

Due on October 5 by 14.15.

1. **Temperature of non-relativistic species.** Show that for a non-relativistic particle species, the distribution function retains the thermal shape as the universe expands, with  $T \propto a^{-2}$  and  $\mu - m \propto a^{-2}$ .
2. **Matter–radiation equality.** Today the energy density of matter is  $\rho_{m0} = \Omega_{m0}\rho_{c0}$  and the energy density of radiation is  $\rho_{r0} = \rho_{\gamma0} + \rho_{\nu0}$ , where  $\rho_{\gamma0} = AT_0^4$  is the contribution of the microwave background ( $T_0 = 2.725$  K) and  $\rho_{\nu0} = (21/8)AT_{\nu0}^4$  is the contribution of the neutrino background (we assume neutrinos are massless). Here  $A = \pi^2/15$  and  $T_{\nu0} = (4/11)^{1/3}T_0$ .
  - a) What was the age of the universe  $t_{\text{eq}}$  when  $\rho_m = \rho_r$ ? (Note that at these early times –but not today– you can ignore the curvature and vacuum terms in the Friedmann equation; you don't have to make other assumptions about the values of  $\Omega_0$  or  $\Omega_{\Lambda0}$ , since the answer does not depend on them.) Give the numerical value (in years) for the cases  $\Omega_{m0} = 0.1, 0.3$ , and  $1.0$ , assuming  $H_0 = 70$  km/s/Mpc.
  - b) What is the temperature  $T_{\text{eq}} \equiv T(t_{\text{eq}})$ ? Give the numerical value (in K) in the three different cases.
3. **Transparency of the universe.** We say that the universe is transparent when the photon mean free path  $\lambda_\gamma$  is larger than the Hubble length  $l_H = H^{-1}$ , and opaque when  $\lambda_\gamma < l_H$ . The photon mean free path is determined mainly by the scattering of photons by free electrons, so  $\lambda_\gamma = 1/(\sigma_T n_e)$ , where  $n_e = xn_e^*$  is the number density of free electrons,  $n_e^*$  is the total number density of electrons, and  $x$  is the ionization fraction. The cross section for photon-electron scattering is independent of energy for  $E_\gamma \ll m_e$  and is then given by the Thomson cross section,  $\sigma_T = \frac{8\pi}{3}(\alpha/m_e)^2$ , where  $\alpha$  is the fine-structure constant. In recombination  $x$  falls from 1 to  $10^{-4}$ . Assume instant recombination at  $1+z = 1300$ .
  - a) Show that the universe is opaque before recombination and transparent after recombination. (Neglect radiation; see below for the parameter values.)
  - b) Interstellar baryonic matter gets later reionised (to  $x \sim 1$ ) by light from the first stars. What is the earliest redshift when this can happen without making the universe opaque again? (You can assume that most ( $\sim$  all) matter has remained interstellar.)

Calculate for  $\Omega_{m0} = 1.0$  and  $\Omega_{m0} = 0.3$  (note that  $\Omega_m$  includes nonbaryonic matter). Use  $\Omega_K = 0$  (so  $\Omega_{\Lambda0} = 1 - \Omega_{m0}$ ),  $h = 0.7$  and  $\eta = 6 \times 10^{-10}$ .
4. **Hubble length at the QCD transition.**
  - a) What is the Hubble length  $H^{-1}$  in meters during the QCD crossover, at  $T = 150$  MeV?
  - b) To what comoving scale does that distance correspond to today, at  $T = 2.7$  K?