Q16 The electric field inside a capacitor is shown on the left in Figure 15.50. Which option (1–5) best represents the electric field at location A?

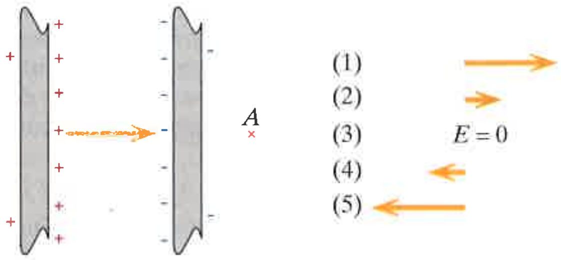

Figure 15.50

Q17 A solid spherical plastic ball was rubbed with wool in such a way that it acquired a uniform negative charge all over the surface. Make a sketch showing the polarization of molecules inside the plastic ball, and explain briefly.

Q18 A student said, "The electric field inside a uniformly charged sphere is always zero." Describe a situation where this is not true.

Q19 Give an example of a configuration of charges that yields an electric field or force whose magnitude varies approximately with distance as specified: (1) Field independent of distance, (2) Field proportional to $1/r$, (3) Field proportional to $1/r^2$, (4) Field proportional to $1/r^3$, (5) Force (not field) that is proportional to $1/r^5$.

PROBLEMS

Section 15.1

•P20 If the total charge on a uniformly charged rod of length 0.4 m is 2.2 nC, what is the magnitude of the electric field at a location 3 cm from the midpoint of the rod?

•P21 A thin rod lies on the x axis with one end at $-A$ and the other end at A, as shown in Figure 15.51. A charge of $-Q$ is spread uniformly over the surface of the rod. We want to set up an integral to find the electric field at location $\langle 0, y, 0\rangle$ due to the rod. Following the procedure discussed in this chapter, we have cut up the rod into small segments, each of which can be considered as a point charge. We have selected a typical piece, shown in red on the diagram.

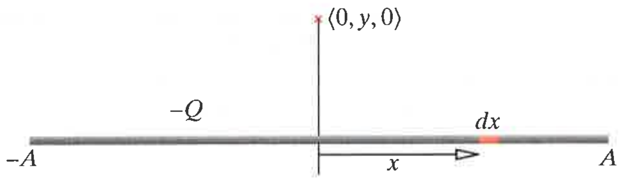

Figure 15.51

Answer using the variables x, y, dx, A, Q as appropriate. Remember that the rod has charge $-Q$.
(a) In terms of the symbolic quantities given above and on the diagram, what is the charge per unit length of the rod? **(b)** What is the amount of charge dQ on the small piece of length dx? **(c)** What is the vector from this source to the observation location? **(d)** What is the distance from this source to the observation location? **(e)** When we set up an integral to find the electric field at the observation location due to the entire rod, what will be the integration variable?

•P22 A plastic rod 1.7 m long is rubbed all over with wool, and acquires a charge of -2×10^{-8} C (Figure 15.52). We choose the center of the rod to be the origin of our coordinate system, with the x axis extending to the right, the y axis extending up, and the z axis out of the page. In order to calculate the electric field at location $A = \langle 0.7, 0, 0\rangle$ m, we divide the rod into eight pieces, and approximate each piece as a point charge located at the center of the piece.

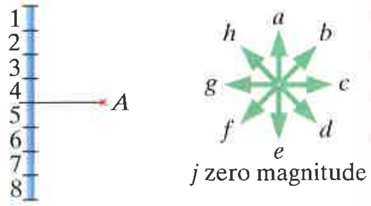

Figure 15.52

(a) What is the length of one of these pieces? **(b)** What is the location of the center of piece number 3? **(c)** How much charge is on piece number 3? (Remember that the charge is negative.) **(d)** Approximating piece 3 as a point charge, what is the electric field at location A due only to piece 3? **(e)** To get the net electric field at location A, we would need to calculate $\Delta\vec{E}$ due to each of the eight pieces, and add up these contributions. If we did that, which arrow (a–h) would best represent the direction of the net electric field at location A?

•P23 A clear plastic pen 12 cm long is rubbed all over with wool, and acquires a negative charge of -2 nC. You want to figure out the electric field a distance of 18 mm from the pen, near the middle of the pen. **(a)** You decide to model the pen as a rod consisting of a series of five segments, each of which you will consider to be approximately point-like. What is the length of each segment in meters? **(b)** What is the amount of charge ΔQ on each of the five segments? **(c)** In general, if the rod has a length L and total charge Q, and you divide the rod into N segments, what is the amount of charge ΔQ on each piece? **(d)** If the length of each segment is dL, write a symbolic expression for the number of pieces N in terms of the length of the rod L and the length of one piece dL. **(e)** Now write a symbolic expression for the amount of charge on each piece in terms of the length of the rod and the length of a small piece.

•P24 A thin glass rod of length 80 cm is rubbed all over with wool and acquires a charge of 60 nC, distributed uniformly over its surface. Calculate the magnitude of the electric field due to the rod at a location 7 cm from the midpoint of the rod. Do the calculation two ways, first using the exact equation for a rod of any length, and second using the approximate equation for a long rod.

•••P25 A water molecule is a permanent dipole with a known dipole moment p $(=qs)$. There is a water molecule in the air a very short distance x from the midpoint of a long glass rod of length L carrying a uniformly distributed positive charge $+Q$ (Figure 15.53). The axis of the dipole is perpendicular to the rod. Note that $s \ll x \ll L$. (The charged rod induces an increase in the dipole moment, but the induced portion of the dipole moment is completely negligible compared to p. It is convenient to use the "binomial expansion" that you may have learned in calculus, that $(1+\epsilon)^n \approx 1+n\epsilon$ is $\epsilon \ll 1$. Note that n can be negative.)

