Due on Monday April 15 by 14.15.

1. **Circular polarisation.** Consider a monochromatic plane wave travelling in the z direction. We use the transverse gauge. The wave is called circularly polarised if

$$s_{ij} = \alpha \begin{pmatrix} 1 & \pm i & 0 \\ \pm i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,$$

   where α is a real constant. Consider a test particle at constant coordinate position $x^i = (x, y, 0)$.

   a) Show that the particle moves in a circle.

   b) The polarisation is called right-handed if the particle moves counterclockwise as seen from the direction where the wave is traveling to. Does this correspond to $+$ or $-$ in the above?

2. **Lorenz gauge.**

   a) Write the Lorenz gauge condition $\partial_\alpha \bar{h}^{\alpha\beta} = 0$ in terms of the irreducible representation of the metric perturbations.

   b) Starting from an arbitrary coordinate system, what conditions does the Lorenz gauge condition set on the components of ξ^α in the irreducible representation? Express the residual gauge freedom in terms of these components.

3. **Transverse traceless gauge.** Consider vacuum, where $\Box \bar{h}_{\alpha\beta} = 0$. Show that the residual gauge freedom of the Lorenz gauge can be used to also choose the transverse traceless gauge defined by $\bar{h}_{0\alpha} = 0$, $\bar{h} = 0$. Does this leave any residual gauge freedom?

4. **LIGO gravitational wave detector.** A simplified* picture of the operation of the LIGO detector is as follows. Laser light is sent down two straight tunnels. It is reflected back by mirrors at the end of the tunnels. The phase difference of the returned light waves as a function of time is observed. Let's find how it is affected by gravitational waves.

   Take the points where the light rays are combined to be at the origin $(x, y, z) = (0, 0, 0)$, and the mirrors to be at $(L, 0, 0)$ and $(0, L, 0)$. Take the initial beams sent along the arms, with wavelength λ, to be perfectly in phase. Take the background spacetime to be Minkowski. A gravitational plane wave, with $+$ polarization, amplitude h_+, and frequency $f = \omega/(2\pi)$ travels in the z direction, passing through the instrument. We use the transverse gauge.

   a) What is the observed phase shift $\Delta\phi$? (The light travels on a geodesic.) Note that the answer depends on the phase of the gravitational wave at the time t_e when the light beams are sent out.

   b) Take $L = 4$ km, $\lambda = 1064$ nm, and assume that $\Delta\phi = 10^{-10}$ is the smallest phase shift you can observe. What is the smallest amplitude h_+ that can be detected? For which frequency f does the instrument achieve this sensitivity?

   (* Here L and λ are the real numbers, but the phase shift is in reality amplified by bouncing the light beams back and forth in the tunnels about 300 times before they are rejoined.)