## Contents



# 5 Big Bang nucleosynthesis

One quarter (by mass) of baryonic matter in the universe is helium, heavier elements make up a few percent, and the rest is hydrogen. The building blocks of atomic nuclei, *nucleons* (meaning protons and neutrons) are formed in the QCD crossover at $T \sim 150$ MeV and $t \sim 30$ $\mu$s. Elements heavier than lithium, up to iron, cobalt, and nickel have been made from lighter elements by fusion reactions in stars. The fusion reactions provide the energy source for the stars. Elements heavier than these have been formed in supernova explosions and in the collisions of stars. However, the amount of helium and some other light isotopes in the universe cannot be understood by these mechanisms. It turns out that $^2$H, $^3$He, $^4$He, and $^7$Li were mainly produced already during the first hour of the universe, in a process called Big Bang nucleosynthesis (BBN).

Nucleons and antinucleons annihilated each other soon after the QCD crossover, and the small excess of nucleons left over from annihilation did not have a significant effect on the expansion and thermodynamics of the universe until much later, when the universe became matter-dominated (at $t_{\mathrm{eq}} \approx 1000\,\omega_m^{-2}$ years $\approx 50\,000$ years). The ordinary matter in the present universe comes from this small excess of nucleons. Let us now consider what happened to it in the early universe. We will focus on the period when the temperature fell from $T \sim 10$ MeV to $T \sim 10$ keV (from $t \sim 0.01$ s to a few hours).

## 5.1 Equilibrium

The total number of nucleons minus antinucleons stays constant due to baryon number conservation. In the temperature range under consideration, the number density of antinucleons is negligible. The baryon number can be in the form of protons and neutrons or atomic nuclei. Weak nuclear reactions convert neutrons and protons into each other and strong nuclear reactions build nuclei from them.

During the period of interest the nucleons and nuclei are nonrelativistic ($T \ll m_{\mathrm{p}}$). Assuming thermal equilibrium, the number density of nucleus type $i$ is

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{\frac{\mu_i - m_i}{T}} \;. \tag{5.1}$$

The proton and neutron masses are

$$m_{\rm p} = 938.272 \text{ MeV} \ , \qquad m_{\rm n} = 939.565 \text{ MeV} \ . \tag{5.2}$$

If the nuclear reactions needed to build nucleus $i$ (with mass number $A_i$ and charge $Z_i$) from the nucleons,

$$(A_i - Z_i)\mathrm{n} + Z_i\mathrm{p} \quad \leftrightarrow \quad i \ ,$$

occur at sufficiently high rate to maintain chemical equilibrium, we have

$$\mu_i = (A_i - Z_i)\mu_{\rm n} + Z_i\mu_{\rm p} \tag{5.3}$$

for the chemical potentials. For free nucleons we have

$$\begin{aligned} n_{\rm p} &= 2\left(\frac{m_{\rm p}T}{2\pi}\right)^{3/2} e^{\frac{\mu_{\rm p}-m_{\rm p}}{T}} \\ n_{\rm n} &= 2\left(\frac{m_{\rm n}T}{2\pi}\right)^{3/2} e^{\frac{\mu_{\rm n}-m_{\rm n}}{T}} \ , \end{aligned} \tag{5.4}$$

| ${}^{A}_{Z}i$ | $B_i$ | $g_i$ |
|---|---|---|
| ${}^{2}$H | 2.22 MeV | 3 |
| ${}^{3}$H | 8.48 MeV | 2 |
| ${}^{3}$He | 7.72 MeV | 2 |
| ${}^{4}$He | 28.3 MeV | 1 |
| ${}^{12}$C | 92.2 MeV | 1 |

Table 1. Some of the lightest nuclei and their binding energies.

so we can express $n_i$ in terms of the neutron and proton densities,

$$n_i = g_i A_i^{\frac{3}{2}} 2^{-A_i} \left(\frac{2\pi}{m_{\rm N}T}\right)^{\frac{3}{2}(A_i-1)} n_{\rm p}^{Z_i} n_{\rm n}^{A_i-Z_i} e^{B_i/T} \ , \tag{5.5}$$

where

$$B_i \equiv Z_i m_{\rm p} + (A_i - Z_i)m_{\rm n} - m_i \tag{5.6}$$

is the binding energy of the nucleus. Here we have approximated $m_{\rm p} \approx m_{\rm n} \approx m_i/A$ outside the exponent, and denoted it by $m_{\rm N}$, the *nucleon mass*.

The different number densities add up to the total baryon number density

$$\sum A_i n_i = n_{\rm b} \ . \tag{5.7}$$

We can express the baryon number density $n_{\rm b}$ in terms of two observational quantities: the photon density

$$n_\gamma = \frac{2}{\pi^2}\zeta(3)T^3 \tag{5.8}$$

and the *baryon/photon ratio*

$$\frac{n_{\rm b}}{n_\gamma} = \frac{g_{*s}(T)}{g_{*s}(T_0)}\eta \tag{5.9}$$

to get

$$n_{\rm b} = \frac{g_{*s}(T)}{g_{*s}(T_0)}\eta\frac{2}{\pi^2}\zeta(3)T^3 \ . \tag{5.10}$$

After electron-positron annihilation we have $g_{*s}(T) = g_{*s}(T_0)$ and $n_{\rm b} = \eta n_\gamma$. Note that $\eta$ is the baryon/photon ratio today, not as a function of time; its value is $6 \times 10^{-10}$.

For temperatures $m_{\rm N} \gg T \gtrsim B_i$ we have

$$(m_{\rm N}T)^{3/2} \gg T^3 \gg n_{\rm b} > n_{\rm p}, n_{\rm n} \ ,$$

so (5.5) implies that

$$n_i \ll n_{\rm p}, n_{\rm n}$$

for $A_i > 1$. Thus initially there are only free neutrons and protons in large numbers.