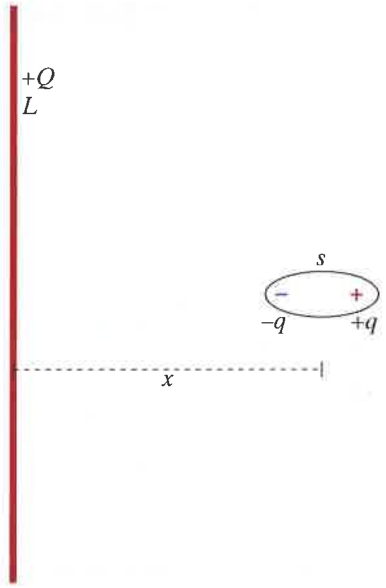

Figure 15.53

(a) Find the magnitude and direction of the electric force acting on the water molecule. Your final result for the magnitude of the force must be expressed in terms of Q, L, x, and p. You can use q and s in your calculations, but your final result must not include q or s, since it is only their product $p = qs$ that is measurable for a water molecule. Explain your work carefully, including appropriate diagrams. **(b)** If the electric field of the rod has the magnitude 1×10^6 N/C at the location of the water molecule, 1 cm from the rod, what is the magnitude of the horizontal component of the acceleration of the water molecule? The measured dipole moment for H_2O is 6.2×10^{-30} C · m, and the mass of one mole is 18 g (1 + 1 + 16). Be sure to check units in your calculation!

•••P26 An electrostatic dust precipitator that is installed in a factory smokestack includes a straight metal wire of length $L = 0.8$ m that is charged approximately uniformly with a total charge $Q = 0.4 \times 10^{-7}$ C. A speck of coal dust (which is mostly carbon) is near the wire, far from both ends of the wire; the distance from the wire to the speck is $d = 1.5$ cm. Carbon has an atomic mass of 12 (6 protons and 6 neutrons in the nucleus). A careful measurement of the polarizability of a carbon atom gives the value

$$\alpha = 1.96 \times 10^{-40} \frac{\text{C} \cdot \text{m}}{\text{N/C}}$$

(a) Calculate the initial acceleration of the speck of coal dust, neglecting gravity. Explain your steps clearly. Your answer must be expressed in terms of Q, L, d, and α. You can use other quantities in your calculations, but your final result must not include them. Don't put numbers into your calculation until the very end, but then show the numerical calculation that you carry out on your calculator. It is convenient to use the "binomial expansion" that you may have learned in calculus, that $(1+\epsilon)^n \approx 1+n\epsilon$ is $\epsilon \ll 1$. Note that n can be negative. **(b)** If the speck of coal dust were initially twice as far from the charged wire, how much smaller would be the initial acceleration of the speck?

Section 15.2

••P27 Consider a thin plastic rod bent into an arc of radius R and angle α (Figure 15.54). The rod carries a uniformly distributed negative charge $-Q$.

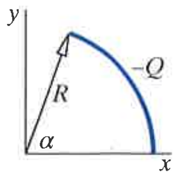

Figure 15.54

What are the components E_x and E_y of the electric field at the origin? Follow the standard four steps: **(a)** Use a diagram to explain how you will cut up the charged rod, and draw the $\Delta\vec{E}$ contributed by a representative piece. **(b)** Express algebraically the contribution each piece makes to the x and y components of the electric field. Be sure to show your integration variable and its origin on your drawing. (*Hint:* An arc of radius R and angle $\Delta\theta$ measured in radians has a length $R\Delta\theta$.) **(c)** Write the summation as an integral, and simplify the integral as much as possible. State explicitly the range of your integration variable. Evaluate the integral. **(d)** Show that your result is reasonable. Apply as many tests as you can think of.

••P28 A strip of invisible tape 0.12 m long by 0.013 m wide is charged uniformly with a total net charge of 3 nC (nano = 1×10^{-9}) and is suspended horizontally, so it lies along the x axis, with its center at the origin, as shown in Figure 15.55. Calculate the approximate electric field at location $\langle 0, 0.03, 0 \rangle$ m (location A) due to the strip of tape. Do this by dividing the strip into three equal sections, as shown in Figure 15.55, and approximating each section as a point charge.

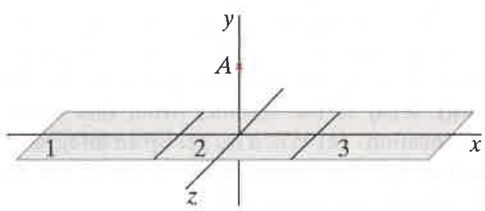

Figure 15.55

(a) What is the approximate electric field at A due to piece 1? **(b)** What is the approximate electric field at A due to piece 2? **(c)** What is the approximate electric field at A due to piece 3? **(d)** What is the approximate net electric field at A? **(e)** What could you do to improve the accuracy of your calculation?

••**P29** Consider a thin glass rod of length L lying along the x axis with one end at the origin. The rod carries a uniformly distributed positive charge Q.

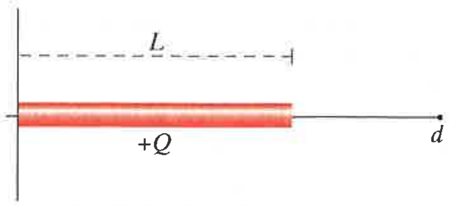

Figure 15.56

At a location $d > L$, on the x axis to the right of the rod in Figure 15.56, what is the electric field due to the rod? Follow the standard four steps. **(a)** Use a diagram to explain how you will cut up the charged rod, and draw the $\Delta\vec{E}$ contributed by a representative piece. **(b)** Express algebraically the contribution each piece makes to the electric field. Be sure to show your integration variable and its origin on your drawing. **(c)** Write the summation as an integral, and simplify the integral as much as possible. State explicitly the range of your integration variable. Evaluate the integral. **(d)** Show that your result is reasonable. Apply as many tests as you can think of.

••**P30** Consider a thin plastic rod bent into a semicircular arc of radius R with center at the origin (Figure 15.57). The rod carries a uniformly distributed negative charge $-Q$.

