Backreaction status report

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The Friedmann-Robertson-Walker equations

- Assuming general relativity (GR) + exact spatial homogeneity and isotropy, the universe is described by the FRW equations:

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G_N \rho - 3\frac{K}{a^2}$$

$$3\frac{\ddot{a}}{a} = -4\pi G_N(\rho + 3p)$$

- Such a model with normal matter (dust and radiation) works well at early times.
  - The spatially flat model with dust is called the Einstein-de Sitter (EdS) model.

Looking for a factor of 2

- Around 10 billion years, the expansion rate rises by about 50% relative to the EdS model. (From H_0t_0=2/3 to H_0t_0≈1.)

- Three possibilities:

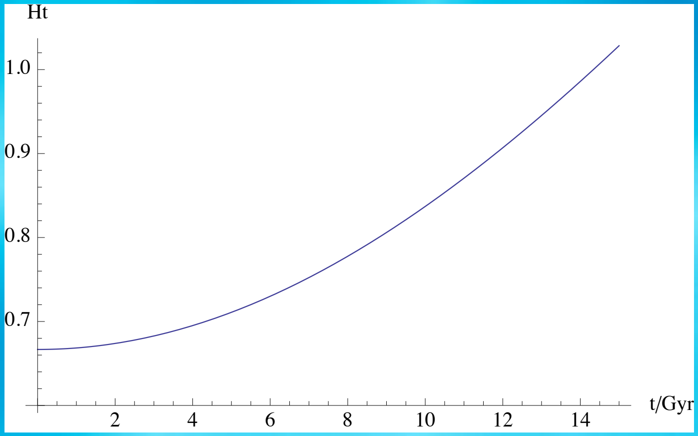
  1) There is matter with negative pressure.

  2) General relativity does not hold.

  3) The homogeneous and isotropic approximation is not valid.

Cosmological constant

- Observations are consistent with an FRW model with added cosmological constant Λ.
  - The coincidence problem: why 10 billion years?



- The posterior for any model that did not predict small deviation from ΛCDM is lower than it was 20 years ago.
- Large deviations from from ΛCDM are still allowed.

Our clumpy universe

- At late times, the universe is only *statistically* homogeneous and isotropic, on scales >100 Mpc.
- The average evolution of a clumpy spacetime is not the same as the evolution of a smooth spacetime, a feature known as **backreaction**. (Ellis 1984, Buchert and SR: 1112.5335)
- Structures affect expansion rate, light propagation and their relationship.
- **The backreaction conjecture**: the reason for the failure of the exactly homogeneous and isotropic dust model is the known breakdown of local homogeneity and isotropy.

The Buchert equations

- The Buchert equations (Buchert: gr-qc/9906015)

$$\begin{cases} 3\frac{\ddot{a}}{a} = -4\pi G\langle\rho\rangle + Q \\ 3\frac{\dot{a}^2}{a^2} = 8\pi G\langle\rho\rangle - \frac{1}{2}\langle{}^{(3)}R\rangle - \frac{1}{2}Q \\ \partial_t\langle\rho\rangle + 3\frac{\dot{a}}{a}\langle\rho\rangle = 0 \end{cases}$$

$$3\frac{\ddot{a}}{a} = -4\pi G\rho$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G\rho - 3\frac{k}{a^2}$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}\rho = 0$$

- The backreaction variable is

$$Q \equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle.$$

$$\langle f\rangle \equiv \frac{\int d^3x\sqrt{{}^{(3)}g}\,f}{\int d^3x\sqrt{{}^{(3)}g}}$$

- The average expansion can accelerate, even though the local expansion decelerates.

Understanding acceleration

- The average expansion rate can increase, because the fraction of volume in faster regions grows.
- Structure formation involves overdense regions decelerating more and underdense regions decelerating less.
- Acceleration can be demonstrated with a toy model which has one overdense and one underdense region. (SR: astro-ph/0607626)

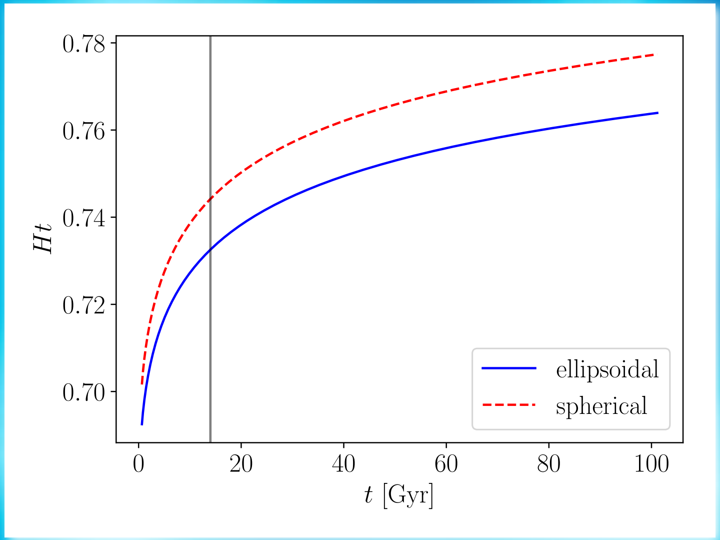
$$H \equiv \frac{\dot{a}}{a} = \frac{a_1^3}{a_1^3 + a_2^3} H_1 + \frac{a_2^3}{a_1^3 + a_2^3} H_2 = v_1 H_1 + v_2 H_2$$

