Due on November 24 by 14.00.

1. **WIMP miracle.** Consider a WIMP that has a constant thermally averaged self-annihilation cross section times velocity, $\langle \sigma v \rangle = \sigma_0$, mass m and two internal degrees of freedom. Assume that the WIMP decouples instantaneously when $\Gamma = H$, that $m \gg T_d$ and $g_*(T_d) = g_{*S}(T_d) = 100$, and that $\eta = 6 \times 10^{-10}$. Find the dark matter energy density today ρ_{dm0} relative to the baryon energy density today ρ_{b0}, as a function of σ_0 and m. Give the numerical value for $\sigma_0 = 10^{-38}$ cm^2 and $m = 100$ GeV.

2. **Baryon catastrophe.** Consider a universe with zero baryon number. In analogy with the WIMP calculation above, find the present value of the energy density of nucleons plus antinucleons left over from annihilation, relative to photons. Use $g = 4$, $m_N = 0.94$ GeV, $\langle \sigma v \rangle = m_{\pi^0}^{-2}$ with $m_{\pi^0} = 0.135$ GeV and $g_*(T_d) = 10.75$.

3. **The oldness problem.** Assume that the universe contains only radiation and $g_* = 100$. Suppose that at the Planck density $\rho = M_{\mathrm{Pl}}^4$ the density parameter was less than one, and not extremely close to one, say $\Omega = 0.9$.

   a) What is the age of the universe when Ω becomes smaller than 10^{-2}?
   b) What is the age of the universe when the temperature falls below $T = 2.7$ K?