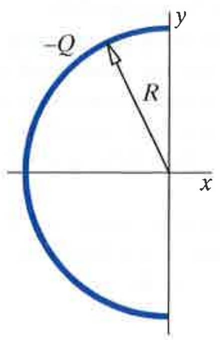

Figure 15.57

(a) Determine the electric field $\vec{E}$ at the origin contributed by the rod. Include carefully labeled diagrams, and be sure to check your result. **(b)** An ion with charge $-2e$ and mass M is placed at rest at the origin. After a *very* short time Δt the ion has moved only a very short distance but has acquired some momentum $\vec{p}$. Calculate $\vec{p}$.

Section 15.3

•**P31** Two rings of radius 5 cm are 24 cm apart and concentric with a common horizontal x axis. The ring on the left carries a uniformly distributed charge of +31 nC, and the ring on the right carries a uniformly distributed charge of −31 nC. **(a)** What are the magnitude and direction of the electric field on the x axis, halfway between the two rings? **(b)** If a charge of −9 nC were placed midway between the rings, what would be the force exerted on this charge by the rings?

•**P32** Two rings of radius 4 cm are 12 cm apart and concentric with a common horizontal x axis. The ring on the left carries a uniformly distributed charge of +40 nC, and the ring on the right carries a uniformly distributed charge of −40 nC. **(a)** What is the electric field due to the right ring at a location midway between the two rings? **(b)** What is the electric field due to the left ring at a location midway between the two rings? **(c)** What is the net electric field at a location midway between the two rings? **(d)** If a charge of −2 nC were placed midway between the rings, what would be the force exerted on this charge by the rings?

•**P33** Two rings of radius 2 cm are 20 cm apart and concentric with a common horizontal x axis. What is the magnitude of the electric field midway between the rings if both rings carry a charge of +35 nC?

•••**P34** A thin-walled hollow circular glass tube, open at both ends, has a radius R and length L. The axis of the tube lies along the x axis, with the left end at the origin (Figure 15.58). The outer sides are rubbed with silk and acquire a net positive charge Q distributed uniformly. Determine the electric field at a location on the x axis, a distance w from the origin. Carry out all steps, including checking your result. Explain each step. (You may have to refer to a table of integrals.)

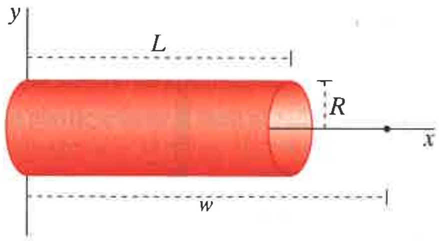

Figure 15.58

Section 15.4

•**P35** Suppose that the radius of a disk is 21 cm, and the total charge distributed uniformly all over the disk is 5.0×10^{-6} C. **(a)** Use the exact result to calculate the electric field 1 mm from the center of the disk. **(b)** Use the exact result to calculate the electric field 3 mm from the center of the disk. **(c)** Does the field decrease significantly?

•**P36** For a disk of radius $R = 20$ cm and $Q = 6 \times 10^{-6}$ C, calculate the electric field 2 mm from the center of the disk using all three equations:

$$E = \frac{(Q/A)}{2\varepsilon_0}\left[1 - \frac{z}{(R^2+z^2)^{1/2}}\right],$$

$$E \approx \frac{Q/A}{2\varepsilon_0}\left[1 - \frac{z}{R}\right], \quad \text{and} \quad E \approx \frac{Q/A}{2\varepsilon_0}.$$

How good are the approximate equations at this distance? For the same disk, calculate E at a distance of 5 cm (50 mm) using all three equations. How good are the approximate equations at this distance?

•**P37** A disk of radius 16 cm has a total charge 4×10^{-6} C distributed uniformly all over the disk. **(a)** Using the exact equation, what is the electric field 1 mm from the center of the disk? **(b)** Using the same exact equation, find the electric field 3 mm from the center of the disk. **(c)** What is the percent difference between these two numbers?

•**P38** For a disk of radius 20 cm with uniformly distributed charge 7×10^{-6} C, calculate the magnitude of the electric field on the axis of the disk, 5 mm from the center of the disk, using each of the following equations:

(a) $E = \dfrac{(Q/A)}{2\varepsilon_0}\left[1 - \dfrac{z}{(R^2+z^2)^{1/2}}\right],$

(b) $E \approx \dfrac{Q/A}{2\varepsilon_0}\left[1 - \frac{z}{R}\right]$,

(c) $E \approx \dfrac{Q/A}{2\varepsilon_0}$.

(d) How good are the approximate equations at this distance?

(e) At what distance does the least accurate approximation for the electric field give a result that is closest to the most accurate: at a distance $R/2$, close to the disk, at a distance R, or far from the disk?

••P39 A large, thin plastic disk with radius $R = 1.5$ m carries a uniformly distributed charge of $-Q = -3 \times 10^{-5}$ C as shown in Figure 15.59. A circular piece of aluminum foil is placed $d = 3$ mm from the disk, parallel to the disk. The foil has a radius of $r = 2$ cm and a thickness $t = 1$ mm.

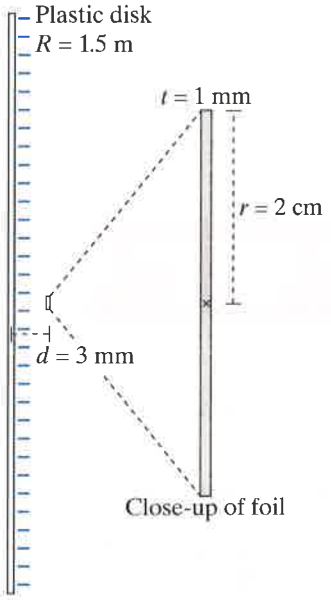

Figure 15.59

(a) Show the charge distribution on the close-up of the foil.

(b) Calculate the magnitude and direction of the electric field at location × at the center of the foil, inside the foil.

(c) Calculate the magnitude q of the charge on the left circular face of the foil.