## 5.2 Neutron-proton ratio

What can we say about $n_{\rm p}$ and $n_{\rm n}$? Protons and neutrons are converted into each other by the weak interaction in the reactions

$$\begin{array}{rcl} \mathrm{n} + \nu_{\rm e} & \leftrightarrow & \mathrm{p} + \mathrm{e}^- \\ \mathrm{n} + \mathrm{e}^+ & \leftrightarrow & \mathrm{p} + \bar{\nu}_{\rm e} \\ \mathrm{n} & \leftrightarrow & \mathrm{p} + \mathrm{e}^- + \bar{\nu}_{\rm e} \ . \end{array} \tag{5.11}$$

If these reactions are in equilibrium, we have $\mu_{\rm n} + \mu_{\nu_{\rm e}} = \mu_{\rm p} + \mu_{\rm e}$, and the neutron/proton ratio is

$$\frac{n_{\rm n}}{n_{\rm p}} \equiv \frac{n}{p} = e^{-Q/T + (\mu_{\rm e} - \mu_{\nu_{\rm e}})/T} \ , \tag{5.12}$$

where $Q \equiv m_{\rm n} - m_{\rm p} = 1.293$ MeV.

We need to know the chemical potentials of electrons and electron neutrinos. The universe is electrically neutral, so the number density of electrons (or $n_{\rm e^-} - n_{\rm e^+}$) equals the number density of protons, and $\mu_{\rm e}$ can be calculated in terms of $\eta$ and $T$. We give below an estimate in the ultrarelativistic limit ($T \gg m_{\rm e}$):

$$n_{\rm e^-} - n_{\rm e^+} = \frac{2T^3}{6\pi^2}\left[\pi^2 \frac{\mu_{\rm e}}{T} + \left(\frac{\mu_{\rm e}}{T}\right)^3\right] = n_{\rm p}^* < n_{\rm b} \approx \eta n_\gamma = \eta \frac{2}{\pi^2}\zeta(3) T^3. \tag{5.13}$$

Here $n_{\rm p}^*$ includes the protons inside nuclei. Since $\eta$ is small, we have $\mu_{\rm e} \ll T$, and we can drop the $(\mu_{\rm e}/T)^3$ term, so

$$\frac{\mu_{\rm e}}{T} \lesssim \frac{6}{\pi^2}\zeta(3)\eta. \tag{5.14}$$

Thus $\mu_{\rm e}/T \sim \eta \sim 10^{-9}$. The nonrelativistic limit can be dealt with in a similar manner. It turns out that $\mu_{\rm e}$ rises as $T$ falls, and somewhere between $T = 30$ keV and $T = 10$ keV it becomes larger than $T$, and, in fact, comparable to $m_e$.

For $T \gtrsim 30$ keV, we have $\mu_{\rm e} \ll T$, and we can drop $\mu_{\rm e}$ in (5.12).

Since we have not measured the cosmic neutrino background (and probably will not do so in the near future, as neutrinos interact so weakly), we don't know the neutrino chemical potentials. Usually it is assumed that the neutrino asymmetry is small, like the baryon asymmetry, so that $|\mu_{\nu_{\rm e}}| \ll T$. The observational upper limit from BBN is $|\mu_{\nu_{\rm e}}|/T \lesssim 0.1$; if neutrinos are their own antiparticles, their chemical potentials are exactly zero. Thus, we ignore both $\mu_{\rm e}$ and $\mu_{\nu_{\rm e}}$, and get the equilibrium neutron/proton ratio

$$\frac{n}{p} = e^{-Q/T} \ . \tag{5.15}$$

This equation is not valid for $T \lesssim 30$ keV, since then $\mu_{\mathrm{e}}$ is no longer small, but we will use it only at higher temperatures.

We can thus express the number densities of all nuclei in terms of the free proton number density $n_{\mathrm{p}}$, as long as chemical equilibrium holds.

## 5.3 Bottlenecks

We define the *mass fraction* of nucleus $i$ as

$$X_i \equiv \frac{A_i n_i}{n_{\mathrm{b}}} \ . \tag{5.16}$$

Since

$$n_{\mathrm{b}} = \sum_i A_i n_i, \tag{5.17}$$

(where the sum includes protons and neutrons), we have $\sum X_i = 1$.

Using also the normalisation condition (5.17), we get all equilibrium abundances as a function of $T$ (they also depend on the value of the parameter $\eta$).[1] There are two items to note:

1. The normalisation condition (5.17) includes all nuclei up to uranium and beyond. Thus we would get a huge polynomial equation from which to solve $X_p$.

2. In practice we don't have to care about item 1, since as the temperature falls the nuclei no longer follow their equilibrium abundances. The reactions are in equilibrium only at high temperatures, when the other equilibrium abundances except $X_{\mathrm{p}}$ and $X_{\mathrm{n}}$ are small, and we can use the approximation $X_{\mathrm{n}} + X_{\mathrm{p}} = 1$.

In the early universe the baryon density is too low and the time available is too short for reactions involving three or more incoming nuclei to occur at any appreciable rate. The heavier nuclei have to be built sequentially from lighter nuclei in two-particle reactions, so deuterium is formed first in the reaction

$$\mathrm{n} + \mathrm{p} \quad \to \quad \mathrm{d} + \gamma \ .$$

Only when deuterium nuclei are available can helium nuclei be formed, and so on. This process has bottlenecks: the lack of sufficient densities of lighter nuclei hinders the production of heavier nuclei, and prevents them from following their equilibrium abundances.

As the temperature falls, the equilibrium abundances rise fast. They become large later for nuclei with small binding energies. Since deuterium is formed directly from neutrons and protons it can follow its equilibrium abundance as long as there are large numbers of free neutrons available. Since the deuterium binding energy is rather small, the deuterium abundance becomes large rather late (at $T < 100$ keV). Therefore heavier nuclei with larger binding energies, whose equilibrium abundances would become large earlier, cannot be formed. This is the *deuterium bottleneck*[2].

[1] For $n_{\mathrm{p}}$ and $n_{\mathrm{n}}$ we know just their ratio, since we do not know $\mu_{\mathrm{p}}$ and $\mu_{\mathrm{n}}$, only that $\mu_{\mathrm{p}} = \mu_{\mathrm{n}}$. Therefore this extra equation is needed to solve all $n_i$.

