert to SI units.)

••P42 A spring with a relaxed length of 25 cm and a stiffness of 11 N/m stands vertically on a table. A block of mass 70 g is attached to the top of the spring. You pull the block upward, stretching the spring until its length is now 28 cm, hold the block at rest for a moment, and then release it. Using a time step of 0.1 s, predict the position and momentum of the block at a time 0.2 s after you release the block.

••P43 A block is attached to the top of a spring that stands vertically on a table. The spring stiffness is 55 N/m, its relaxed length is 23 cm, and the mass of the block is 350 g. The block is oscillating up and down as the spring stretches and compresses. At a particular time you observe that the velocity of the block is $\langle 0, 0.0877, 0\rangle$ m/s, and the position of the block is $\langle 0, 0.0798, 0\rangle$ m, relative to an origin at the base of the spring. Using a time step of 0.1 s, determine the position of the block 0.2 s later.

••P44 A paddle ball toy consists of a flat wooden paddle and a small rubber ball that are attached to each other by an elastic band (Figure 2.61). You have a paddle ball toy for which the mass of the ball is 0.015 kg, the stiffness of the elastic band is 0.9 N/m, and the relaxed length of the elastic band is 0.30 m. You are holding the paddle so the ball hangs suspended under it, when your cat comes along and bats the ball around, setting it in motion. At a particular instant the the momentum of the ball is $\langle -0.02, -0.01, -0.02\rangle$ kg · m/s, and the moving ball is at location $\langle -0.2, -0.61, 0\rangle$ m relative to an origin located at the point where the elastic band is attached to the paddle. **(a)** Determine the position of the ball 0.1 s later, using a Δt of 0.1 s. **(b)** Starting with the same initial position ($\langle -0.2, -0.61, 0\rangle$ m) and momentum ($\langle -0.02, -0.01, -0.02\rangle$ kg · m/s), determine the position of the ball 0.1 s later, using a Δt of 0.05 s. **(c)** If your answers are different, explain why.

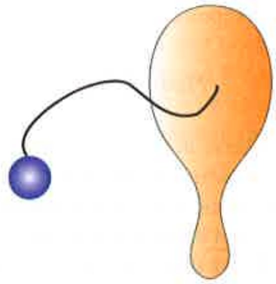

Figure 2.61

••P45 A block of mass 0.5 kg is placed at rest on a relaxed vertical spring-like device of length 0.1 m which exerts an upward force of $F_y = -2 \times 10^5 \times s^3$, where s is the stretch. With a time step of 0.04 s, calculate the length L of the device and the y component of the velocity of the block at $t = 0.04s$ and $t = 0.08s$.

Section 2.9

•P46 A proton has mass 1.7×10^{-27} kg. What is the magnitude of the impulse required to increase its speed from $0.990c$ to $0.994c$?

••P47 SLAC, the Stanford Linear Accelerator Center, located at Stanford University in Palo Alto, California, accelerates electrons through a vacuum tube 2 mi long (it can be seen from an overpass of the Junipero Serra freeway that goes right over the accelerator). Electrons that are initially at rest are subjected to a continuous force of 2×10^{-12} newton along the entire length of 2 mi (1 mi is 1.6 km) and reach speeds very near the speed of light. **(a)** Determine how much time is required to increase the electrons' speed from $0.93c$ to $0.99c$. (That is, the quantity $|\vec{v}|/c$ increases from 0.93 to 0.99.) **(b)** Approximately how far does the electron go in this time? What is approximate about your result?

COMPUTATIONAL PROBLEMS

You should do the introductory computational problems at the end of Chapter 1 before doing these problems.

More detailed and extended versions of some of these computational modeling problems may be found in the lab activities included in the *Matter & Interactions, 4th Edition*, resources for instructors.

A note on graphs: Graphs in VPython appear in a separate window. A graph is dynamic; it appears point by point as the program runs, and scales itself automatically. To make a graph in VPython requires three things, as shown in the program shown below:

(1) At the beginning of the program, type these two lines:
```
from visual import *
from visual.graph import *
```
(2) Create one or more `gcurve` objects, such as the one called `speed` in the program shown below.

(3) Inside the computational loop, add one or more `plot` operations specifying the pair of values to plot, such as the statement `speed.plot( pos=(t, v) )` in the program shown below.

