

Due on Monday January 19 by 14.15. Answers are returned via Moodle.  
 Doing homework exercises with machine learning algorithms (“AI”) is not allowed.

1. **Boost.** Show that the matrix

$$\Lambda^\alpha{}_\beta = \begin{pmatrix} \cosh \psi & -\sinh \psi & 0 & 0 \\ -\sinh \psi & \cosh \psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

satisfies the condition  $\Lambda^T \eta \Lambda = \eta$ .

2. **The twin non-paradox.** Consider the twins Alice and Betty. Alice stays on Earth. Betty leaves Alice and travels to Alpha Centauri (distance 4 light years) at the speed  $v = 0.8$ , turns instantly around, and returns at the same speed. How much have Alice and Betty aged when they meet again? Draw a spacetime diagram of the worldlines of Alice and Betty

- a) in the frame of Alice ( $K$ ),
- b) in the frames of Betty. (Note that at midpoint Betty changes to a frame of constant speed coming back ( $K''$ ), while the frame where she travelled towards  $\alpha$ Cen ( $K'$ ) keeps on going.)

3. **Null and timelike vectors.**

- a) Show that the sum of two future-pointing null vectors is a future-pointing timelike vector, except when the null vectors are parallel.
- b) Show that any timelike vector can be expressed as the sum of two null vectors. For a given timelike vector, the null vectors are not uniquely determined: what is the freedom in choosing them?

4. **Rotating coordinate system.** Minkowski space metric in Cartesian coordinates is

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2.$$

Find the metric in the merry-go-round coordinates defined by the transformation

$$\begin{aligned} t &= t' \\ x' &= \sqrt{x^2 + y^2} \cos(\varphi - \omega t) \\ y' &= \sqrt{x^2 + y^2} \sin(\varphi - \omega t) \\ z' &= z, \end{aligned}$$

where  $\varphi \equiv \arctan(y/x)$ , and  $\omega$  is a constant.