Cosmology II Homework 3 Fall 2026

Due on November 16 by 14.15.

1. **Running of the spectral index.** Assuming slow-roll, calculate $\mathrm{d}\varepsilon/\mathrm{d}(\ln k)$, $\mathrm{d}\eta/\mathrm{d}(\ln k)$, and α_s to leading order in the slow-roll parameters.

2. **Conservation in the super-Hubble limit.**

   a) Starting from eqs. (9.8) and (9.10), derive eqs. (9.18) and (9.19).
   b) Show that if $w = v^2$, then $\delta_{\boldsymbol{k}} = -2\Phi_{\boldsymbol{k}} =$ constant is a solution in the long-wavelength limit $k \ll aH$. (You may assume that $a \propto t^n$ with $n > 1/4$ and $v^2 =$ constant ≥ 0.)
   c) Is this the only solution? If so, explain why. If not, is it the dominant solution at late times?

3. **No growth of radiation perturbations.** Assuming there is only radiation ($v^2 = w = \frac{1}{3}$), derive the evolution of $\Phi_{\boldsymbol{k}}$ and $\delta_{\boldsymbol{k}}$ in the deep sub-Hubble regime ($k \gg aH$).

4. **Growth of matter perturbations.** Assuming there is only matter ($v^2 = w = 0$), derive the evolution of $\Phi_{\boldsymbol{k}}$ and $\delta_{\boldsymbol{k}}$ in the deep sub-Hubble regime ($k \gg aH$).