••P40 A thin circular sheet of glass of diameter 3 m is rubbed with a cloth on one surface and becomes charged uniformly. A chloride ion (a chlorine atom that has gained one extra electron) passes near the glass sheet. When the chloride ion is near the center of the sheet, at a location 0.8 mm from the sheet, it experiences an electric force of 5×10^{-15} N, toward the glass sheet. It will be useful to you to draw a diagram on paper, showing field vectors, force vectors, and charges, before answering the following questions about this situation.

Which of the following statements about this situation are correct? Select all that apply. (1) The electric field that acts on the chloride ion is due to the charge on the glass sheet and to the charge on the chloride ion. (2) The electric field of the glass sheet is equal to the electric field of the chloride ion. (3) The charged disk is the source of the electric field that causes the force on the chloride ion. (4) The net electric field at the location of the chloride ion is zero. (5) The force on the chloride ion is equal to the electric field of the glass sheet.

In addition to an exact equation for the electric field of a disk, the text derives two approximate equations. In the current situation we want an answer that is correct to three significant figures. Which of the following is correct? We should not use an approximation if we have enough information to do an exact calculation. (1) $R \gg z$, so it is adequate to use the most approximate equation here. (2) z is nearly equal to R, so we have to use the exact equation. (3) $z \ll R$, so we can't use an approximation.

How much charge is on the surface of the glass disk? Give the amount, including sign and correct units.

Section 15.5

•P41 If the magnitude of the electric field in air exceeds roughly 3×10^6 N/C, the air breaks down and a spark forms. For a two-disk capacitor of radius 47 cm with a gap of 1 mm, what is the maximum charge (plus and minus) that can be placed on the disks without a spark forming (which would permit charge to flow from one disk to the other)?

•P42 Consider a capacitor made of two rectangular metal plates of length L and width W, with a very small gap s between the plates. There is a charge $+Q$ on one plate and a charge $-Q$ on the other. Assume that the electric field is nearly uniform throughout the gap region and negligibly small outside. Calculate the attractive force that one plate exerts on the other. Remember that one of the plates doesn't exert a net force on itself.

•P43 A capacitor consists of two large metal disks of radius 1.1 m placed parallel to each other, a distance of 1.2 mm apart. The capacitor is charged up to have an increasing amount of charge $+Q$ on one disk and $-Q$ on the other. At about what value of Q does a spark appear between the disks?

••P44 If the magnitude of the electric field in air exceeds roughly 3×10^6 N/C, the air breaks down and a spark forms. For a two-disk capacitor of radius 51 cm with a gap of 2 mm, if the electric field inside is just high enough that a spark occurs, what is the strength of the fringe field just outside the center of the capacitor?

••P45 In a cathode-ray tube, an electron travels in a vacuum and enters a region between two deflection plates where there is an upward electric field of magnitude 1×10^5 N/C (Figure 15.60).

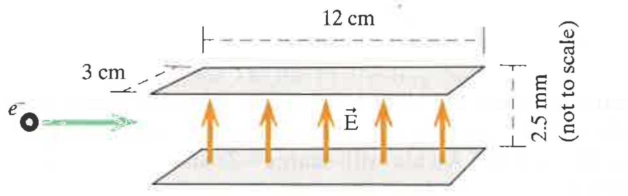

Figure 15.60

(a) Sketch the trajectory of the electron, continuing on well past the deflection plates (the electron is going fast enough that it does not strike the plates).

(b) Calculate the acceleration of the electron while it is between the deflection plates.

(c) The deflection plates measure 12 cm by 3 cm, and the gap between them is 2.5 mm. The plates are charged equally and oppositely. What are the magnitude and sign of the charge on the upper plate?

••P46 In Figure 15.61 are two uniformly charged disks of radius R that are very close to each other (gap $\ll R$). The disk on the left has a charge of $-Q_{\text{left}}$ and the disk on the right has a charge of $+Q_{\text{right}}$ (Q_{right} is greater than Q_{left}). A uniformly charged thin rod of length L lies at the edge of the disks, parallel to the axis of the disks and centered on the gap. The rod has a charge of $+Q_{\text{rod}}$.

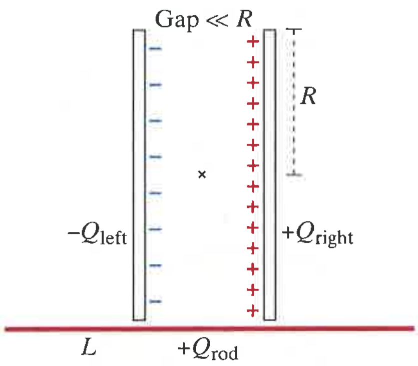

Figure 15.61

(a) Calculate the magnitude and direction of the electric field at the point marked × at the center of the gap region, and explain briefly, including showing the electric field on a diagram. Your results must not contain any symbols other than the given quantities R, Q_{left}, Q_{right}, L, and Q_{rod} (and fundamental constants), unless you define intermediate results in terms of the given quantities. **(b)** If an electron is placed at the center of the gap region, what are the magnitude and direction of the electric force that acts on the electron?

••P47 A capacitor made of two parallel uniformly charged circular metal disks carries a charge of $+Q$ and $-Q$ on the inner surfaces of the plates and very small amounts of charge $+q$ and $-q$ on the outer surfaces of the plates. Each plate has a radius R and thickness t, and the gap distance between the plates is s. How much charge q is on the outside surface of the positive disk, in terms of Q?

•••P48 This is a challenging problem that requires you to construct a model of a real physical situation, to make idealizations, approximations, and assumptions, and to work through a detailed analysis based on physical principles. Be sure to allow yourself enough time to think it through. You may need to measure, estimate, or look up various quantities.

