COSMOLOGY I Homework 4 Fall 2024

Due on Monday September 30 by 14.00.

1. **Age of the positively curved universe.** Find the age-redshift relation for a positively curved universe ($K > 0$) with only matter (no radiation or vacuum energy). Calculate t_0 if $H_0 = 70$ km/s/Mpc and a) $\Omega_0 = 1.1$, b) $\Omega_0 = 2$.

2. **The ΛCDM model.** Suppose that we have $H_0 = 70$ km/s/Mpc, $\Omega_{m0} = 0.3$ and $\Omega_{\Lambda 0} = 0.7$, so that $\Omega = \Omega_m + \Omega_\Lambda = 1$ and the universe is spatially flat.

   a) Find the age of the universe today and at redshift $z = 1090$.

   b) When is the matter density equal to the vacuum energy density? (Give both t and z.)

   c) The scale factor has an inflection point, where $\ddot{a} = 0$, at which the expansion starts to accelerate. When does this happen, in t and in z?

   (Hint: Use the substitution $x^{3/2} = b \sinh \phi$ for the integral $\int \frac{x^{1/2} dx}{\sqrt{b^2 + x^3}}$.)

3. **Dynamical dark energy.** Find $H(z)$ for a non-flat ($\Omega_K \neq 0$) universe, which contains matter, radiation, and dark energy (that is not vacuum energy), where the dark energy equation of state is

$$w(a) = w_0 + w_a(1 - a) \ ,$$

   where w_0 and w_a are constants. (This is not a theoretically motivated dark energy model: it is just a commonly used phenomenological parametrization, which may be a reasonable approximation over an observationally relevant redshift range).