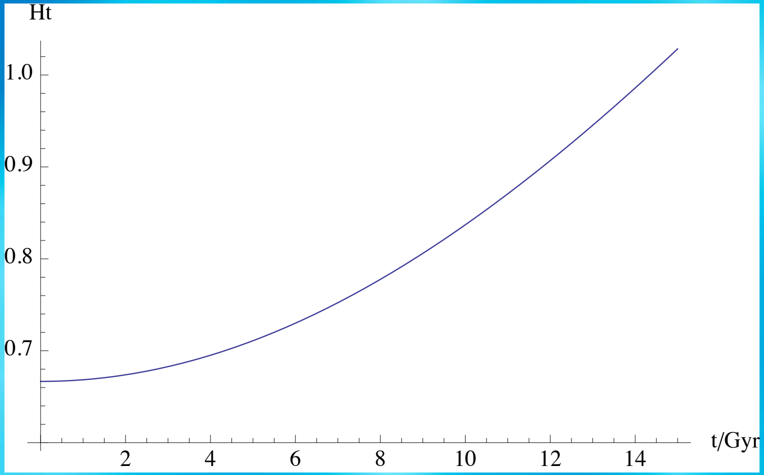
$$\frac{\ddot{a}}{a} = v_1 \frac{\ddot{a}_1}{a_1} + v_2 \frac{\ddot{a}_2}{a_2} + 2 v_1 v_2 (H_1 - H_2)^2$$

A simple estimate

- Take a smooth background with an initial Gaussian linear density field.
- Identify structures with ellipsoidal isolated peaks of the smoothed density field. (SR: 0801.2692, Montanari and SR: 1710.02451)
- Each peak evolves separately.
- The peak number density as a function of time is determined by the power spectrum.
- The expansion rate is $$H(t) = \int_{-1}^{\infty} d\delta \, v_\delta(t) H_\delta(t).$$

Two things right





- The peak model gets amplitude and timing roughly right.
- $2/3 < Ht < 1$ because the volume is dominated by underdense voids.
- The timescale $t \sim A^{-3/2} t_{eq} \sim 10^{11}$ yr is imprinted on the perturbation spectrum, where $A = 3\times10^{-5}$ is the primordial amplitude of perturbations.

Towards reality beyond Newton

- Acceleration due to structures is possible: is it realised in the universe?
- Non-linear evolution is usually studied with *N*-body simulations.
  - Simulations use Newtonian gravity with periodic boundary conditions.
- In Newtonian gravity, backreaction reduces to a boundary term. (Buchert, Ehlers: astro-ph/9510056)
- This is not true in GR. (Buchert: gr-qc/9906015)

So how about the details?

Can the Acceleration of Our Universe Be Explained by the Effects of Inhomogeneities?

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February 4, 2008

Abstract

No. It is simply not plausible that cosmic acceleration could arise within the context of general relativity from a back-reaction effect of inhomogeneities in our universe, without the presence of a cosmological constant or "dark energy." We point out that our universe appears to be described very accurately on all scales by a Newtonianly perturbed FLRW metric. (This assertion is entirely consistent with the fact that we commonly encounter $\delta\rho/\rho > 10^{30}$.) If the universe is accurately described by a Newtonianly perturbed FLRW metric, then the back-reaction of inhomogeneities on the dynamics of the universe is negligible. If not, then it is the burden of an alternative model to account for the observed properties of our universe. We emphasize with concrete examples that it is *not* adequate to attempt to justify a model by merely showing that some spatially averaged quantities behave the same way as in FLRW models with acceleration. A quantity representing the "scale factor" may "accelerate" without there being any physically observable consequences of this acceleration. It also is *not* adequate to calculate the second-order stress energy tensor and show that it has a form similar to that of a cosmological constant of the appropriate magnitude. The second-order stress energy tensor is gauge dependent, and if it were large, contributions of higher perturbative order could not be neglected. We attempt to clear up the apparent confusion between the second-order stress energy tensor arising in perturbation theory and the "effective stress energy tensor" arising in the "shortwave approximation."

