Due on Monday April 13 by 14.15.

1. **Irreducible components of the Einstein tensor.** Decompose the Einstein tensor (8.42) into the irreducible scalar, vector and tensor parts (without fixing the gauge).

2. **A covariant gauge condition.**

   a) Write the gauge condition $\partial_\alpha h^{\alpha\beta} = 0$ in terms of the irreducible representation of the metric perturbations.

   b) Starting from an arbitrary coordinate system, what conditions does this gauge condition set on the irreducible parts of of ξ^α? Express the residual gauge freedom in terms of these irreducible parts.

3. **Transverse traceless gauge.** Consider vacuum, where the Einstein tensor given in (8.41) is zero. Consider the gauge introduced in the previous problem. Show that its residual gauge freedom can be used to simultaneously choose the transverse traceless gauge defined by $h_{0\alpha} = 0$, $h = 0$. Does this leave any residual gauge freedom?

4. **Circular polarisation.** Consider a monochromatic plane wave travelling in the z direction. We use the transverse gauge. The wave is called circularly polarised if

$$s_{ij} = \alpha \begin{pmatrix} 1 & \pm i & 0 \\ \pm i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,$$

   where α is a real constant. Consider a test particle at constant coordinate position $x^i = (x, y, 0)$.

   a) Show that the particle moves in a circle.

   b) The polarisation is called right-handed if the particle moves counterclockwise as seen from the direction where the wave is traveling to. Does this correspond to $+$ or $-$ in the above?