

Due on Monday September 28 by 14.15.

- Age of the positively curved universe.** Find the age-redshift relation for a positively curved universe ($K > 0$) with only matter (no radiation or vacuum energy). Calculate t_0 if $H_0 = 70$ km/s/Mpc and a) $\Omega_0 = 1.1$, b) $\Omega_0 = 2$.
- The Λ CDM model.** Suppose that we have $H_0 = 70$ km/s/Mpc, $\Omega_{m0} = 0.3$ and $\Omega_{\Lambda 0} = 0.7$, so that $\Omega = \Omega_m + \Omega_\Lambda = 1$ and the universe is spatially flat.
 - Find the age of the universe today and at redshift $z = 1090$.
 - When is the matter density equal to the vacuum energy density? (Give both t and z .)
 - The scale factor has an inflection point, where $\ddot{a} = 0$, at which the expansion starts to accelerate. When does this happen, in t and in z ?
(Hint: Use the substitution $x^{3/2} = b \sinh \phi$ for the integral $\int \frac{x^{1/2} dx}{\sqrt{b^2 + x^3}}$.)
- Dynamical dark energy.** Consider a non-flat ($\Omega_K \neq 0$) universe that contains matter, radiation, and dark energy (with positive energy density) with the equation of state (3.88).
 - Find $H(z)$.
 - What are the different possibilities for the fate of the universe? How do they depend on the density parameters?
- Thermal distributions in the non-relativistic limit.** Derive the following equations for non-relativistic Maxwell-Boltzmann statistics ($T \ll m$ and $T \ll |m - \mu|$).

$$\begin{aligned}
 n &= g \left(\frac{mT}{2\pi} \right)^{3/2} e^{-\frac{m-\mu}{T}} \\
 \rho &= n \left(m + \frac{3T}{2} \right) \\
 p &= nT \\
 \langle E \rangle &= m + \frac{3T}{2} \\
 n - \bar{n} &= 2g \left(\frac{mT}{2\pi} \right)^{\frac{3}{2}} e^{-\frac{m}{T}} \sinh \frac{\mu}{T} .
 \end{aligned}$$