Cosmology I & II

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Preface

These are lecture notes for the courses Cosmology I and II at the University of Helsinki. They owe a heavy debt to lecture notes prepared by Hannu Kurki-Suonio. Credit for some figures belongs to Elina Keihänen, Jussi Väliviita and Reijo Keskitalo.

One problem in teaching cosmology is that some central aspects rely on advanced physics. Nevertheless, the main applications of these concepts can be discussed in relatively simple terms, so first learning general relativity, quantum field theory and so on in detail is not necessary. Thus only standard bachelor level theoretical physics background (mechanics, special relativity, statistical physics, mathematical methods) will be assumed in Cosmology I, with some more advanced Master's level mathematical methods and bachelor's level quantum mechanics will be assumed in Cosmology II. The more advanced theories that cosmology relies on are reviewed as part of these notes to the required extent, but this necessarily means that some results have to be accepted without proper derivation.

1 Introduction

1.1 Overview of modern cosmology

Cosmology is the study of the universe as a whole, its structure, origin and evolution. Cosmology is grounded on observations, many of them astronomical, and laws of physics measured in the Solar system. These lead naturally to the standard framework of modern cosmology, *the hot big bang theory*, according to which the universe expands and cools down.

As a science, cosmology has a rare, if not singular, restriction: we can observe only one universe. We cannot observationally make comparative or statistical studies among many universes (and there can even be difficulties with the concept of "many universes"). We also cannot move around our universe, but are restricted to (on cosmological scales) almost a single point in space and time. As a result, cosmology has rlied more on model-dependent interpretations than many other branches of physics.

Nevertheless, the last few decades have seen remarkable progress, as a significant body of observational data has become available with modern instruments. We now have a good observational handle of the overall history of the universe for all times between one second and the present time – the universe is today about 14 billion years old. Theoretically, we understand the evolution of the universe at all times between 10^{-11} s and a few billion years, and we have good ideas and some observational support for the evolution going back to earlier eras, perhaps all the way to 10^{-36} s or so.

Important questions remain, in particular about the nature of *dark matter*, *dark energy* and the processes of *inflation* and *baryogenesis*. In the first part of the course, we will consider dark energy and dark matter, and in the second part we will look at will look at inflation. Baryogenesis will be mentioned only in passing.

One historically important observation in support of the big bang theory is the *redshift* of distant galaxies. Their spectra are shifted towards longer wavelengths. The further out they are, the larger the shift. This implies that the galaxies are receding from us: their distance from us increases. According to general relativity, this is because of the expansion of space, which in turn is an aspect of the curvature of spacetime, not because gaalxies would move in space. As space expands, the wavelength of the light travelling through it stretches.

The expansion appears to be uniform over large scales. While there are deviations of order unity in the expansion rate on small scales (for example, our galaxy does not expand), the average expansion rate on scales larger than galaxy clustering, 100 Mpc or so, is almost the same everywhere. In homogeneous and isotropic cosmological models, the expansion is simply described by a time-dependent *scale factor* $a(t)$. Starting from the observed present expansion rate, $H \equiv \dot{a}/a$, we can use general relativity to calculate $a(t)$ as a function of time, given our understanding of the physics of the matter content in the universe. (We will discuss this in detail.) The result is that $a(t) \to 0$ and the density of the universe $\rho \to \infty$ about 14 billion years ago. At this *singularity*, time and space begin, and we can choose it as the origin of the time coordinate, $t = 0$. However, we do not expect general relativity, which governs the evolution of spacetime, nor the Standard Model of particle physics, which governs the behaviour of matter, to be applicable at extremely high energy densities. At the so called *Planck density*, $\rho_{\mathrm{Pl}} \sim 10^{97}$ kg/m^3, at the latest, quantum gravity effects are expected to be large. To describe the earliest times, the *Planck era*, we would need a theory of *quantum gravity*, which we do not have.[1] At present we don't know what happened at $t = 0$ or in its immediate vicinity at the *Planck time* 10^{-42} s. It seems likely that a major breakthrough in our understanding of physics is required before we can make any definite statements about that primordial era.

Thus, the big bang theory does not actually apply all the way to the beginning of time and a "big bang". Rather, it is a valid description of the history of the universe starting from some early time when the universe was very hot, very dense, and expanded rapidly. The universe was at early times filled with an almost homogeneous "soup" of particles which was in thermal equilibrium for a long time. We can therefore describe the state of the early universe with a small number of thermodynamic variables, and this makes the evolution of the universe calculable.