A clear plastic ball-point pen is rubbed thoroughly with wool. The charged plastic pen is held above a small, uncharged disk-shaped piece of aluminum foil, smaller than a hole in a sheet of three-ring binder paper. **(a)** Make a clear physics diagram of the situation, showing charges, fields, forces, and distances. Refer to this physics diagram in your analysis. **(b)** Starting from fundamental physical principles, predict quantitatively how close you must move the pen to the foil in order to pick up the foil (that is, predict an actual numerical distance). State explicitly all approximations and assumptions that you make. **(c)** Try the experiment and compare your observation to your prediction.

Section 15.6

•P49 A solid metal ball of radius 1.5 cm bearing a charge of −17 nC is located near a solid plastic ball of radius 2 cm bearing a uniformly distributed charge of +7 nC (Figure 15.62) on its outer surface. The distance between the centers of the balls is 9 cm. **(a)** Show the approximate charge distribution in and on each ball. **(b)** What is the electric field at the center of the metal ball due only to the charges on the plastic ball? **(c)** What is the net electric field at the center of the metal ball? **(d)** What is the electric field at the center of the metal ball due only to the charges on the surface of the metal ball?

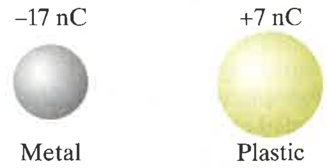

Figure 15.62

•P50 You stand at location A, a distance d from the origin, and hold a small charged ball. You find that the electric force on the ball is 0.08 N. You move to location B, a distance $2d$ from the origin, and find the electric force on the ball to be 0.04 N. What object located at the origin might be the source of the field? (1) A point charge, (2) A dipole, (3) A uniformly charged rod, (4) A uniformly charged ring, (5) A uniformly charged disk, (6) A capacitor, (7) A uniformly charged hollow sphere, (8) None of the above

If the force at B were 0.0799 N, what would be your answer? If the force at B were 0.01 N, what would be your answer? If the force at B were 0.02 N, what would be your answer?

•P51 A thin plastic spherical shell of radius 5 cm has a uniformly distributed charge of −25 nC on its outer surface. A concentric thin plastic spherical shell of radius 8 cm has a uniformly distributed charge of +64 nC on its outer surface. Find the magnitude and direction of the electric field at distances of 3 cm, 7 cm, and 10 cm from the center. See Figure 15.63.

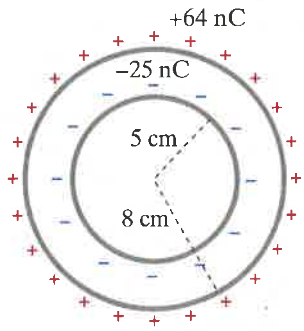

Figure 15.63

•P52 A hollow ball of radius 7 cm, made of very thin glass, is rubbed all over with a silk cloth and acquires a negative charge of -9×10^{-8} C that is uniformly distributed all over its surface. Location A in Figure 15.64 is inside the sphere, 1 cm from the surface. Location B in Figure 15.64 is outside the sphere, 2 cm from the surface. There are no other charged objects nearby.

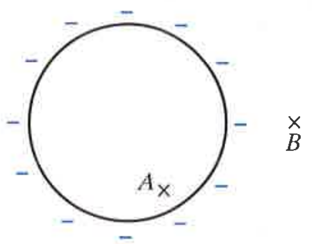

Figure 15.64

Which of the following statements about E_{ball}, the magnitude of the electric field due to the ball, are correct? Select all that apply. **(a)** At location A, E_{ball} is 0 N/C. **(b)** All of the charges on the surface of the sphere contribute to E_{ball} at location A. **(c)** A hydrogen atom at location A would polarize because it is close to the negative charges on the surface of the sphere. What is E_{ball} at location B?

••P53 A thin hollow spherical glass shell of radius 0.17 m carries a uniformly distributed positive charge $+6 \times 10^{-9}$ C, as shown in Figure 15.65. To the right of it is a horizontal permanent dipole with charges $+3 \times 10^{-11}$ and -3×10^{-11} C separated by a distance 2×10^{-5} m (the dipole is shown greatly enlarged for clarity). The dipole is fixed in position and is not free to rotate. The distance from the center of the glass shell to the center of the dipole is 0.6 m.

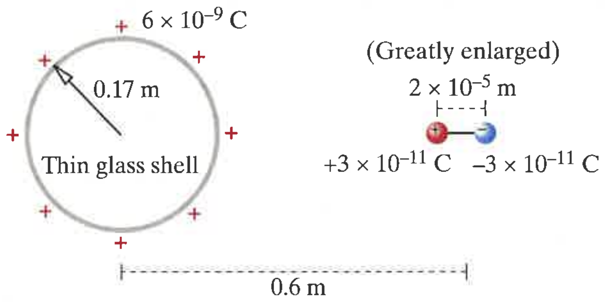

Figure 15.65

(a) Calculate the net electric field at the center of the glass shell. **(b)** If the sphere were a solid metal ball with a charge $+6 \times 10^{-9}$ C, what would be the net electric field at its center? **(c)** Draw the approximate charge distribution in and/or on the metal sphere.

••P54 Two large, thin, charged plastic circular plates each of radius R are placed a short distance s apart; s is much smaller than the dimensions of a plate (Figure 15.66). The right-hand plate has a positive charge of $+Q$ evenly distributed over its inner surface (Q is a positive number). The left-hand plate has a negative charge of $-2Q$ evenly distributed over its inner surface. A very thin plastic spherical shell of radius r is placed midway between the plates (and shown in cross section). It has a uniformly distributed positive charge of $+q$. You can ignore the contributions to the electric field due to the polarization of the thin plastic shell and the thin plastic plates.

