Due on Monday January 27 by 14.15.

1. **Four-force and three-force.** The three-force is defined as

$$F^i \equiv \frac{1}{\gamma} f^i \,,$$

where f^i is the spatial part of the four-force

$$f^\mu = \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = ma^\mu \ .$$

Show that $f^0 = \gamma \vec{F} \cdot \vec{v}$, where $\vec{v}$ is the three-velocity.

2. **Index gymnastics.** Consider tensor A and vector $\underline{v}$ in Minkowski space, with components

$$A^{\alpha\beta} = \begin{pmatrix} 1 & 0 & -2 & -1 \\ -1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 0 \\ -2 & 1 & 1 & 2 \end{pmatrix} \,, \qquad v^\alpha = (2, 1, -1, 0) \ .$$

Find the components a) $A^\alpha{}_\beta$, b) $A_\alpha{}^\beta$, c) $A^{(\alpha\beta)}$, d) $A_{[\alpha\beta]}$, e) $A^\alpha{}_\alpha$, f) $v^\alpha v_\alpha$, g) $v_\alpha A^{\alpha\beta}$, and h) $v_\beta A^{\alpha\beta}$.

Symmetrisation and antisymmetrisation (indicated by () and []) are defined in section 2.3.4 of the lecture notes.

3. **Lorentz transformation of the electric and magnetic field components.** Derive the transformation properties (1.60) for the electric and the magnetic field under Lorentz transformations.

4. **Newtonian gravity.** Show that for a system of a finite number of point particles in infinite 3-dimensional Euclidean space, Newton's second law plus the inverse square law are equivalent to the Poisson equation and $\ddot{x}^i = -\delta^{ij}\partial_j\phi$ (where dot stands for time derivative).

If there are particles all the way to infinity, does the proof of equivalence still go through? If not, what problems are there? (You don't have to solve them.)

5. **Tangent vectors and gradients.** In 3-dimensional Euclidean space, let p be the point with Cartesian coordinates $(x, y, z) = (1, 0, -1)$. Consider the following curves that pass through p:

$$\begin{aligned} x^i(\lambda) &= \left(\lambda, (\lambda-1)^2, -\lambda\right) \\ x^i(\mu) &= (\cos\mu, \sin\mu, \mu - 1) \\ x^i(\sigma) &= \left(\sigma^2, \sigma^3 + \sigma^2, \sigma\right) \ . \end{aligned}$$

a) Calculate the components of the tangent vectors of these curves at p in the coordinate basis $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$.

b) Take $f = x^2 + y^2 - yz$. Calculate $\mathrm{d}f/\mathrm{d}\lambda$, $\mathrm{d}f/\mathrm{d}\mu$, and $\mathrm{d}f/\mathrm{d}\sigma$.