The fact that the scale factor tends to zero at early times does not imply that the universe would have started from a point. The part of the universe which we

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[1] *String theory* is a candidate for the theory of quantum gravity, and applying it to the early universe is an active area of research. However, we do not have a complete formulation of the theory, and it is not known whether string theory is correct, so its applications are necessarily rather speculative.

can observe today was indeed very small at very early times, possibly smaller than 1 mm in diameter at the earliest times that can be sensibly discussed within the big bang framework. And if the inflationary scenario is correct, it would have been much smaller than that before inflation. However, the universe extends beyond what can be observed today (beyond our *horizon*), and may even be infinite. In the current cosmological models, if the universe is infinite, it has always been infinite, except at the moment $t = 0$, when the size is not defined. We do not know, theoretically or observationally, whether the universe is finite or infinite.

As the universe expands, it is not expanding into some space "around" the universe. The universe by definition consists of all space: as the universe expands, the volume of space grows. (In the case of an infinite universe, it is meaningless to talk about change of the total volume. However, we can say that the volume of any sufficiently large finite portion of the universe grows in time. The disclaimer "sufficiently large" is needed because on small scales, space can collapse, as happens during structure formation.

In order to describe behaviour at high temperatures and energies, we have to treat matter in terms of quantum fields rather than classical particles. In this course, we will skip a lot of particle physics details, but we will need some quantities like masses, spins and interaction cross-sections. The Standard Model of particle physics is based on the symmetry group $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_W \otimes \mathrm{U}(1)_Y$, where C refers to the colour mediated by gluons, W to the weak interaction mediated by the W and Z bosons, and Y to the weak hypercharge. From the viewpoint of the Standard Model, we live today in a low-energy universe, where part of the symmetry of the theory is broken. The natural energy scale of the Standard Model is reached when the temperature of the universe exceeds 100 GeV (about 10^{15} K), which was the case when the universe was younger than 10^{-11} s. Then the primordial soup of particles consisted of free massless fermions (quarks and leptons) and massless gauge bosons mediating the interactions (colour and electroweak) between these fermions. The final piece of the Standard Model, the *Higgs boson*, was discovered in 2012.

The Higgs field is responsible for breaking the electroweak symmetry. In the electroweak transition the electroweak interaction splits into two separate interactions: 1) the weak interaction mediated by the massive gauge bosons $W^\pm$ and Z^0, and 2) the electromagnetic interaction mediated by the massless gauge boson γ, the photon.[2] Fermions and W and Z bosons get their masses in the EW transition (with the possible exception of neutrinos, the origin of their masses is an open question). The mass is due to the interaction of the particle with the Higgs field. The EW transition took place when the universe cooled to the critical temperature $T_c = 160$ GeV at $t \sim 10^{-11}$ s.

The order of the electroweak transition depends on the particle physics on those scales. In the Standard Model, the transition is a smooth crossover, not a phase transition. However, in extensions of the Standard Model, there can be a first order electroweak phase transition (so the two phases have different energy densities at the critical temperature), in which case it proceeds through the formation of bubbles of the new phase. The phase transition can then have interesting effects, and baryogenesis, the generation of the observed matter-antimatter asymmetry in

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[2] More accurately, the interaction before the symmetry breaking had the two parts $\mathrm{SU}(2)_W \otimes \mathrm{U}(1)_Y$, and afterwards the broken weak interactions and the unbroken electromagnetic symmetry each have parts of both.

baryons, may occur in that process. Observable amounts of gravitational waves may also be generated at such a phase transition.

The QCD transition (also a crossover, not a phase transition), also known as the quark-hadron transition, took place at $t \sim 10^{-5}$ s. The critical temperature of the QCD transition is $T_c \sim 150$ MeV. Quarks, which had been free until this time, formed hadrons: baryons (which contain three quarks), e.g. the nucleons n and p, and mesons (which contain a quark and an antiquark), e.g. π and K. The matter filling the universe was converted from a quark-gluon plasma to a hadron gas. All mesons and most baryons are unstable with rather short lifetimes, so only protons and neutrons remain at late times.

For every type of particle there is a corresponding *antiparticle*, which has the same mass and quantum numbers as the particle, except for charges (like the electric charge or colour charge), which have the opposite sign. Particles that do not have any charges, such as photons, are their own antiparticles. (Or we can say that they do not have antiparticles.) At high temperatures, $T \gg m$, where m is the mass of the particle, particles and antiparticles are constantly created and annihilated in various reactions, and there is roughly the same number of particles and antiparticles. When $T \ll m$, particles and antiparticles may still annihilate each other (and decay, if they are unstable), but there is no more thermal production of particle-antiparticle pairs. Therefore the number density of heavy particles decreases rapidly. Their annihilation reactions produce lighter particles and antiparticles. If the universe would originally have had an equal number of particles and antiparticles, only photons and neutrinos (of the known particles) would be left over today in any significant quantity. The presence of matter today indicates that in the early universe there must have been slightly more nucleons and electrons than antinucleons and positrons, and this excess was left over. The lightest known charged massive particle is the electron, so the last annihilation event was the electron-positron annihilation which took place when $T \sim m_e \sim 0.5$ MeV and $t \sim 1$ s.[3] After this the only remaining antiparticles were the antineutrinos, and the primordial soup consisted of a large number of photons (who are their own antiparticles) and neutrinos (and antineutrinos) as well as a much smaller number of left-over protons, neutrons, and electrons. Dark matter was left out of this story; in many dark matter models, an equal number of dark matter particles and antiparticles remain in the late universe. We will come back to this later.

