

# Cosmic parallax

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# Change of viewpoint

- Parallax is the change in the angular position of objects when seen from different locations.
- Can measure distance without knowing source properties.  $D_p^{-1} = \delta\varphi / \delta x$ .
- Problem for the heliocentric system in the 1500's.
- Best current measurement by Hipparcos (1989-1993) at 100 pc.
- Can be used as a cosmological test. (McCrea 1935)

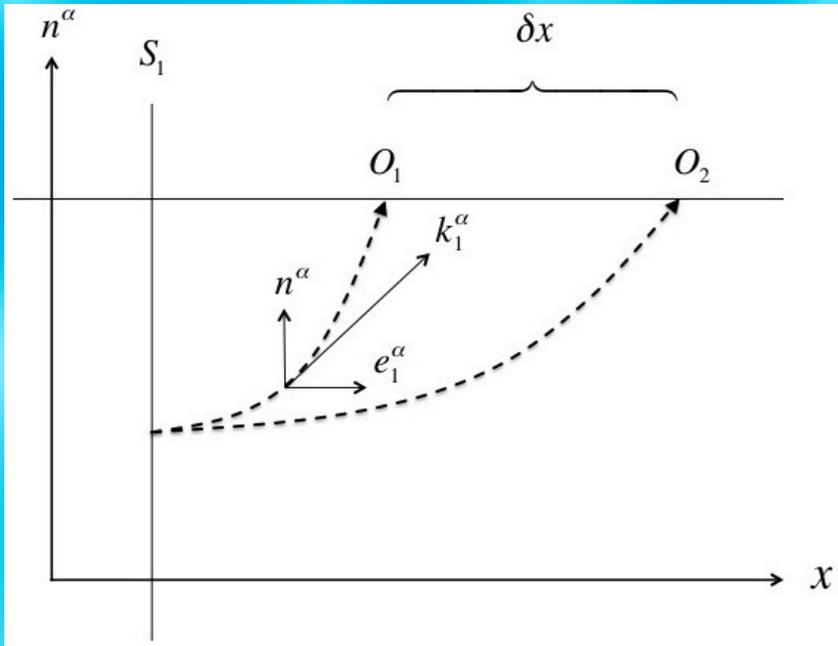
# Motion parallax

- Classic definition of parallax distance:  $D_p^{-1} = \delta\varphi / \delta x$ .
- Naively, maximum  $\delta x = 2$  AU.
- However, motion wrt. CMB frame is 78 AU/yr.  
(Kardashëv 1986)
- For adiabatic perturbations, CMB rest frame is the same as the rest frame of cosmological sources.

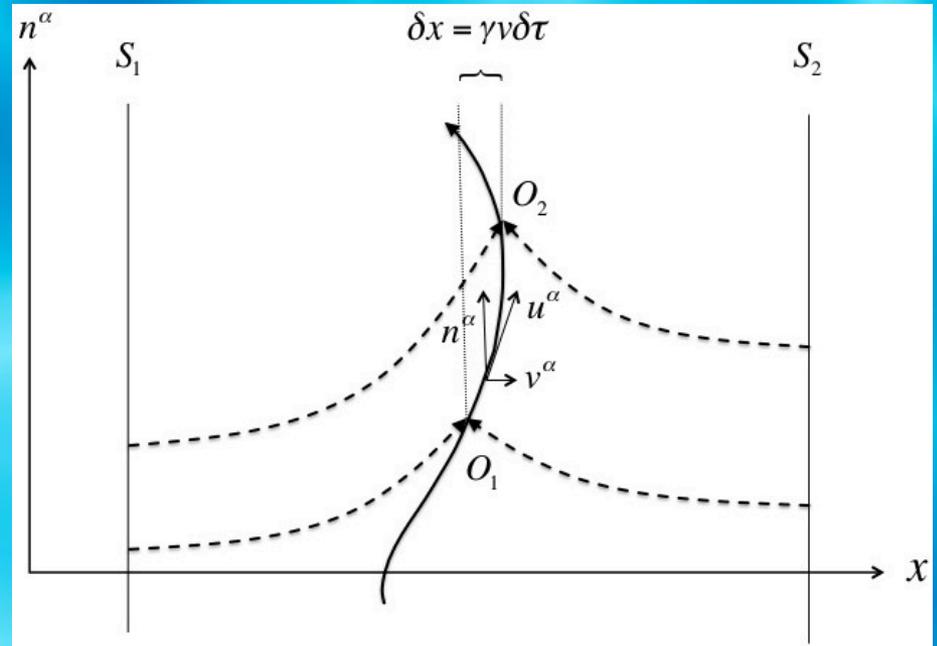
# Intrinsic parallax

- Assume that both sources and observers move along the same timelike curves.
- The angle between a pair of sources is constant for all pairs and all observers at all times if and only if the spacetime is conformally stationary. (Hasse and Perlick 1988)
- Dust matter: spacetime is FRW or stationary.
- Same condition as vanishing CMB anisotropy.