Consult the VPython help for full details on graphing.

```
from visual import *
# Import graphing:
from visual.graph import *
...
# Create a graphing curve:
speed = gcurve(color=color.yellow)
...
while ship.pos.x < L:
    rate(20000)
    # update position and momentum:
    ...
    # Calculate speed v, and
    # add to the graphing curve:
    speed.plot( pos=(t, v) )
    # Update the time:
    t = t + dt
```

••P48 Write an iterative computational model that predicts and displays (as a real-time animation) the motion of a fan cart on a low-friction track. A typical track is 2 m long, and a typical fan cart (which can be represented by a box object) is about 10 cm long and has a mass of about 0.8 kg (including the fan). Use a time step $\Delta t = 0.01$ s. Run your program often while writing it. It is much easier to find and fix errors when creating the program incrementally. **(a)** Start the cart at the left edge of the track, and give it an initial velocity of $\langle 0.5,0,0\rangle$ m/s. **(b)** With the fan off ($\vec{F}_{\text{net}} = \vec{0}$) determine how long it takes for the cart to reach the right end of the track. **(c)** To model the behavior of the cart with the fan on, add a constant force to your model. By experimenting with your program, find values for the force and the initial velocity that lead to this behavior: The cart starts at the right end of the track and moves to the left, gradually slowing down. It stops at the left end of the track, and moves back to the right, gradually speeding up. **(d)** What happens when the initial velocity of the cart has a nonzero y component? Make the cart leave a trail so you can see its path clearly. Explain why your model predicts this behavior.

••P49 Starting with the program you wrote in Problem P48, **(a)** Add commands to create a graph of the x component of the cart's position (`cart.pos.x`) vs. time as the cart moves. Explain the shape of the graph. **(b)** Change your graph to plot the x component of the cart's momentum vs. time. Explain the shape of the graph.

••P50 Write an iterative computational model that predicts and displays the 3D motion of a 55 g tennis ball that leaves a tennis racket with a speed of 55 m/s. Neglect air resistance (at this speed, this approximation is actually not realistic). Experiment by changing the initial velocity of the ball to see what kinds of different trajectories can be produced.

•••P51 The nearest stars are a group of three stars orbiting each other, Alpha Centauri A, Alpha Centauri B, and Proxima Centauri, located about 4.3 light years from Earth (one light year is the distance light travels in one Earth year). Suppose a space tug is able to pull a cargo ship with a constant force of $g = 9.8$ N/kg times the mass of the cargo ship, for many years. Starting from rest, speed up the cargo ship until you're halfway to the nearest stars, then pull back with the same force to slow the cargo ship back to rest when you reach the nearest stars. Determine the maximum speed attained (which occurs at the halfway point) and how many Earth years the trip takes. You must use the relativistic position update equation, because the speed approaches the speed of light. It is interesting to make a graph of the speed as a function of time, which does not consist of straight lines as it would for nonrelativistic motion. Time passes slowly on board the cargo ship, a relativistic effect that we'll discuss in Chapter 20.

ANSWERS TO CHECKPOINTS

1 (1) $\langle 60,-60,0\rangle$ N; **(2) (a)** $\langle 0,-2,0\rangle$ kg·m/s **(b)** $\langle 0,-0.667,0\rangle$ N
2 (a) By estimating the distance one student "traveled" during the collision–that is, how much the student's body was compressed. Using the approximation that $v_{\text{avg}} \approx (v_i+0)/2$ we divided distance by average speed to estimate contact time. **(b)** Only the z component of $\vec{p}$ changed, so we were able to deduce that $\vec{F}_{\text{net}}$ was in the z direction.
3 (1) b **(2)** a
4 (1) c **(2)** c
5 (a) $\langle -10,7.12,-5\rangle$ m/s **(b)** $\langle 3,6.04,-8\rangle$ m **(c)** 8.62 m
6 (1) 0.275 N **(2) (a)** 3571 N/m **(b)** 71.4 N
7 (1) **(a)** $\langle 0,1,0\rangle$, **(b)** $\langle 0,1,0\rangle$; (2) **(a)** $\langle 0,0.11,0\rangle$ m, **(b)** 0.11 m, **(c)** $\langle 0,1,0\rangle$, **(d)** -0.04 m, **(e)** $\langle 0,3.8,0\rangle$ N
8 (a) The momentum of the block a short time in the future depends on two things: its momentum now and the net force acting on it now. At the beginning of time step 2 the net force is downward but the momentum is upward. The sum of the momentum now plus the downward impulse over the next Δt gives a new momentum that is smaller in magnitude, but still upward, so the block moves upward.
9 a
10 a, b, c, d