When the universe was a few minutes old, $T \sim 0.1$ MeV $\sim 10^9$ K, protons and neutrons formed nuclei of light elements. This process is known as Big Bang Nucleosynthesis (BBN), and it produced about 75% (of the total mass in ordinary matter) ^{1}H, 25% ^{4}He, 10^{-4} ^{2}H, 10^{-4} ^{3}He, and 10^{-9} ^{7}Li. (Heavier elements formed much later in stars.) At this time matter was completely ionised, all electrons were free. In this plasma the mean free path of a photon was short, so the universe was opaque.

The universe became transparent when it was 380 000 years old. At a temperature $T \sim 3000$ K (~ 0.3 eV), electrons and nuclei formed neutral atoms, and the photon mean free path became longer than the radius of the observable universe. This event is called *recombination*. (This name, taken from thermodynamics, is

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[3] Neutrinos also have very small masses. However, at temperatures less than the neutrino mass, the neutrino interactions are so weak that the neutrinos and antineutrinos cannot annihilate each other. It is also possible that neutrinos are their own antiparticles, like photons.

misleading, since this is actually the first known time when electrons and nuclei combined.) A less misleading name is *last scattering*, when photons and electrons last interacted. Since then the primordial photons have been travelling through space almost without scattering. We can observe them today as the *cosmic microwave background* (CMB). The CMB is a photograph of the universe at 380 000 years of age, modified by its passage to us through 14 billion years.

The CMB shows that the early universe was locally very homogeneous and isotropic, unlike the present universe, where matter has accumulated into stars, galaxies, clusters, filaments and superclusters. However, even in today's universe, the distribution of structures in the late time universe is *statistically* homogeneous and isotropic, though there are large local variations. At the time of last scattering density variations in baryonic matter and photons were of the order 10^{-5},[4] and we see these as small intensity variations of the CMB (the CMB *anisotropy*). Due to gravity, slight overdensities in the matter grew in time, and eventually formed galaxies, which formed larger structures such as galaxy groups, clusters (large gravitationally bound groups), superclusters, filaments, and walls, separated by large, relatively empty voids. This process is called cosmological *structure formation*. Observations of this present-day *large-scale structure* of the universe form a significant body of data, which our cosmological theories are able to explain in detail.

There are two parts to structure formation:

1. The origin of the primordial density fluctuations, the seeds of galaxies. The scenario that has been most successful in explaining the observed properties of the primordial fluctuations is cosmic *inflation*, which probably occurred well before the electroweak crossover. We will discuss inflation in the second part of the course and calculate how primordial perturbations are generated from quantum fluctuations.

2. The growth of fluctuations in time. The growth is due to gravity, and it depends on the composition and amount (average density) of matter, as well as on how the universe expands.

One of the main open questions in cosmology today is that most of the matter and energy content of the universe appears to be in some unknown forms, called *dark matter* and *dark energy*. The dark matter issue dates back to 1930s, and the dark energy problem arose in the 1990s.

From the motions of galaxies we can deduce that matter made of baryonic particles, i.e. protons and neutrons[5] is only a small fraction of the total mass which affects the galaxy motions through gravity. The rest is dark matter, which we have observed only via its gravity, because it interacts so weakly with light (or is made of very massive and therefore rare clumps) There are also many lines of evidence for dark matter, including the motions of stars and gas in galaxies, the motions of galaxies and gas in clusters, gravitational lensing the structure of the CMB anisotropies, the pattern of large-scale structure, the change of the expansion rate of the universe with time, and gravitational lensing. Since all evidence for dark matter comes from its gravitational effect, it might in principle be possible to explain the observations

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[4] There were larger density variations in dark matter. The amplitude depends on the scale, but for relevant cosmological scales they were of the order 10^{-3} – we will come to this later in the second part of the course.