[2] Some people prefer to make clear distinction between the nucleus and the atom, calling the former deuteron and the latter deuterium, and similarly for other isotopes.

Only when there is enough deuterium ($X_\mathrm{d} \sim 10^{-3}$) can helium be produced in large numbers.

The nuclei are positively charged so they repel each other electromagnetically. The nuclei need large kinetic energies to overcome this Coulomb barrier and get within the range of the strong interaction. Thus the cross sections for these fusion reactions fall rapidly with energy and the nuclear reactions freeze out when the temperature falls below $T \sim 30$ keV. For this reason there is less than one hour available for nucleosynthesis. Because of the short duration and additional bottlenecks (e.g. there are no stable nuclei with $A = 8$), only very small amounts of elements heavier than helium are formed.

## 5.4 Calculation of the helium abundance

Let us now calculate the numbers. For $T > 0.1$ MeV, we still have $X_\mathrm{n} + X_\mathrm{p} \approx 1$, so the equilibrium abundances are

$$X_\mathrm{n} = \frac{e^{-Q/T}}{1+e^{-Q/T}} \qquad \text{and} \qquad X_\mathrm{p} = \frac{1}{1+e^{-Q/T}}. \tag{5.18}$$

Nucleons follow these equilibrium abundances until neutrinos decouple at $T_D \sim 0.8$ MeV, shutting off the weak n $\leftrightarrow$ p reactions. After this, free neutrons decay, so

$$X_\mathrm{n}(t) = X_\mathrm{n}(t_\mathrm{dec}) e^{-(t-t_\mathrm{dec})/\tau_\mathrm{n}} \ , \tag{5.19}$$

where $\tau_\mathrm{n} = 878.4 \pm 0.5$ s is the mean lifetime of a free neutron [1][3]. (The half-life is $\tau_{1/2} = (\ln 2)\tau_\mathrm{n}$.) In reality, the decoupling and thus the shift from behaviour (5.18) to behaviour (5.19) is not instantaneous, but an approximation where one takes it to be instantaneous at time $t_\mathrm{dec}$ when $T_\mathrm{dec} = 0.8$ MeV, so $X_\mathrm{n}(t_\mathrm{dec}) = 0.1657$, gives a fairly accurate final result.

The equilibrium mass fractions are, from (5.5),

$$X_i = \frac{1}{2} X_\mathrm{p}^{Z_i} X_\mathrm{n}^{A_i - Z_i} g_i A_i^{\frac{5}{2}} \epsilon^{A_i - 1} e^{B_i/T} \tag{5.20}$$

where

$$\epsilon \equiv \frac{1}{2}\left(\frac{2\pi}{m_\mathrm{N} T}\right)^{3/2} n_\mathrm{b} \;=\; \frac{1}{\pi^2}\zeta(3)\left(\frac{2\pi T}{m_\mathrm{N}}\right)^{3/2} \frac{g_{*s}(T)}{g_{*s}(T_0)}\eta \sim \left(\frac{T}{m_\mathrm{N}}\right)^{3/2} \eta.$$

The factors which change rapidly with $T$ are $\epsilon^{A_i-1} e^{B_i/T}$. For temperatures $m_\mathrm{N} \gg T \gg B_i$ we have $e^{B_i/T} \sim 1$ and $\epsilon \ll 1$. Thus $X_i \ll 1$ for others ($A_i > 1$) than protons and neutrons. As temperature falls, $\epsilon$ becomes even smaller and at $T \sim B_i$ we have $X_i \ll 1$ still. The temperature has to fall below $B_i$ by a large factor before the factor $e^{B_i/T}$ wins and the equilibrium abundance becomes large.

Deuterium has $B_\mathrm{d} = 2.22$ MeV, so we get $\epsilon e^{B_\mathrm{d}/T} = 1$ at $T_\mathrm{d} = 0.06$ MeV–0.07 MeV (assuming $\eta = 10^{-10} - 10^{-9}$), so the deuterium abundance becomes large close to this temperature. Since $^4$He has a much higher binding energy, $B_4 = 28.3$ MeV, the corresponding situation $\epsilon^3 e^{B_4/T} = 1$ occurs at a higher temperature $T_4 \sim 0.3$ MeV. But we noted earlier that only deuterium stays close to its equilibrium

[3]The error bar may not be an accurate reflection of the uncertainty in the neutron lifetime, as there are large differences between measurements.

abundance once it gets large. Helium begins to form only when there is sufficient deuterium available, in practice slightly above $T_\mathrm{d}$. Helium then forms rapidly. The available number of neutrons sets an upper limit to $^4$He production. Since helium has the highest binding energy per nucleon (of all isotopes below $A = 12$), almost all neutrons end up in $^4$He, and only small amounts of the other light isotopes, $^2$H, $^3$H, $^3$He, $^7$Li, and $^7$Be, are produced.

The Coulomb barrier shuts off the nuclear reactions before there is time for heavier nuclei ($A > 8$) to form. We get a fairly good approximation for $^4$He production by assuming instantaneous nucleosynthesis at $T = T_\mathrm{ns} \sim 1.1T_\mathrm{d} \sim 70$ keV, with all neutrons ending up in $^4$He, so that

$$X_4 \approx 2X_\mathrm{n}(T_\mathrm{ns}) \ . \tag{5.21}$$

After electron-positron annihilation ($T \ll m_\mathrm{e} = 0.511$ MeV) the time-temperature relation is

$$t \approx \frac{2.42}{\sqrt{g_*}} \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \mathrm{s} \ , \tag{5.22}$$

where $g_* = 3.363$. Since most of the time in $T = 0.8$ MeV–0.07 MeV is spent at the lower part of this temperature range, this formula gives a good approximation for the time,

$$t_\mathrm{ns} - t_\mathrm{dec} = 267 \text{ s} \qquad\qquad \text{(in reality 264.3 s).}$$

Thus we get for the final $^4$He abundance

$$X_4 = 2X_\mathrm{n}(t_\mathrm{dec})e^{-(t_\mathrm{ns}-t_\mathrm{dec})/\tau_\mathrm{n}} = 24.5 \ \%. \tag{5.23}$$

The result of numerical calculations for $X_4$ as a function of $\eta$ is shown in figure 4 (there called $Y$).

