Cosmology I Homework 3 Fall 2026

Due on Monday September 21 by 14.15.

1. **Redshift.** Derive the relation between the scale factor and redshift using conformal time.

2. **Einstein–de Sitter model.** Consider the case $a \propto t^{2/3}$ and $K = 0$. (This corresponds to a spatially flat universe filled with non-relativistic matter.)

   a) Calculate the age-redshift relationship $t(z)$ and the angular diameter distance $D_A(z)$. (Express the age and distance in units of the Hubble time H_0^{-1}.)

   b) What is the particle horizon today in units of H_0^{-1}? (Defined as the proper distance to $z = \infty$.)

   c) What is the age of the universe (in years) today and at $z = 1090$ if $H_0 = 70$ km/s/Mpc?

   d) What is the angular diameter distance (in Mpc) to $z = 1090$ if $H_0 = 70$ km/s/Mpc?

   e) The function $D_A(z)$ has a maximum. At what redshift is it? What does this mean for observations?

3. **Continuity equation.**

   a) Derive the continuity equation

   $$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}$$

   from the Friedmann equations.

   b) What does this equation say about conservation of energy?