[5] And electrons. Although electrons are not baryons (they are leptons), cosmologists refer to matter made out of protons, electrons, and neutrons as "baryonic'". Electrons are anyway so light that their contribution to the total mass is tiny.

without dark matter by modifying the laws of gravity instead. However, trying to fit all the different observations has required resort to rather baroque models, and since the early 2000s no modified gravity model has been able to explain all the observations, whereas dark matter has explained and predicted many observations via one simple hypothesis. The focus is now on determining what dark matter is made of, not whether it exists.

The only stable and massive non-baryonic particles in the Standard Model are neutrinos. If they had suitable mass, ~ 1 eV, neutrinos left over from the early universe would have sufficient mass density to explain all dark matter. We now know that the neutrino masses are smaller. Also, neutrinos have moved too fast to produce the structures seen in the universe: they are called *hot dark matter (HDM).* Instead, according to observations we must have *warm dark matter* (WDM) or *cold dark matter* (CDM). The difference between *hot dark matter* (HDM) and *cold dark matter* (CDM) is that HDM is made of particles whose velocities were large when structure formation began, but CDM particles had negligible velocities; WDM is in-between. As the Standard Model does not contain particles that could explain all of the dark matter, it appears that most of the matter in the universe is made out of some unknown particles, or black holes formed at early times (rather than collapse of stars). Usually, neutrinos are excluded from the definition of dark matter (one could also say that they form a small subdominant component), so that the term "dark matter" refers only to the unknown, exotic part of the dark matter.

Particle physicists have independently come to the conclusion that the Standard Model is not the final word in particle physics. Many proposed extensions of the Standard Model contain suitable dark matter particle candidates (e.g. *neutralinos*, *technibaryons*, *axions*, right-handed neutrinos). Their interactions have to be rather weak to explain why they have not been detected so far, which imposes constraints on models of particle physics. Dark matter is a significant area of overlap between the physics of the very small and the very large.

The situation of *dark energy* is different. While cosmologists consider dark matter quite natural, many of them find dark energy puzzling. The increasing breadth and precision of cosmological observations has made it possible to determine the distance scales and the expansion of the universe accurately. In the context of the homogeneous and isotropic models based on general relativity, a form of matter called dark energy is required to fit these observations. Unlike dark matter, which is clustered, dark energy is relatively uniform in the observable universe. And while dark matter has negligible pressure, dark energy has a large negative pressure. The simplest possibility for dark energy is the *cosmological constant* or *vacuum energy.* Quantum field theory indicates that the the vacuum has an energy density, but it is difficult to understand the small energy scale $\sim$ meV that is required to explain the observations. Another possible explanation is modification of the law of gravity at large distances. In the dark energy case, this is less difficult than for dark matter, as the only observable effect of dark energy is to increase the expansion rate of the universe at late times, whereas the effect of dark matter is seen in various physical systems on different scales and in different eras of the universe. Nevertheless, constructing models that would explain the observations on large scales while being consistent with the precision tests of general relativity in the solar system has proven to be difficult (apart from the cosmological constant). This remains an active area of research. The third possibility is that the homogeneous and isotropic approximation is not good enough at late times due to the formation of non-linear structures. Theoretical work so far, however, shows that this approximation is surprisingly good, and the vacuum energy explanation has also been very successful in explaining and predicting observations, although the issue has not yet been definitely settled.

1.2 Units

Natural units. We use natural units in which $c = \hbar = k_B = 1$.

$c = 1$

The theory of relativity unifies space and time into a single concept: spacetime. It is thus natural to use the same units for measuring spatial distance and time. Since the (vacuum) speed of light is $c = 299\ 792\ 458$ m/s, we set 1 s $\equiv$ 299 792 458 m, so that $c = 1$ and 1 second = 1 light second and 1 year = 1 light year. (Note that in SI units, this value of c is exact, as it is used to define the meter.) Velocity is thus dimensionless, and smaller than unity for massive objects. Energy and mass have the same dimension, so for example the relation between mass m and energy E for free particles $E^2 = m^2c^4 + p^2c^2$ is simply $E^2 = m^2 + p^2$, where p is momentum.

$k_B = 1$

Temperature T is a parameter that describes a thermal equilibrium distribution. The formula for the occupation number of energy level E includes the exponential form $e^{\beta E}$, where $\beta = 1/(k_B T)$. The only role of the Boltzmann constant, $k_B = 1.380649 \times 10^{-23}$ J/K, is to convert temperature into energy units. (Again, in SI units, this value is exact, as it part of the definition of the system of units.) We give temperature directly in energy units, making k_B unnecessary. We define $k_B = 1$, so 1 K $= 1.380649 \times 10^{-23}$ J, or

$$1\ \text{eV} = 11600\ \text{K} = 1.78 \times 10^{-36}\ \text{kg} = 1.60 \times 10^{-19}\ \text{J}\ . \tag{1.1}$$

Thus the exponential form is just $e^{E/T}$.

