Due on Monday April 22 by 14.15.

1. **Gravitational waves from a binary system.** On September 14 2015, LIGO made the first direct detection of gravitational waves. The source was a black hole binary system. Both black holes had mass $30M_\odot$, and the distance to the system was 1 billion light years.

   Take the black holes to be on a circular Newtonian orbit with radius $r = \lambda r_\mathrm{s}$, where r_s is the Schwarzschild radius and $\lambda > 1$. Take the orbit to be on the xy-plane, its centre to be at the origin and the observer to be on the z-axis. Approximate the black holes as pointlike and non-rotating, and take the background spacetime to be Minkowski.

   Find the frequency and the amplitude of the emitted gravitational waves as a function of λ.

2. **Energy loss of a binary system.** (This problem is worth double the usual points.) Consider the binary system of the previous problem.

   a) Starting from (8.142), derive (8.143).

   b) Approximating that the orbit remains circular, find the decay of λ as a function of time due to gravitational wave emission. What is the lifetime of the system –defined here as the time to reach $\lambda = 1$, where our approximation must break down– if the initial radius is 1) one astronomical unit or 2) $10r_\mathrm{s}$? How close do the black holes have to start from in order to merge within 10^{10} years?

   c) Find the velocity as a function of λ. Given that we use Newtonian orbits, is there a point before $\lambda = 1$ when the approximation is no longer reliable?

   d) What is the total radiated energy (from the initial radius to $\lambda = 1$) in cases 1) and 2), in units of $M_\odot$?

3. **Gravity vs electromagnetism.** Consider a thin metal rod of mass M, length L, and cross-sectional area $A \ll L^2$, centred on the origin and spinning on the xy-plane with constant angular frequency ω.

   a) Calculate the power emitted in gravitational waves.

   b) Calculate the power emitted in electromagnetic waves by the slight excess of electrons pushed towards the ends of the rod by the rotation. (Hint: calculate the resulting charge density ρ_Q by balancing the centrifugal effect and the electrostatic force. Approximate the quadrupole moment as $I_Q = \rho_Q L^2$, and the electric power as $(\omega^3 I_Q)^2$.)

   c) If the density of the rod is 10^4 kg/m^3 and the angular frequency is 1 kHz, which is more important in slowing down the rotation, gravitational or electromagnetic radiation?