This calculation of the helium abundance $X_4$ involves a bit of cheating in the sense that we have used results of accurate numerical calculations to infer that we need to use $T = 0.8$ MeV as the neutrino decoupling temperature, and $T_\mathrm{ns} = 1.1T_\mathrm{d}$ as the “instantaneous nucleosynthesis” temperature, to best approximate the correct behaviour. However, it gives us a quantitative description of what is going on, and an understanding of how the helium yield depends on various things.

**Exercise:** Find the dependence of $X_4$ on $\eta$, i.e. calculate $\mathrm{d}X_4/\mathrm{d}\eta$.

## 5.5 Why so late?

Let us return to the question of why the temperature has to fall so much below the binding energy before the equilibrium abundances become large. From the energetics we might conclude that when typical kinetic energies, $\langle E_k \rangle \approx \frac{3}{2}T$, are smaller than the binding energy, it would be easy to form nuclei but difficult to break them. Above we saw that the smallness of the factor $\epsilon \sim (T/m_\mathrm{N})^{3/2}\eta$ is the reason why this is not so. Here $\eta \sim 10^{-9}$ and $(T/m_\mathrm{N})^{3/2} \sim 10^{-6}$ (for $T \sim 0.1$ MeV). The main culprit is thus the small baryon/photon ratio. Since there are $10^9$ photons for each baryon, there is a sufficient amount of photons who can disintegrate a nucleus in the high-energy tail of the photon distribution, even at rather low temperatures. This is similar to how atoms only form at $T \sim 0.3$ eV, even though hydrogen ionisation energy is 13.6 eV. We can also express this result in terms of entropy. High photon/baryon ratio corresponds to high entropy per baryon, and high entropy favours free nucleons.

## 5.6 The most important reactions

In reality, neither neutrino decoupling nor nucleosynthesis are instantaneous processes. Accurate results require a rather large numerical computation where one uses the cross sections of all the relevant weak and strong interactions. These cross sections are energy-dependent. Integrating them over the energy and velocity distributions and multiplying with the relevant number densities leads to temperature-dependent reaction rates. The most important reactions are the weak $n \leftrightarrow p$ reactions (5.11) and the following strong reactions (see figure 1)[4]:

$$
\begin{array}{lcl}
\mathrm{p} + \mathrm{n} & \rightarrow & {}^{2}\mathrm{H} + \gamma \\
{}^{2}\mathrm{H} + \mathrm{p} & \rightarrow & {}^{3}\mathrm{He} + \gamma \\
{}^{2}\mathrm{H} + {}^{2}\mathrm{H} & \rightarrow & {}^{3}\mathrm{H} + \mathrm{p} \\
{}^{2}\mathrm{H} + {}^{2}\mathrm{H} & \rightarrow & {}^{3}\mathrm{He} + \mathrm{n} \\
\mathrm{n} + {}^{3}\mathrm{He} & \rightarrow & {}^{3}\mathrm{H} + \mathrm{p} \\
\mathrm{p} + {}^{3}\mathrm{H} & \rightarrow & {}^{4}\mathrm{He} + \gamma \\
{}^{2}\mathrm{H} + {}^{3}\mathrm{H} & \rightarrow & {}^{4}\mathrm{He} + \mathrm{n} \\
{}^{2}\mathrm{H} + {}^{3}\mathrm{He} & \rightarrow & {}^{4}\mathrm{He} + \mathrm{p} \\
{}^{4}\mathrm{He} + {}^{3}\mathrm{He} & \rightarrow & {}^{7}\mathrm{Be} + \gamma \\
{}^{4}\mathrm{He} + {}^{3}\mathrm{H} & \rightarrow & {}^{7}\mathrm{Li} + \gamma \\
{}^{7}\mathrm{Be} + \mathrm{n} & \rightarrow & {}^{7}\mathrm{Li} + \mathrm{p} \\
{}^{7}\mathrm{Li} + \mathrm{p} & \rightarrow & {}^{4}\mathrm{He} + {}^{4}\mathrm{He}
\end{array}
$$

In principle, all of these nuclear cross sections are determined by just a few parameters in QCD. However, calculating these cross sections from first principles is too difficult in practice. Instead cross sections measured in the laboratory are used. In contrast, cross sections of the weak reactions (5.11) are known from theory. The relevant reaction rates are now known sufficiently accurately so that the nuclear abundances produced in BBN (for a given value of $\eta$) can be calculated with better accuracy than the present abundances can be measured from astronomical observations.

The reaction chain proceeds along stable and long-lived (compared to the nucleosynthesis timescale—minutes) isotopes towards larger mass numbers. At least one of the two incoming nuclei must be an isotope which is abundant during nucleosynthesis, i.e. n, p, ${}^{2}\mathrm{H}$ or ${}^{4}\mathrm{He}$. The mass numbers $A = 5$ and $A = 8$ form bottlenecks, since they have no stable or long-lived isotopes. The $A = 5$ bottleneck is crossed with the reactions ${}^{4}\mathrm{He}+{}^{3}\mathrm{He}$ and ${}^{4}\mathrm{He}+{}^{3}\mathrm{H}$, which form a small number of ${}^{7}\mathrm{Be}$ and ${}^{7}\mathrm{Li}$. Their abundances remain so small that we can ignore the reactions that cross the $A = 8$ bottleneck (e.g. ${}^{7}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{11}\mathrm{C} + \gamma$ and ${}^{7}\mathrm{Li} + {}^{4}\mathrm{He} \rightarrow {}^{11}\mathrm{B}$). Numerical calculations also show that the production of the other stable lithium isotope, ${}^{6}\mathrm{Li}$ is several orders of magnitude smaller than that of ${}^{7}\mathrm{Li}$.