$\hbar = 1$

The third simplification in the natural system of units is to set the reduced Planck constant to unity, $\hbar = h/2\pi = 1$. In SI units we have $\hbar = 6.62607015 \times 10^{-34}/(2\pi) \approx 1.054571817 \times 10^{-34}$Js, so in the natural system of units the dimensions of mass and energy are equal to the dimension of 1/time or 1/distance. (In SI units, this value of h is exact.) This is convenient, because the typical time and distance scales of quantum mechanics are associated with particle energy. For example, the energy of a photon $E = \hbar\omega = \omega$ is equal to its angular frequency. We have

$$1\,\text{eV} = 5.07 \times 10^{6}\,\text{m}^{-1} = 1.52 \times 10^{15}\,\text{s}^{-1}\ . \tag{1.2}$$

A useful relation to remember is

$$\hbar = 197\ \text{MeV}\,\text{fm} = 1\ , \tag{1.3}$$

where we have the energy scale $\sim$ 100 MeV and length scale $\sim$1 fm of strong interactions.

Equations now become simpler and the physical relations more transparent, since we do not have to include the above fundamental constants. However, still have to do conversions among different units because the preferred units used in particle units and cosmology (not to mention astrophysics) are different.

Astronomical units. A common unit of mass and energy is the solar mass, $M_\odot = 1.99 \times 10^{30}$ kg, and a common unit of length is the parsec, 1 pc = 3.26 light years = 3.09×10^{16} m. One parsec is defined as the distance from which 1 astronomical unit AU, the distance between the Earth and the Sun) forms an angle of one arcsecond, 1".[6] (Astronomers and cosmologists only use light years when talking to outsiders.) A common scale in cosmology is 1 Mpc = 10^6 pc, which is roughly the typical distance between galaxies at the present time.

1.3 The observable universe

The observations relevant to cosmology are mainly astronomical. The speed of light is finite, and therefore, when we look away in space, we also look back in time. The universe has been transparent since the formation of atoms at about 380 000 years, so we can see more than 99.99% of the history of the universe.

The most important channel of observation is the electromagnetic radiation (light, radio waves, X-rays, etc.) coming from space. We also observe charged particles (protons, electrons, nuclei), called *cosmic rays*, as well as neutrinos. In addition, the composition of matter in the solar system has cosmological significance. An important channel is gravitational waves. The first direct detection of gravitational waves was made on September 14 2015 by the Advanced LIGO detector, opening the era of gravitational wave astronomy, followed up with nearly a hundred detections by the LIGO/Virgo/KAGRA collaboration.

1.4 Redshift and the Hubble law

One of the starting points of modern observational cosmology was the discovery by Lemaître in 1927 (theoretically) and Hubble in 1929 (observationally) that the *redshift* of galaxies is proportional to their distance. Redshift refers to the fact that the visible light is redder (has longer wavelength) when it arrives to us than when it was emitted. This redshift can be determined with high accuracy from spectral lines. These lines are caused by transitions between different energy states of atoms and molecules, so their original wavelengths λ_0 can be measured in the laboratory on Earth[7]. The redshift z is defined as

$$z \equiv \frac{\lambda - \lambda_0}{\lambda_0} \qquad \text{or} \qquad 1 + z = \frac{\lambda}{\lambda_0} \tag{1.4}$$

where λ is the observed wavelength. The redshift is observed to be independent of wavelength and for small z follows the approximate relation

$$z = H_0 D \ , \tag{1.5}$$

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[6] One degree is divided into 60 arc minutes, denoted by 60', and one arc minute is divided into 60 arc seconds, denoted by 60".

[7] Assuming that the laws of physics are the same here and at the emission event. Put another way, spectral lines offer a sensitive test of the change of the laws of quantum electrodynamics and nuclear physics in time and space. No deviations from the laws observed on Earth have been found so far.

where D is the distance to the galaxy and z is its redshift. (The speed of light c is set to unity, as noted above.) This relation was first discovered by Lemaître, and it is called the *Hubble law*[8]. The proportionality constant H_0 is correspondingly called *Hubble constant*. It was introduced, its value was first determined from observations, and it was interpreted as the expansion rate by Lemaître.

