Due on Monday April 7 by 14.15.

1. **Irreducible components of the Einstein tensor.** Decompose the Einstein tensor (8.42) into the irreducible scalar, vector and tensor parts (without fixing the gauge). (See the irreducible perturbation variables introduced in (8.12).)

2. **Lorenz gauge.**

   a) Write the Lorenz gauge condition $\partial_\alpha \bar{h}^{\alpha\beta} = 0$ in terms of the irreducible representation of the metric perturbations.

   b) Starting from an arbitrary coordinate system, what conditions does the Lorenz gauge condition set on the components of ξ^α in the irreducible representation? Express the residual gauge freedom in terms of these components.

3. **Transverse traceless gauge.** Consider vacuum, where $\Box \bar{h}_{\alpha\beta} = 0$. Show that the residual gauge freedom of the Lorenz gauge can be used to simultaneously choose the transverse traceless gauge defined by $\bar{h}_{0\alpha} = 0$, $\bar{h} = 0$. Does this leave any residual gauge freedom?

4. **Circular polarisation.** Consider a monochromatic plane wave travelling in the z direction. We use the transverse gauge. The wave is called circularly polarised if

$$s_{ij} = \alpha \begin{pmatrix} 1 & \pm i & 0 \\ \pm i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,$$

   where α is a real constant. Consider a test particle at constant coordinate position $x^i = (x, y, 0)$.

   a) Show that the particle moves in a circle.

   b) The polarisation is called right-handed if the particle moves counterclockwise as seen from the direction where the wave is traveling to. Does this correspond to $+$ or $-$ in the above?