Thus BBN produces the isotopes ${}^{2}\mathrm{H}$, ${}^{3}\mathrm{H}$, ${}^{3}\mathrm{He}$,${}^{4}\mathrm{He}$, ${}^{7}\mathrm{Li}$ and ${}^{7}\mathrm{Be}$. Of these, ${}^{3}\mathrm{H}$ (half-life 12.3 a) and ${}^{7}\mathrm{Be}$ (half-life 53 d) are unstable and decay after nucleosynthesis into ${}^{3}\mathrm{He}$ and ${}^{7}\mathrm{Li}$. (Actually, ${}^{7}\mathrm{Be}$ becomes ${}^{7}\mathrm{Li}$ through electron capture ${}^{7}\mathrm{Be} + \mathrm{e}^{-} \rightarrow$

[4] The reaction chain that produces helium from hydrogen in BBN is not the same that occurs in stars. The conditions in stars are different: there are no free neutrons and the temperatures are lower, but the densities are higher and there is more time available. In addition, second and later generations of stars contain heavier nuclei (C, N, O) that act as catalysts in helium production.

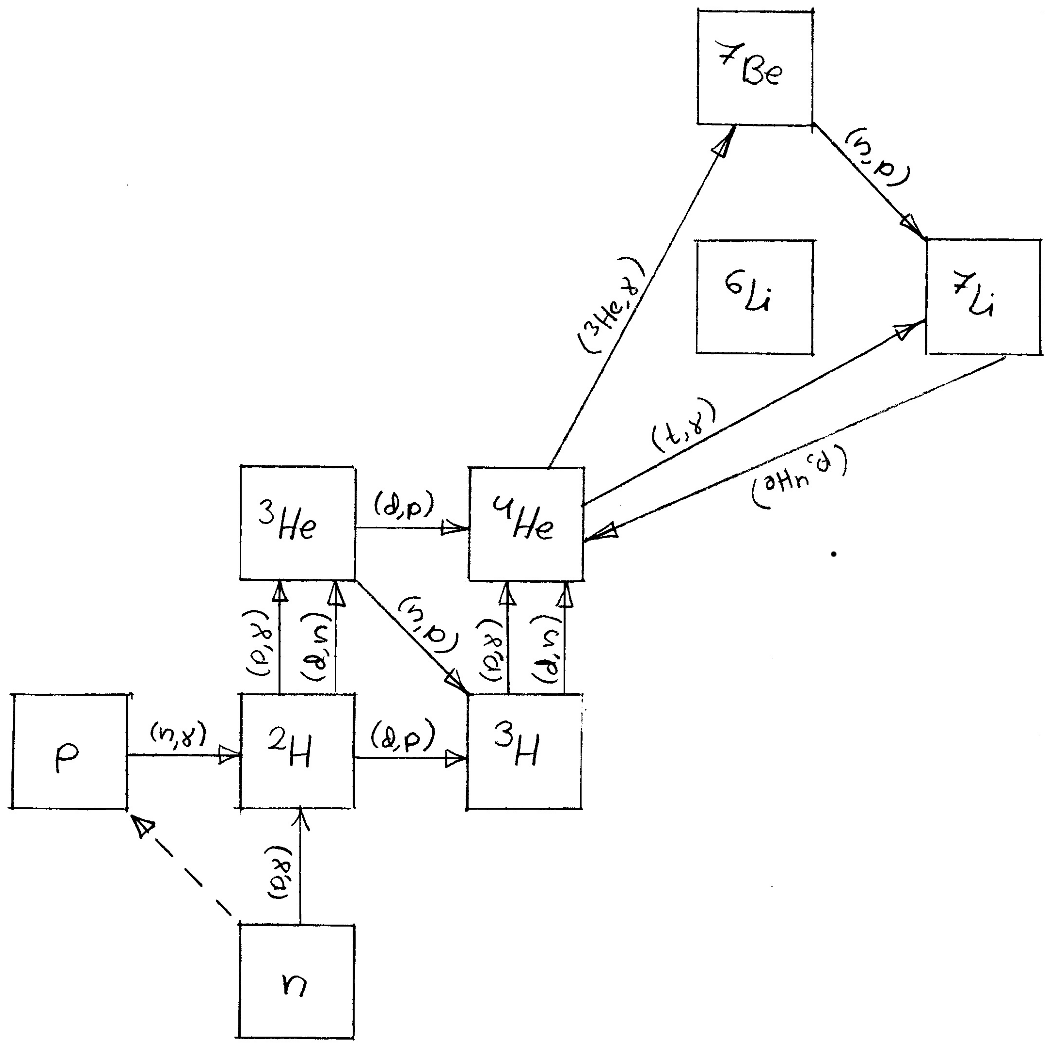


Figure 1: The 12 most important nuclear reactions in big bang nucleosynthesis.

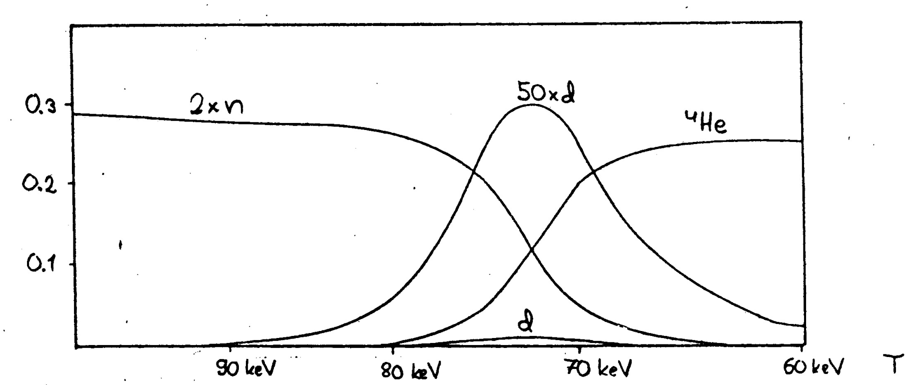


Figure 2: The time evolution of the n, $^2$H (written as d) and $^4$He abundances during BBN. Notice how the final $^4$He abundance is determined by the n abundance before nuclear reactions begin. Only a small part of the neutrons decay or end up in other nuclei. Before becoming $^4$He, all neutrons pass through $^2$H. To improve the visibility of the deuterium curve, we have plotted it also as multiplied by a factor of 50. The other abundances (except p) remain so low, that to see them the figure should be redrawn in logarithmic scale. This plot is for $\eta = 6 \times 10^{-10}$. The time at $T = (90, 80, 70, 60)$ keV is $(152, 199, 266, 367)$ s. Thus the action peaks at about $t = 4$ min.