While the redshift can be readily determined with high accuracy, it is more difficult to determine the distance D. Measurements of distances in general and the Hubble parameter in particular have been the subject of much work and controversy over decades. Distance determinations used to be exclusively based on the *cosmic distance ladder*. This refers to a series of relative distance determinations between more nearby and faraway objects. The first step of the ladder is made of nearby stars, whose absolute distance can be determined from their *parallax*, their apparent motion on the sky due to our motion around the Sun. The other steps require "standard candles", classes of objects with the same absolute luminosity (radiated power), so that their relative distances are inversely related to the square roots of their "brightness" or apparent luminosity (received flux). (In practice, standard candles do not exist, but there are "standardisable candles" – objects whose luminosity varies in a way that is at least empirically understood.) Nowadays there are also measurements that do not use the cosmic distance ladder, but rather fit for data on for example the cosmic microwave background or the clustering of galaxies on the sky, especially due to *baryonic acoustic oscillations*, the galaxy counterpart of the CMB anisotropies due to inhomogeneities in matter at last scattering. The drawback of these measurements is that the ones with high accuracy so far are dependent on assuming a certain cosmological model – we will discuss this in detail. A key distance ladder determination based on measurements by the Hubble Space Telescope is [1]

$$H_0 = 73.2 \pm 1.04 \text{ km/s/Mpc} \ . \tag{1.6}$$

The error represents the range where the real result is with a 68% probability. Doubling the error bar gives the 95% probability range. (Unless otherwise noted, all errors bars in the lecture notes correspond to the 68% range.) In contrast, a determination of H_0 from the CMB gives (we will later discuss how H_0 is determined from the CMB) [2]

$$H_0 = 67.4 \pm 0.5 \text{ km/s/Mpc} \ . \tag{1.7}$$

The probability that the difference in these numbers is a statistical accident, naively estimated assuming that the probability is Gaussian (i.e. follows the normal distribution), is 2×10^{-8}. More careful assessments of the probability do not improve the situation. This discrepancy between model-independent measurements from the Hubble flow in the local universe and model-dependent measurements spanning larger distances is also bone out by other observations. This *Hubble tension* is one of the biggest open questions in cosmology.

For small redshifts ($z \ll 1$), the redshift is sometimes spoken of if it was a Doppler effect due to relative motion of the source and the observer. The distant

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[8] In 2018, the International Astronomical Union voted to recommend the name Hubble–Lemaître law. It has not caught on.

galaxies are thus apparently receding from us with the velocity

$$v = z \ . \tag{1.8}$$

The further the galaxies are, the faster they seem to move. Astronomers sometimes report the redshift in units of velocity (by reintroducing c in (1.8), $z = v/c$).

However, according to general relativity, movement in space is not the proper way understand the redshift. The galaxies are not moving, the distance between the galaxies increases because the space between the galaxies is expanding. We will later derive the redshift from general relativity. It turns out that equations (1.5) and (1.8) hold only at the limit $z \ll 1$, and the general result, $D(z)$, which relates distance D and redshift z is more complicated. In particular, the distance reaches a finite value as z goes to infinity – though we should be careful about what we mean by distance! We will look at this in detail in the next chapter. The redshift is directly related to the expansion. The easiest way to understand the cosmological redshift is that the wavelength of travelling light expands with the universe. Thus the universe has expanded by a factor $1 + z$ during the time light travelled from an object with redshift z to us.

The expansion rate H changes on the cosmological timescale. The time-dependent function $H(t)$ (where t is the age of the universe) is called the Hubble rate or the parameter, and its present value is called the Hubble constant, H_0. In cosmology, it is customary to denote the present value of a quantity with the subscript 0. Thus $H_0 \equiv H(t_0)$.

The galaxies are not exactly at rest in the expanding space. Each galaxy has its own *peculiar motion* v_{gal}, caused by the gravity of nearby mass concentrations, such as other galaxies. Neighbouring galaxies can fall towards each other, orbit each other and so on[9].

Thus the redshift of an individual galaxy is the sum of the cosmic and the peculiar redshift.

$$z = H_0 D + v_{\text{gal}} \qquad (\text{when } z \ll 1) \ . \tag{1.9}$$

Usually only the redshift is known precisely. Typically v_{gal} is around $300 \ldots 500$ km/s. In large galaxy clusters, where galaxies orbit each other, it can be thousands of km/s. For nearby galaxies, the peculiar redshift can be larger than the cosmological redshift, and the total redshift can be negative, i.e. a *blueshift*. For distant galaxies, $H_0 D \gg v_{\text{gal}}$. The larger the redshift, the younger the universe was when the light left.

1.5 The horizon

Because of the finite speed of light and the finite age of the universe, only a finite part of the universe is observable. Our *horizon* is at the distance from which light has just had time to reach us during the entire age of the universe. Were it not for the expansion of the universe, the distance to this horizon r_{hor} would equal the age of the universe, around 14 billion light years (4300 Mpc). Expansion complicates

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[9] Alternatively, we may say that the galaxies remain in place, but there are local deviations in the expansion rate. This is the more natural interpretation from the point of view of general relativity. However, the idea of peculiar velocities is perhaps simpler to understand, as it is closer to Newtonian gravity in the way it is usually formulated.

the situation; we will calculate the horizon distance later. For large distances the redshift grows faster than (1.5). At the horizon $z \to \infty$, i.e. $D_{\text{hor}} = D(z = \infty)$. The universe has been transparent only for $z < 1090$ (after last scattering), so the "practical horizon", i.e. the limit to what we can see (at least using light; gravitational waves are not subject to this restriction), lies already at $z = 1090$. The distances $D(z = 1090)$ and $D(z = \infty)$ are very close to each other; $z = 4$ is about halfway from here to the horizon.