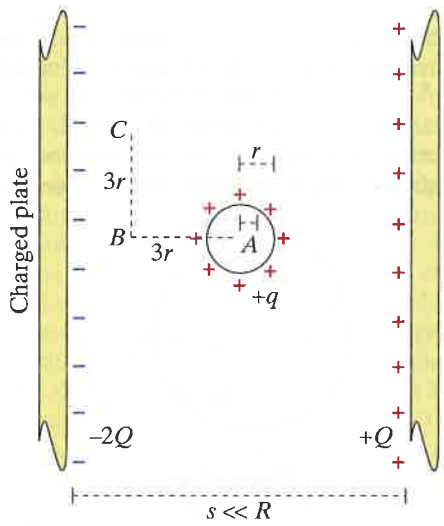

Figure 15.66

(a) Calculate the x and y components of the electric field at location A, a horizontal distance $r/2$ to the right of the center of the sphere. **(b)** Calculate the x and y components of the electric field at location B, a horizontal distance $3r$ to the left of the center of the sphere. **(c)** Calculate the x and y components of the electric field at location C, a horizontal distance $3r$ to the left and a vertical distance $3r$ above the center of the sphere.

••P55 A glass sphere carrying a uniformly distributed charge of $+Q$ is surrounded by an initially neutral spherical plastic shell (Figure 15.67).

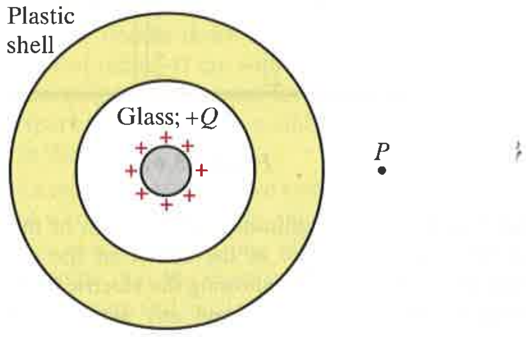

Figure 15.67

(a) Qualitatively, indicate the polarization of the plastic. **(b)** Qualitatively, indicate the polarization of the inner glass sphere. Explain briefly. **(c)** Is the electric field at location P outside the plastic shell larger, smaller, or the same as it would be if the plastic weren't there? Explain briefly. **(d)** Now suppose that the glass sphere carrying a uniform charge of $+Q$ is surrounded by an initially neutral metal shell (Figure 15.68). Qualitatively, indicate the polarization of the metal.

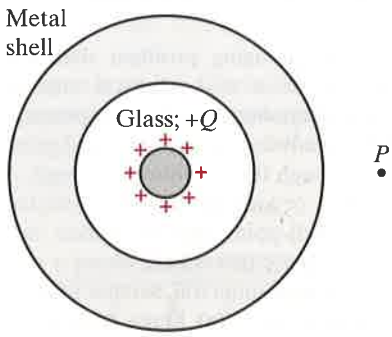

Figure 15.68

(e) Now be *quantitative* about the polarization of the metal sphere and prove your assertions. **(f)** Is the electric field at location P outside the metal shell larger, smaller, or the same as it would be if the metal shell weren't there? Explain briefly.

••P56 Breakdown field strength for air is roughly 3×10^6 N/C. If the electric field is greater than this value, the air becomes a conductor. **(a)** There is a limit to the amount of charge that you can put on a metal sphere in air. If you slightly exceed this limit, why would breakdown occur, and why would the breakdown occur very near the surface of the sphere, rather than somewhere else? **(b)** How much excess charge can you put on a metal sphere of 10-cm radius without causing breakdown in the neighboring air, which would discharge the sphere? **(c)** How much excess charge can you put on a metal sphere of only 1-mm radius? These results hint at the reason why a highly charged piece of metal tends to spark at places where the radius of curvature is small, or at places where there are sharp points.

••P57 Two thin plastic spherical shells (shown in cross section in Figure 15.69) are uniformly charged. The center of the larger

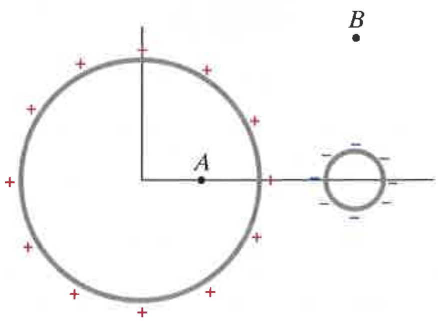

Figure 15.69

sphere is at (0,0); it has a radius of 12 cm and a uniform positive charge of $+4 \times 10^{-9}$ C. The center of the smaller sphere is at (25 cm, 0); it has a radius of 3 cm and a uniform negative charge of -1×10^{-9} C. **(a)** What are the components $E_{A,x}$ and $E_{A,y}$ of the electric field $\Delta\vec{E}$ at location A (6 cm to the right of the center of the large sphere)? Neglect the small contribution of the polarized molecules in the plastic, because the shells are very thin and don't contain much matter. **(b)** What are the components $E_{B,x}$ and $E_{B,y}$ of the electric field at location B (15 cm above the center of the small sphere)? Neglect the small contribution of the polarized molecules in the plastic, because the shells are very thin and don't contain much matter. **(c)** What are the components F_x and F_y of the force on an electron placed at location B?

••P58 A solid plastic sphere of radius R_1 has a charge $-Q_1$ on its surface (Figure 15.70). A concentric spherical metal shell of inner radius R_2 and outer radius R_3 carries a charge Q_2 on the inner surface and a charge Q_3 on the outer surface. Q_1, Q_2, and Q_3 are positive numbers, and the total charge $Q_2 + Q_3$ on the metal shell is greater than Q_1.

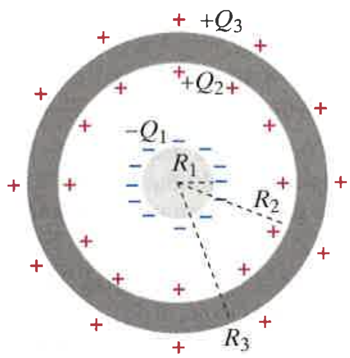

Figure 15.70

At an observation location a distance r from the center, determine the magnitude and direction of the electric field in the following regions, and explain briefly in each case. For parts (a)–(d), be sure to give both the direction and the magnitude of the electric field, and explain briefly: **(a)** $r < R_1$ (inside the plastic sphere), **(b)** $R_1 < r < R_2$ (in the air gap), **(c)** $R_2 < r < R_3$ (in the metal), **(d)** $r > R_3$ (outside the metal). **(e)** Suppose $-Q_1 = -5$ nC. What is Q_2? Explain fully on the basis of fundamental principles. **(f)** What can you say about the molecular polarization in the plastic? Explain briefly. Include a drawing if appropriate.