$^7$Li + $\nu_e$.)

In the end BBN has produced cosmologically significant (compared to present abundances) amounts of the four isotopes, $^2$H, $^3$He, $^4$He and $^7$Li (the fifth isotope $^1$H=p we had already before BBN). Their production in the BBN can be calculated, and there is only one free parameter, the baryon/photon ratio. It is directly related to the baryon density as follows:

$$\eta \equiv \frac{n_{b0}}{n_{\gamma 0}} = \frac{\rho_{b0}}{\rho_{\gamma 0}} \frac{\rho_{\gamma 0}}{m_N n_{\gamma 0}} = \frac{\omega_b}{\omega_\gamma} \frac{\pi^4 T_0}{30\zeta(3) m_N} = 2.74 \times 10^{-8} \omega_b ,$$

where $\rho_{b0}$ is the density of baryonic matter today, and $\omega_b \equiv \Omega_{b0} h^2$ as before.

## 5.7 BBN as a function of time

Nucleosynthesis as a function of time (or temperature) is shown in figure 2, and in more detail on a logarithmic scale in figure 3 The nuclei $^2$H and $^3$H are intermediate states through which reactions proceed towards $^4$He. Therefore their abundance first rises, is highest at the time when $^4$He production is fastest, and then falls as baryonic matter ends up in $^4$He. $^3$He is also an intermediate state, but the main channel from $^3$He to $^4$He is via $^3$He+n→$^3$H+p, which is extinguished early as the free neutrons are used up. Therefore the abundance of $^3$He does not fall the same way as $^2$H and $^3$H. The abundance of $^7$Li also rises at first and then falls via $^7$Li+p→$^4$He+$^4$He. Since $^4$He has a higher binding energy per nucleon, $B/A$, than $^7$Li and $^7$Be have, these also want to return into $^4$He. This does not happen to $^7$Be, however, since, just like for $^3$He, the free neutrons needed for the reaction $^7$Be+n→$^4$He+$^4$He have almost disappeared.

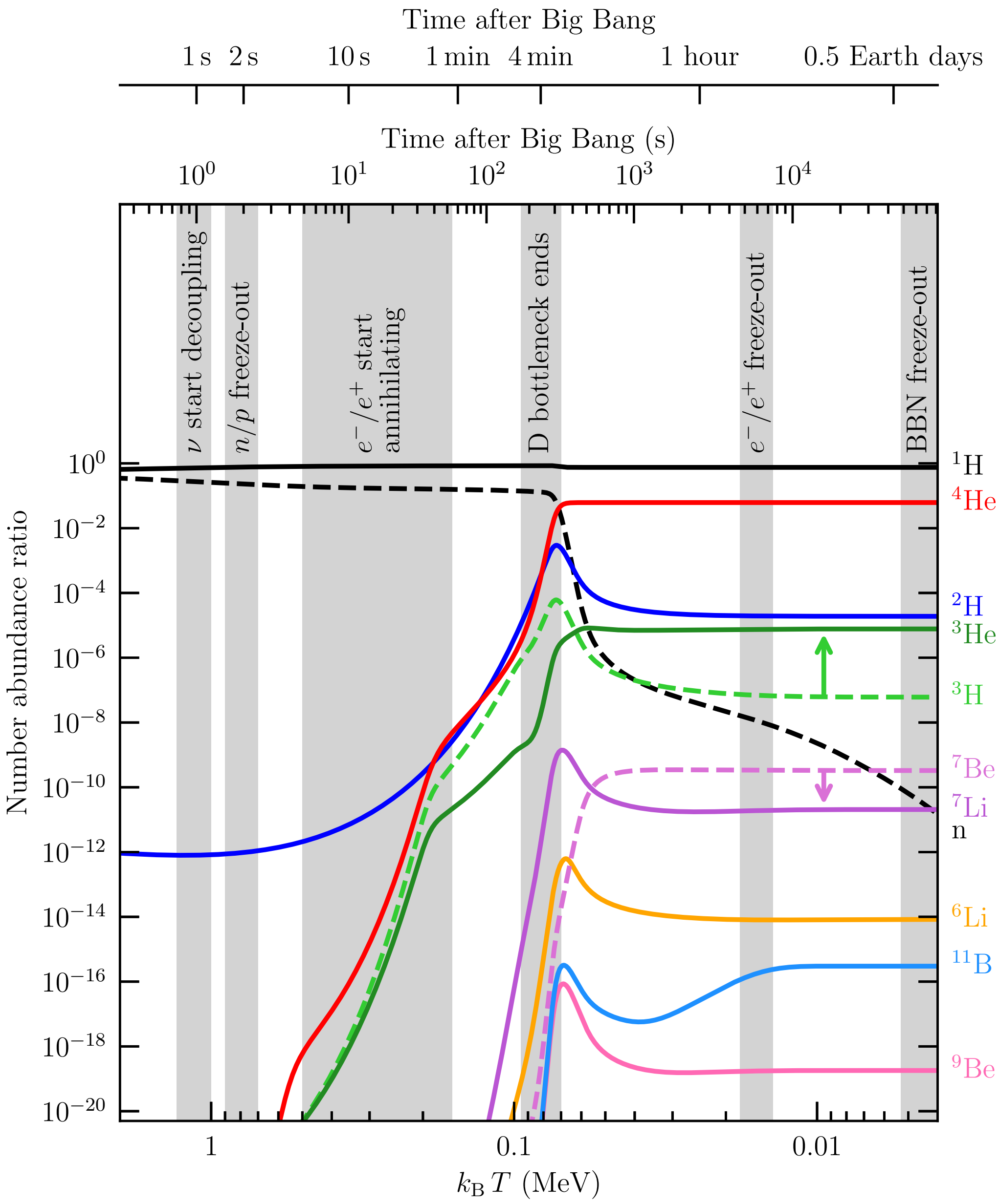


Figure 3: The time evolution of the abundance of n and various nuclei on a logarithmic scale during BBN, with various cosmological milestones marked with gray bands [2]. Note that the $e^+ - e^-$ annihilation in reality lasts throughout BBN, and only ends at around 2 hours, after nuclear fusion reactions have shut down.