Therefore we can only observe a finite region of the universe, enclosed in the sphere with radius D_{hor}. The universe can extend far beyond that, and it may even be infinite. Sometimes the word "universe" is used to denote just this observable part of the whole' universe. Then we can say that the universe contains some 10^{12} galaxies and 10^{23} stars. When the expansion of space decelerates, the horizon recedes and parts of the universe that are beyond our present horizon become observable. When the expansion accelerates, as it seems to have done during the past few billion years, the observable region dows not grow, and if the acceleration continues, in the distant future galaxies that are now observable will disappear from sight.

1.6 Electromagnetic channel

Interstellar space is transparent, except for radio waves longer than 100 m, which are absorbed by interstellar ionised gas, short-wavelength ultraviolet radiation, which is absorbed by neutral gas and very high-energy gamma rays, which interact with the cosmic microwave background. However, Earth's atmosphere is opaque except for two wavelength ranges, the *optical window* ($\lambda = 300$–800 nm), which includes visible light, and the *radio window* ($\lambda = 1$ mm–20 m). The atmosphere is partially transparent to infrared radiation, which is absorbed by water molecules in the air; high altitude and dry air favours infrared astronomy. Accordingly, the traditional branches of astronomy are optical astronomy and radio astronomy. Observations at other wavelengths have become possible only during the past few decades, from space (with satellites), at very high altitude in the atmosphere (with planes, rockets and balloons) and in very dry regions (such as Antarctica).

From optical astronomy we know that there are *stars* in space. The stars are grouped into *galaxies*. There are different kinds of galaxies, such as irregular, elliptical, and flat disks or spirals. Our own galaxy (the Galaxy, or Milky Way) is a spiral galaxy. The plane of the disk can be seen (on a dark night) as a faint band –the Milky Way– across the sky.

Notable nearby galaxies and dwarf galaxies are the Andromeda galaxy (M31) and the Magellanic clouds (LMC, Large Magellanic Cloud, and SMC, Small Magellanic Cloud). These are the only other galaxies that are visible to the naked eye. (The Magellanic clouds and the center of the Milky Way lie too far south to be seen from Finland.) The number of galaxies that can be seen with powerful telescopes is many billions.

Other observable objects include dust clouds, which hide stars behind them, and gas clouds. Gas clouds absorb starlight at certain frequencies, which excite the gas atoms to higher energy states. As the atoms return to lower energy states they emit photons at the corresponding wavelength. Thus we can determine from the spectrum of light what elements the gas cloud is made of. In the same way the composition of stellar surfaces can be determined.

The earliest ”cosmological observation” was that the night sky is dark. If the universe were eternal and infinitely large, unchanging and similar everywhere, our eye would eventually meet the surface of a star in every direction, and the entire night sky would be as bright as the Sun. This is called *Olbers' paradox*.

Optical astronomy and the large-scale structure. There is a large body of data relevant to cosmology from optical astronomy. By counting the number of stars and galaxies we can estimate the matter density they contribute to the universe. From the different redshifts of galaxies within the same galaxy cluster we obtain their relative motions, which reflect the gravitating mass within the system. The mass estimates for galaxy clusters obtained this way are much larger than those obtained by counting the galaxies and gas in the cluster, pointing to the existence of dark matter.

From the spectral lines of stars and gas clouds we can determine the relative amounts of different elements and their isotopes in the universe.

The distribution of galaxies in space and their relative velocities tell us about the large-scale structure of the universe. The galaxies are not distributed uniformly. There are galaxy groups and clusters. Our own galaxy belongs to a small group of galaxies called the Local Group. The Local Group consists of three large spiral galaxies: M31 (the Andromeda galaxy), M33, and the Milky Way, and about 60 smaller galaxies and dwarf galaxies. The local group's diameter is around 3 Mpc. The nearest large cluster is the Virgo Cluster. The grouping of galaxies into clusters is not as strong as the grouping of stars into galaxies. Rather, galaxies are distributed in a complex pattern called the *cosmic web*, which consists of walls, filaments, clusters and voids (low-density regions). Most galaxies are not part of any well defined cluster.