••P59 A small, thin, hollow spherical glass shell of radius R carries a uniformly distributed positive charge $+Q$. Below it is a horizontal permanent dipole with charges $+q$ and $-q$ separated by a distance s (s is shown greatly enlarged in Figure 15.71 for clarity). The dipole is fixed in position and is not free to rotate. The distance from the center of the glass shell to the center of the dipole is L.

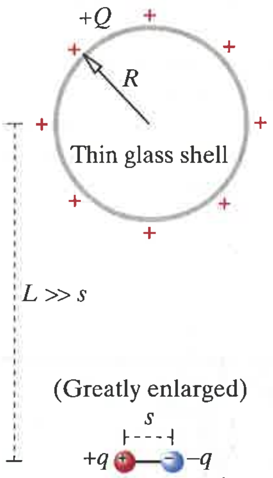

Figure 15.71

(a) Calculate the magnitude and direction of the electric field at the center of the glass shell, and explain briefly, including showing the electric field on the diagram. Your results must not contain any symbols other than the given quantities R, Q, q, s, and L (and fundamental constants), unless you define intermediate results in terms of the given quantities. What simplifying assumption do you have to make? **(b)** If the upper sphere were a solid metal ball with a charge $+Q$, what would be the magnitude and direction of the electric field at its center? Explain briefly. Show the distribution of charges everywhere, and at the center of the ball accurately draw and label the electric field due to the ball charges $\vec{E}_{\text{ball}}$ and the electric field $\vec{E}_{\text{dipole}}$ of the dipole.

••P60 A very thin glass rod 4 m long is rubbed all over with a silk cloth (Figure 15.72). It gains a uniformly distributed charge $+1.3 \times 10^{-6}$ C. Two small spherical rubber balloons of radius 1.2 cm are rubbed all over with wool. They each gain a uniformly distributed charge of -2×10^{-8} C. The balloons are near the midpoint of the glass rod, with their centers 3 cm from the rod. The balloons are 2 cm apart (4.4 cm between centers).

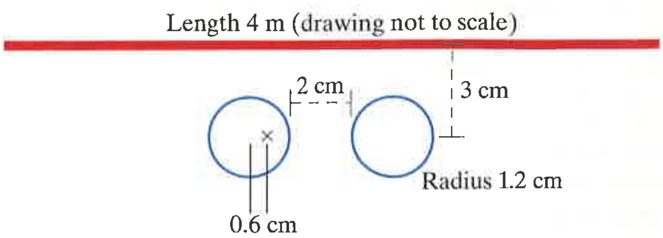

Figure 15.72

(a) Find the magnitude of the electric field at the location marked by the ×, 0.6 cm to the right of the center of the left balloon. Also calculate the angle the electric field makes with the horizontal. Show all your work, including showing vectors. State any approximations you are making. **(b)** Next, a neutral hydrogen atom is placed at that same location (marked by the ×). Draw a diagram showing the effect on the hydrogen atom while it is at that location. The polarizability of atomic hydrogen has been measured to be $\alpha = 7.4 \times 10^{-41}$ C·m/(N/C). What is the distance between the center of the proton and the center of the electron cloud in the hydrogen atom?

••P61 A very long thin wire of length L carries a total charge $+Q$, nearly uniformly distributed along the wire (Figure 15.73). The center of a small neutral metal ball of radius r is located a distance d from the wire, where $d \ll L$. Determine the approximate magnitude of the force that the rod exerts on the ball. Show labeled physics information on the diagram. Explain how you model the situation.

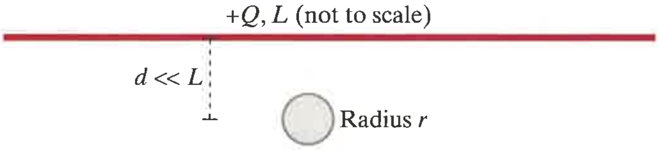

Figure 15.73

Section 15.7

•••P62 A simplified model of a hydrogen atom is that the electron cloud is a sphere of radius R with uniform charge density and total charge $-e$. (The actual charge density in the ground state is nonuniform.) See Figure 15.74.

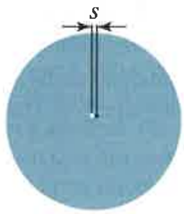

Figure 15.74

(a) For the uniform-density model, calculate the polarizability α of atomic hydrogen in terms of R. Consider the case where the magnitude E of the applied electric field is much smaller than the electric field required to ionize the atom. Suggestions for your analysis: Imagine that the hydrogen atom is inside a capacitor whose uniform field polarizes but does not accelerate the atom. Consider forces on the proton in the equilibrium situation, where the proton is displaced a distance s from the center of the electron cloud ($s \ll R$ in the diagram). **(b)** For a hydrogen atom, R can be taken as roughly 1×10^{-10} m (the Bohr model of the H atom gives $R = 0.5 \times 10^{-10}$ m). Calculate a numerical value for the polarizability α of atomic hydrogen. For comparison, the measured polarizability of a hydrogen atom is $\alpha = 7.4 \times 10^{-41}$ C·m(N/C); see the note below. **(c)** If the magnitude E of the applied electric field is 1×10^6 N/C, use the measured value of α to calculate the shift s shown in Figure 15.74. **(d)** For some purposes it is useful to model an atom as though the nucleus and electron cloud were connected by a spring. Use the measured value of α to calculate the effective spring stiffness k_s for atomic hydrogen. For comparison, measurements of Young's modulus show that the effective spring stiffness of the *interatomic* force in solid aluminum is about 16 N/m. **(e)** If α were twice as large, what would k_s be?