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Ishibashi and Wald: gr-qc/0509108

would not be bound. But if one cannot neglect nonlinear terms in Einstein's equation on small scales, how can one justify neglecting them on large (i.e., ~ 100 Mpc or larger) scales? In addition, since it is not clear exactly what approximations are needed for assumptions (1)–(3) to be valid, it is far from clear as to how one could go about systematically improving these approximations.

Indeed, it is far from obvious, *a priori*, that nonlinearities associated with small-scale inhomogeneities could not produce important effects on the large-scale dynamics of the FLRW model itself, as has been suggested by a number of authors [5–17] as a possible way to account for the effects of "dark energy" without invoking a cosmological constant, a new source of matter, or a modification of Einstein's equation. In fact, the example of gravitational radiation of wavelength much less than the Hubble scale illustrates that it is possible, in principle, for small-scale inhomogeneities in the metric and curvature to affect large-scale dynamics. The dynamics of a FLRW model whose energy content is dominated by gravitational radiation will be very different from one with a similar matter content but no gravitational radiation. It is the nonlinear terms in Einstein's equation associated with the short-wavelength gravitational radiation that are responsible for producing this difference in the large-scale dynamics. Although common-sense estimates indicate that similar effects on large-scale dynamics should not be produced by nonlinear effects of small-scale matter inhomogeneities in our universe, it would be very useful to have a systematic and general approach that can determine exactly what effects small-scale inhomogeneities can and cannot produce on large-scale dynamics.

The main approach that has been taken to investigate the effects of small-scale inhomogeneities on large-scale dynamics has been to consider inhomogeneous models, take spatial averages to define corresponding FLRW quantities, and derive equations of motion for these FLRW quantities [18, 19]. Since, in particular, the spatial average of the square of a quantity does not equal the square of its spatial average, the effective FLRW dynamics of an inhomogeneous universe will differ from that of a homogeneous universe. However, a major difficulty with this approach is that, when the deviations of the metric from that of a FLRW background are not very small, it is not obvious how to interpret the averaged quantities in terms of observable quantities. For example, if the total volume of a spatial region is found to increase with time, this certainly does not imply that observers in this region will find that Hubble's law appears to be satisfied. Further serious difficulties with

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Green and Wald: 1011.4920

So how about the details?

- In 2010, Green & Wald introduced a new formalism for perturbation theory, claiming that it shows that backreaction is small. (Green, Wald: 1011.4920)
- The formalism introduces a family of metrics $g_{\alpha\beta}(x,\lambda)$ where the singular limit $\lambda \to 0$ corresponds to the real universe.
- This limit does not describe spatial averaging. (Buchert et al: 1505.07800)
- The formalism assumes that perturbations are small, from which the conclusion follows without the new formalism. (SR: 1107.1176)

What about the Newtonian limit?

- It has been argued that because a Newtonian solution can be mapped to a perturbed FRW metric (with small corrections to the equations of motion), backreaction is small. (Green and Wald: 1111.2997)

- However, the problem at hand is the reverse: how well is a GR solution approximated by a Newtonian one?
  - Compare to the fact that a Newtonian binary system can be at all times well approximated by a perturbed Minkowski metric (with small corrections)

Analytical work

- Perturbative studies.
  - If we show that the metric remains close to FRW, we will establish that backreaction is small.
  - If we show that it doesn't, this will invalidate the usual analysis, but does not establish that backreaction is large.
- Statistical models.
  - Using collections of regions, it has been shown that backreaction could lead to acceleration.
  - The difference between Newtonian and GR constraints has to be carefully addressed.

Simulations

- Cosmological GR simulations can establish whether backreaction is small or large. (Giblin, Mertens, Starkman: 1511.01105, 1511.01106, 1704.04307; Bentivegna and Bruni: 1511.05124, 1610.05198; Macpherson, Lasky, Price: 1611.05447)
- Simulations so far have not been realistic.
- Intermediate step to a realistic case: showing that the effect can be large in a reasonable toy model.

Observations

- If backreaction is significant, the universe is not on average described by the FRW metric.
- If we can observationally rule out the FRW metric, this would provide strong support for backreaction.
- Backreaction has a unique observational signature: specific deviations from FRW consistency conditions. (Clarkson et al: 0712.3457, SR: 1308.6731, SR et al: 1412.4976)

FRW consistency conditions

- Consistency between angular diameter distance and expansion rate. (Clarkson et al: 0712.3457)