Radio astronomy. The sky looks very different on radio wavelengths than to the naked eye. There are many strong radio sources very far away. These are galaxies which are optically barely observable. They are distributed isotropically, i.e. there are equal numbers of them in every direction, but there are more far away (at $z > 1$) than close by ($z < 1$). The isotropy is evidence of the homogeneity of the universe at the largest scales—there is structure only at smaller scales. The dependence on distance is a time evolution effect in two ways: the radio sources evolve and the volume of the universe evolves. In general, there are more objects at larger distances simply because there is more volume there. However, the evolution in the number counts is not explained only by the change in the volume of the universe, it shows that the radio sources themselves evolve. Some galaxies are strong radio sources when they are young, but become weaker with age by a factor of more than 1000.

Cold gas clouds can be mapped using the 21 cm spectral line of hydrogen. The ground state ($n = 1$) of hydrogen is split into two very close energy levels depending on whether the proton and electron spins are parallel or antiparallel (the hyperfine structure). The separation of these energy levels, the hyperfine structure constant, is 5.9 μeV, corresponding to a photon wavelength of 21 cm, i.e. radio waves. The redshift of this spectral line shows that redshift is independent of wavelength (it is the same for radio waves and visible light), as it should if it is due to the expansion of space.

1.7 Cosmic microwave background

At microwave frequencies the sky is dominated by the *cosmic microwave background* (CMB), which is highly isotropic, i.e. the microwave sky glows almost uniformly, with only small contrasts. The electromagnetic spectrum of the CMB is the black body spectrum with a temperature of $T_0 = 2.72548 \pm 0.00057$ K [3]. It follows the theoretical black body spectrum better than anything else we have observed. It is the remnant of a hot state in the early universe, when matter and light were almost homogeneously and isotropically distributed and in thermal equilibrium. The temperature of the CMB falls as $(1+z)^{-1}$ due to photon redshift, so as as the last scattering surface is a $z = 1090$, the original temperature was about 3000 K.

The state of a system in thermal equilibrium is determined by a small number of thermodynamic variables, in this case the temperature and chemical potentials (for particles with conserved quantum numbers). The observed temperature of the CMB and the observed density of the present universe allows us to fix the evolution of the temperature and the density of the universe, which then allows us to calculate the sequence of events in the early universe.

The small anisotropy on the microwave sky is dominated by the *dipole anisotropy* (one side of the sky is slightly hotter and the other side colder), with an amplitude of 3.346 ± 0.017 mK, or $\Delta T/T_0 = 0.0012$. This is a Doppler effect due to the motion of the observer, i.e. the motion of our Solar System with respect to the radiating matter at our horizon. The velocity of this motion is $v = (\Delta T/T_0)\, c = 369$ km/s, or $v = 0.00123$ and it is directed towards the constellation of Leo (R.A. $11^h 8^m 50^s$, Dec. $-6°37'$), near the autumnal equinox (where the ecliptic and the equator cross on the sky) [4]. It is due to two components, the motion of the Sun around the center of the Galaxy, and the peculiar motion of the Galaxy due to the gravitational pull of matter concentrations up to 100–200 Mpc away[10].

When we subtract the effect of this motion from the observations (and subtract emission from the Milky Way and other nearby sources – the Galaxy emits microwave radiation, but with a non-thermal spectrum) the true anisotropy of the CMB remains, with an amplitude of about 3×10^{-5}, or 80 μK.[11] This anisotropy gives a picture of the small density variations in the early universe, the seeds' of galaxies. Theories of structure formation have to match both the small inhomogeneity of the order 10^{-5} at $z = 1090$, the structure observed today ($z = 0$), and the evolution we measure in-between.

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[10] Sometimes people ask whether there is a contradiction with special relativity — doesn't CMB provide an absolute reference frame? There is no contradiction. Special relativity just says that the laws of physics are the same for all inertial observers. The realisations of these laws, i.e. real physical systems, however do not respect this symmetry, and have reference frames associated with them, such as the center-of-mass frame or the laboratory frame. For road transportation, the surface of the Earth is a natural reference frame. In cosmology, the CMB gives us a useful reference frame—it is closely related to the center-of-mass frame of the observable part of the universe, or rather, a part of it which is close to the horizon (the last scattering surface). The different parts of the plasma from which the CMB originates are moving with different velocities (part of the 10^{-5} anisotropy is due to these velocity variations). If there is something surprising here, it is that these relative velocities are so small, of the order of just a few km/s, reflecting the astonishing homogeneity of the early universe over large scales. We will return to the question of whether these are natural initial conditions when we discuss inflation in the second part of the course.

[11] These numbers refer to the standard deviation of the CMB temperature on the sky. The hottest and coldest spots deviate some 4 or 5 times this amount from the average temperature.

References

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