# Classic parallax



# Observational parallax



# Parallax in general

- Consider a pair of sources located in directions  $r_1$  and  $r_2$ . The angle  $\varphi$  between them is given by

$$g \equiv \cos \varphi = r_1 \cdot r_2.$$

- For observers moving along a timelike geodesic:

$$\begin{aligned} \frac{\delta \varphi}{\delta x} &= -\frac{1}{\sqrt{1-(r_1 \cdot r_2)^2}} \frac{\dot{g}}{v} \\ &= \frac{1}{\sqrt{1-(r_1 \cdot r_2)^2}} \left[ \left( \frac{1}{2v} P_{\perp 11} \cdot r_2 - (r_2 \cdot \hat{v} - r_1 \cdot r_2 \ r_1 \cdot \hat{v}) \frac{1}{2} E_1^{-1} \tilde{\theta}_1 - E_1^{-1} \tilde{\sigma}_{1\alpha\beta} r_2^\alpha \hat{v}^\beta + (1 \leftrightarrow 2) \right) \right] \end{aligned}$$

intrinsic parallax

motion parallax related  
to distance

motion parallax related  
to image deformation  
(small)

# Parallax distance

- Classic definition of parallax distance:  $D_P^{-1} = \frac{\delta\varphi}{\delta x}$
- Suitable for a single source and spacelike separated observation points.
- Real observations: relative angles, timelike separated observation points.
- Modern definition:  $D_P^{-1} = \left( \frac{1}{2} E^{-1} \tilde{\theta} \right)_0$  (Ellis 1971)

# $D_P$ and $D_A$

- ‘Parallax is angular diameter in reverse.’
- For  $D_P$ , beam converges at the source, for  $D_A$  it converges at the observer.

$$\frac{1}{E_0} \frac{dD_P^{-1}}{d\lambda} = D_A^{-2}. \quad (\text{No image deformation}) \quad (\text{Rosquist 1988})$$

- Compare  $D_L = (1+z)^2 D_A$ . (Etherington 1933)

# Parallax in the FRW universe

- In the FRW case there is no intrinsic parallax:

$$\frac{\delta\varphi}{\delta x} = -\frac{1}{\sqrt{1-(r_1 \cdot r_2)^2}} \left[ (r_2 \cdot \hat{v} - r_1 \cdot r_2 \ r_1 \cdot \hat{v}) D_{P1}^{-1} + (r_1 \cdot \hat{v} - r_1 \cdot r_2 \ r_2 \cdot \hat{v}) D_{P2}^{-1} \right].$$

- The parallax distance is (McCrea 1935)

$$d_p(z) = \frac{d(z)}{d(z) + \sqrt{1 - kd(z)^2}}.$$

$$(d_p \equiv H_0 D_p, \quad d \equiv H_0(1+z)D_A, \quad k \equiv H_0^2 K = -\Omega_{K0})$$

- Solving for  $k$ , we have

$$k = \frac{1}{d(z)^2} - \left( \frac{1}{d_p(z)} - 1 \right)^2.$$

# Testing the FRW metric

$$k_p = \frac{1}{d} - \left( \frac{1}{d_p} - 1 \right)^2$$

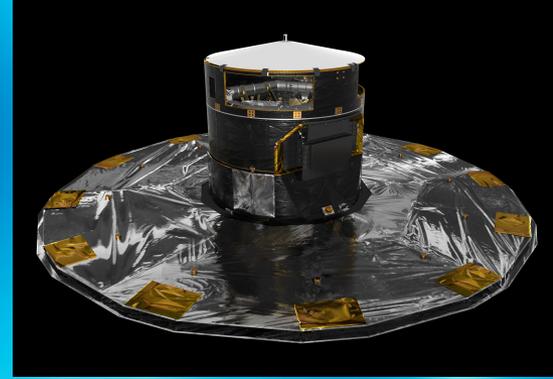
- If  $k_p$  is not constant, universe is not FRW. (More precisely: the optical properties of the universe are not FRW.)
- This test of the  $D$ - $D_p$  relation is independent of matter content or the Einstein equation.
- Test of FRW kinematics using only null geodesics. (Geometrical optics, no dynamics.)

- Similar to the  $D$ - $H$  relation (Clarkson et al 2007):  $k_H = \frac{1 - (HD')^2}{d^2}$ .

# Beyond FRW

- Backreaction: statistical homogeneity and isotropy does not imply FRW.
- Parallax can be used to test specific models violating homogeneity and isotropy, like large voids: real-time cosmology. (Quercellini et al 2008)
- Perturbations have to be taken into account: intrinsic perturbation parallax is of the same order of magnitude as motion parallax.
- Correct treatment of local motion requires non-linear treatment.

# Gaia



- ESA satellite launched on 19.12.2013.
- Will measure 3 million galaxies up to  $z=0.75$  and 500 000 quasars up to  $z=5$ .
- Precision  $100 \mu\text{as}$ .
- Cosmological signal:  $\delta\varphi \sim H_0 \delta x = H_0 v \delta t \sim 10^{-2} \mu\text{as } \delta t/\text{yr}$ .
- Errors down by  $1/(2N)^{1/2} \sim 10^{-3}$ , so feasible? (Rest frame?)

# Future

- Cosmic parallax provides a powerful test of the FRW metric.
- Will be first probed by the Gaia satellite (some parallax data out in 2016, full data in 2022).
- More work needed for the cosmological analysis.