Due on Monday April 28 by 14.15. These are the last exercises.

1. **Killing vectors.** Show that if $\underline{U}$ and $\underline{V}$ are Killing vectors, then $[\underline{U}, \underline{V}]$ is a Killing vector.

2. **No static universe.** Consider a FLRW universe filled with matter with $\rho > 0, p \geq 0$ and $\omega \equiv p/\rho =$ constant.

   a) Show that there are no static solutions for $\Lambda \leq 0$.

   b) Show that there is a static solution for $\Lambda > 0$, and that it is unstable.

3. **Age of the Universe.** The expansion of the universe is well described by the ΛCDM model, where the universe is spatially flat, and there are two main energy components, matter with $w = 0$ and vacuum energy (Λ), with $w = -1$. (Radiation is only important for the first million years or so; we ignore it here.) This model is a good fit to the data if the Hubble constant (the current value of the Hubble parameter $H(t)$) is $H_0 = 67$ km/s/Mpc and the total energy density is 32% matter and 68% vacuum energy today, $\Omega_{m0} = \rho_m(t_0)/\rho(t_0) = 0.32$ and $\Omega_{\Lambda 0} = \rho_{\text{vac}}(t_0)/\rho(t_0) = 0.68$.

   a) Find the age of the universe t_0.

   b) At what time t_Λ were the matter and vacuum energy densities equal?

   c) Today the expansion is accelerating, $\ddot{a} > 0$. When did the acceleration begin ($\ddot{a} = 0$), in redshift and in time?

   (Hint: Use the substitution $x^{3/2} = b \sinh\phi$ in the integral $\int \frac{x^{1/2} dx}{\sqrt{b^2 + x^3}}$.)

4. **Penrose diagram for an accelerating FLRW universe.** Draw the Penrose diagram of a FLRW universe where the equation of state w is constant and $-1 < w < -\frac{1}{3}$, and $K = 0$. Explain the causal structure.