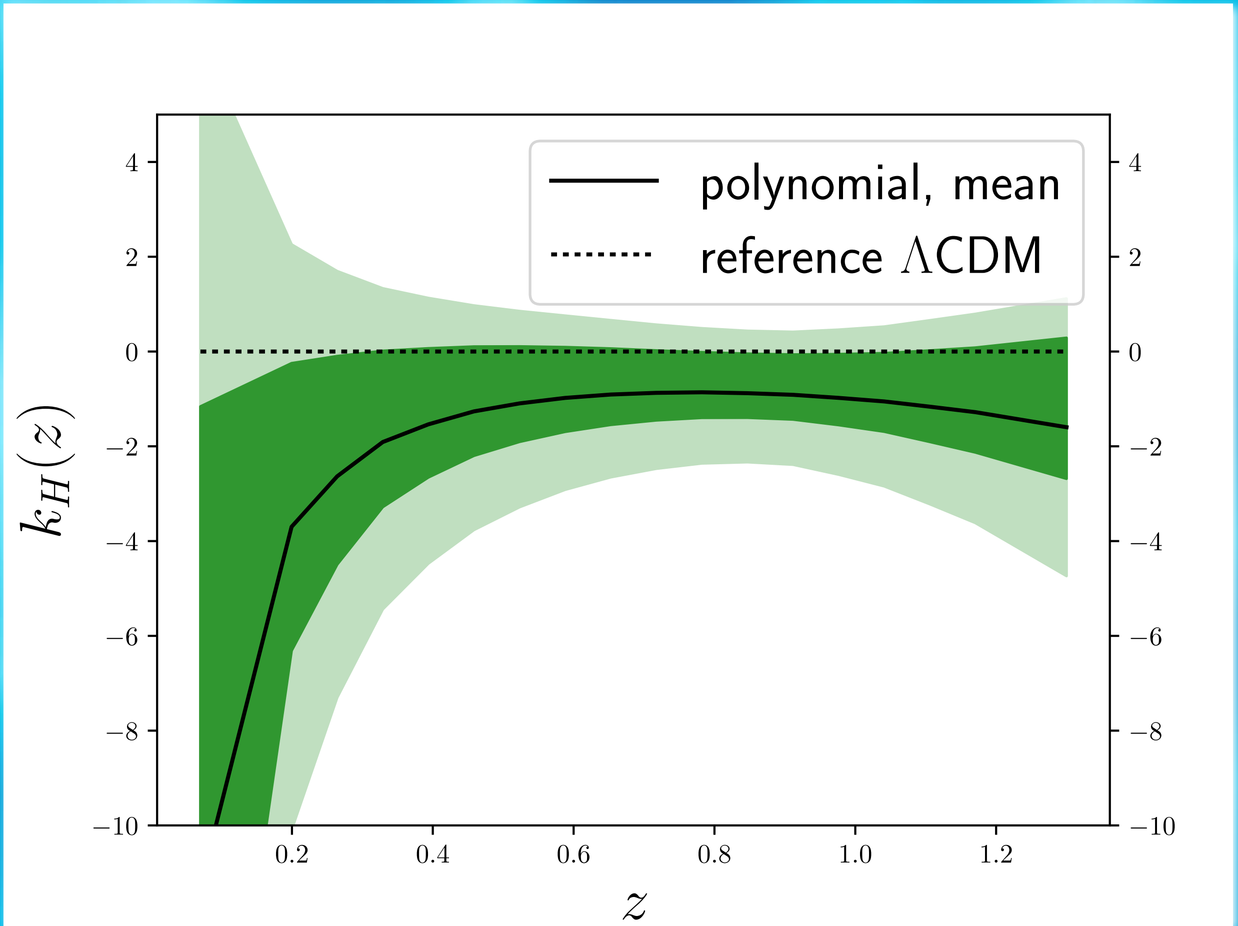
$$d(z) = \frac{1}{\sqrt{-k}} \sinh\left(\sqrt{-k}\int_0^z \frac{dz'}{h(z')}\right) \;\Rightarrow\; k_H = \frac{1 - h(z)^2 d'(z)^2}{d(z)^2}$$

$$d(z) = H_0(1+z)D_A(z)$$
$$h(z) = H(z)/H_0$$

- Also measurements of cosmic parallax and the distance sum rule. (SR: 1308.6731, SR et al: 1412.4976)

- If consistency is pushed to better than 1%, backreaction seems unlikely.

Room for deviation



Montanari and SR: 1709.06022

Conclusions

- Statistically homogeneous and isotropic spaces do not in general expand like FRW.
  - Structure formation has a timescale of 10 billion years.
  - Mechanism for acceleration: volume fraction of faster regions rises.
  - Local variations in the expansion rate are of the same size as the observed deviation from EdS.
- No evidence for deviations from ΛCDM.
  - If the metric is close to FRW, backreaction is small.
  - If non-Newtonian effects can be neglected, backreaction is small.
- It is possible to observationally test the FRW metric.
- Even if backreaction is small, it can be important for precision measurements.