Note: Quantum-mechanical calculations agree with the experimental measurement of α reported in T. M. Miller and B. Bederson, "Atomic and molecular polarizabilities: a review of recent advances," *Advances in Atomic and Molecular Physics* **13**, 1–55, 1977. They use cgs units, so their value is $1/(4\pi\varepsilon_0)$ greater than the value given here.

COMPUTATIONAL PROBLEMS

More detailed and extended versions of some of these computational modeling problems may be found in the lab activities included in the *Matter & Interactions* resources for instructors.

•P63 Start with the program in Section 15.9 that calculates the electric field of a uniformly charged rod at a single location on the midplane. **(a)** Improve the accuracy of the calculation by increasing the number of point charges used to model the rod. Determine experimentally the minimum number of point charges required to get an accurate answer, and explain your criterion for accuracy. **(b)** Use your improved program to calculate the electric field of the rod at location $\langle 0.05, -0.35, 0 \rangle$ m, and display your result with an arrow. Report the value of the electric field you calculated.

•P64 Write a program to model a long thin rod of length 0.5 m with a net charge of −2 nC, lying on the x axis and centered at the origin. **(a)** Calculate the electric field of the rod at location $\langle -0.4, 0.3, 0.3 \rangle$ m, and display the field with an arrow. **(b)** Determine experimentally the minimum number of point charges required to give an accurate result, and explain how you determined this.

••P65 A strip of invisible tape 12 cm by 2 cm is charged uniformly with a total net charge of 4×10^{-8} and hangs vertically. Write a program to calculate and display both the tape and the electric field due to this strip of tape, at a location a perpendicular distance of 3 cm from the center of the tape. Use the same scale factor to display the arrow representing the electric field in all parts of this problem. **(a)** Divide the strip into three sections, each 4 cm high by 2 cm wide, and use numerical summation to calculate the magnitude of the electric field at a perpendicular distance of 3 cm from the center of the tape. **(b)** Divide the strip into eight sections, each 3 cm high by 1 cm wide, and use numerical summation to calculate the magnitude of the electric field at a perpendicular distance of 3 cm from the center of the tape. Compare the result to your previous result. **(c)** Divide the tape into a large number of small sections. Compare your result to your previous results.

•**P66** Start with the program in Section 15.9 that calculates the electric field of a uniformly charged rod, modeled as 10 point charges, at multiple observation locations in a circle around the rod. **(a)** Add more arrows to the list named "observation," so that the electric field of the rod is displayed with 5 rings of arrows centered on the rod, at different locations. **(b)** Increase the number of point charges used to model the rod to increase the accuracy of the calculation. Explain your criteria for determining an appropriate number of point charges.

••**P67** Write a computer program to compute and display the electric field of a thin rod of length 2 m, aligned along the z axis and centered at the origin. A total charge of −60 nC is uniformly distributed along the surface of the rod. Start by modeling the rod as a collection of 5 point charges. **(a)** Increase the number of point charges until you are satisfied with the accuracy of the calculation. Explain the criteria you used to make this decision. **(b)** Display the electric field at locations around the rod with arrows arranged in 6 rings centered on the rod, at different z locations along the rod. Scale the arrows appropriately to fit the display.

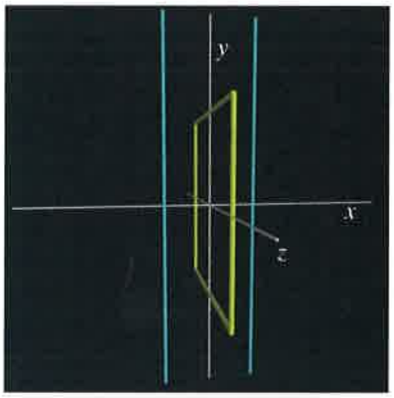

Figure 15.75

•••**P68** Four thin rods, each of length 0.4 m, form a square loop in the yz plane, as shown in yellow in Figure 15.75. A charge of 3 nC is uniformly distributed over the surface of each rod (the net charge of the loop is 12 nC). Write a program to calculate and display the electric field of the loop at 12 observation locations: six that lie on a vertical line that crosses the x axis at $x = -0.1$ m, and six that lie on a vertical line that crosses the x axis at $x = +0.1$ m. These lines are shown in cyan in Figure 15.75.

•••**P69** Model a uniformly charged ring as a set of point charges placed on a circle. Place the ring in the yz plane so that its axis is along the x axis. Give the ring a total charge of $Q = 50 \times 10^{-9}$ C and a radius of $R = 0.1$ m. **(a)** Start an electron from rest at location $\langle 0.15, 0, 0\rangle$ m, and model its motion. In each step you will need to calculate the net electric field at the location of the electron. Describe the motion you observe. **(b)** Vary `deltat` and the number of point charges used to model the uniformly charged ring to make sure your calculation is accurate. **(c)** Experiment with different initial locations that do not lie on the axis of the ring. You may also wish to experiment with giving the electron some initial momentum. Make a screen shot of the most interesting trajectory you find. Figure 15.76 is an example of one such trajectory.

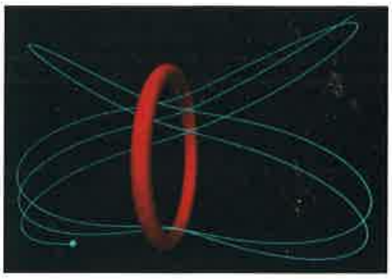

Figure 15.76

ANSWERS TO CHECKPOINTS

1 1.12×10^4 N/C
2 **(a)** 4.51×10^4 N/C to the right; **(b)** 2.25×10^{-4} N to the left; **(c)** zero
3 2.684×10^6 N/C; 2.657×10^6 N/C
4 About 2.1×10^{-5} C; about 3000 N/C
5 Although the other charges are much farther away, there are a lot more of them, and in a sphere the two effects exactly cancel each other. The electric field at location D is zero.