| | **B**(MeV) | **B/A** |
|---|---|---|
| $^2$H | 2.2245 | 1.11 |
| $^3$H | 8.4820 | 2.83 |
| $^3$He | 7.7186 | 2.57 |
| $^4$He | 28.2970 | 7.07 |
| $^6$Li | 31.9965 | 5.33 |
| $^7$Li | 39.2460 | 5.61 |
| $^7$Be | 37.6026 | 5.37 |
| $^{12}$C | 92.1631 | 7.68 |
| $^{56}$Fe | 492.2623 | 8.79 |

## 5.8 Primordial abundances as a function of the baryon-to-photon ratio

Let us consider BBN as a function of $\eta$, shown in figure 4. The larger $\eta$ is, the higher is the number density of nucleons. The reaction rates are faster and the nucleosynthesis can proceed further. This mean that a smaller fraction of intermediate nuclei, $^2$H, $^3$H, and $^7$Li are left over— the burning of nuclear matter into $^4$He is cleaner. Also the $^3$He production falls with increasing $\eta$. However, $^7$Be production increases with $\eta$. In the figure we have plotted the final BBN yields, so that $^3$He is the sum of $^3$He and $^3$H, and $^7$Li is the sum of $^7$Li and $^7$Be. The complicated shape of the $^7$Li($\eta$) curve is due to these two contributions: 1) For small $\eta$ we get lots of directly produced $^7$Li, whereas 2) for large $\eta$ there is very little directly produced $^7$Li, but a lot of $^7$Be is produced. In the middle, at $\eta \sim 3 \times 10^{-10}$, there is a minimum of $^7$Li production where neither channel is very effective.

The $^4$He production increases with $\eta$, since for higher density nucleosynthesis begins earlier, when there are more neutrons left.

## 5.9 Comparison with observations

Abundances of the various isotopes calculated from BBN can be compared to the observed abundances. This is one of the most important tests of the big bang theory. The comparison of theory and observations complicated by *chemical evolution*, i.e. nuclear reactions in stars. BBN gives the primordial abundances of the isotopes. The first stars form with this element composition. In stars, further fusion reactions take place and the composition of the star changes with time. Towards the end of its life, the star ejects its outer parts into interstellar space, and the processed material mixes with primordial material. The next generation of stars forms from this mixed material, and so on.

The observations of present abundances are based on spectra of interstellar clouds and stellar surfaces. To obtain the primordial abundances from the present abundances the effect of chemical evolution has to be estimated. Since $^2$H is so fragile (its binding energy is so low), there is hardly any $^2$H production in stars, rather any pre-existing $^2$H is destroyed early on in stars. Therefore any interstellar $^2$H is primordial. The smaller the fraction of processed material in an interstellar cloud, the higher its $^2$H abundance should be. Thus all observed $^2$H abundances are lower limits to the primordial $^2$H abundance[5]. Conversely, stellar production increases the $^4$He abundance. Thus all $^4$He observations are upper limits to the primordial $^4$He. Moreover, stellar processing produces heavier elements, such as C, N, and O, which are not produced in BBN. Their abundance varies a lot from place to place, giving a measure of how much chemical evolution has happened in various parts of the universe. Plotting $^4$He vs. these heavier elements, we can extrapolate the $^4$He abundance to zero chemical evolution to obtain the primordial abundance. Since $^3$He and $^7$Li are both produced and destroyed in stellar processing, it is more difficult to make estimates of their primordial abundances based on observed present abundances.

There are two clear qualitative signatures of big bang nucleosynthesis in the present universe:

1. All stars and gas clouds observed contain at least 23% $^4$He. If all $^4$He had been produced in stars, we would see similar variations in the $^4$He abundance as we see for the other elements, such for C, N, and O, with some regions containing just a few % or even less $^4$He. This universal minimum amount of $^4$He signifies primordial abundance produced when matter in the universe was uniform.

2. The existence of significant amounts of $^2$H in the universe is a sign of BBN, since there are no known astrophysical sources of large amounts of $^2$H.

The observed abundances of the BBN isotopes, $^2$H, $^3$He and $^4$He indicate the range (mostly determined by the ratio D/H) $\eta = (6.040 \pm 0.118) \times 10^{-10}$ [1]. The agreement between different isotopes and the value determined independently from the CMB provides strong support for this value. However, the abundance of $^7$Li seems to be strongly inconsistent with this value, as shown in figure 4. This *lithium problem* has become less acute in recent years. It seems that the estimates of lithium abundance based on observations of the surfaces of old stars have underestimated how much lithium is recycled between the surface and the inner stellar regions by convection, and hence how much of it is destroyed. The primordial abundance thus seems to be larger than the estimates from stars shown in figure 4.

The value $\eta = (6.040 \pm 0.118) \times 10^{-10}$ corresponds to $100\omega_\mathrm{b} = 2.205 \pm 0.043$. With $h = 0.7$, we get

$$\Omega_{\mathrm{b}0} = 0.04 \ldots 0.05 \ . \tag{5.24}$$

This is much less than the cosmological observational estimates for $\Omega_{\mathrm{m}0}$. Therefore they, together with BBN, provides strong evidence that most matter in the universe is non-baryonic; we discuss this dark matter in the next chapter. We can also compare the value of $\omega_\mathrm{b}$ to the one inferred from the CMB, $100\omega_\mathrm{b} = 2.237 \pm 0.015$, a remarkable agreement [3].

BBN can be used to probe physics beyond the Standard Model. The expansion rate of the universe depends on the energy density of radiation, encoded in $g_*$. During BBN, we have $g_* = 5.5 + 1.75 N_\nu$, where $N_\nu$ is the number of neutrino species with masses so small that they are relativistic during BBN and have weak

[5]This does not apply to sites which have been enriched in $^2$H due to separation of $^2$H from $^1$H. Deuterium binds into molecules more easily than ordinary hydrogen. Since deuterium is heavier than ordinary hydrogen, deuterium and molecules involving deuterium have lower thermal velocities and do not escape from gravity as easily. Thus planets tend to have high deuterium-to-hydrogen ratios.

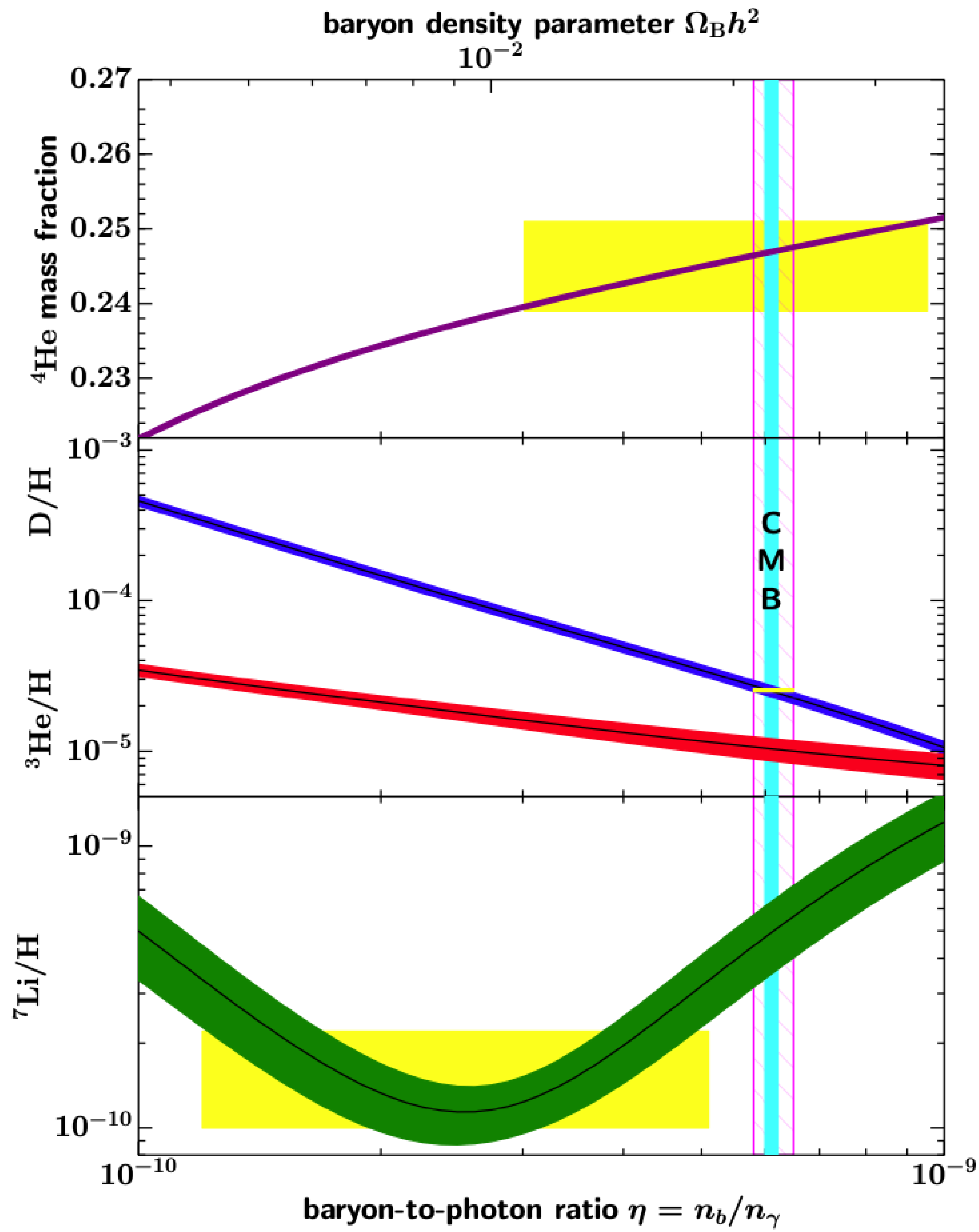


Figure 4: The abundances of $^4$He, D, $^3$He and $^7$Li and the range of $\eta$ determined from BBN (yellow boxes) and the the CMB (blue strip) [1]. Both the BBN and the CMB ranges indicate the 95% confidence level. The narrow vertical band is determined from the observed D and $^4$He abundance.

interactions so that their distribution is coupled to the thermal bath until about $T = 0.8$ MeV. The number of neutrino species can also be left as a free parameter, in which case it parametrises any additional radiation degrees of freedom that may be present. As mentioned in the previous chapter, for the Standard Model we have $N_\nu = 3.043$, because neutrinos are not totally decoupled from the thermal bath when electrons and positrons annihilate, so some of the entropy (and energy density) of electrons and positrons is transferred to the neutrinos, hence the 0.046 correction. If we leave $N_\nu$ as a free parameter and fit the observations (neglecting lithium) we get from light element abundances the range $\eta = (6.088 \pm 0.054) \times 10^{-10}$ and $N_\nu = 2.898 \pm 0.141$ [1]. CMB and large-scale structure data gives a similar constraint $N_\nu = 2.99 \pm 0.17$ [3]. There is no room for an extra neutrino species that would have been in thermal equilibrium. We also know from collider and laboratory experiments that there are only three light neutrinos that interact with the weak interaction. *Sterile neutrinos*, which do not feel the weak interaction remain a possible candidate – they would not be in thermal equilibrium at BBN, so they would contribution a fractional (and possible negligible) number of neutrino species to $N_\nu$. A well-motivated candidate for sterile neutrinos is the right-handed complement of the left-handed neutrinos of the Standard Model.

## References

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