COSMOLOGY II Homework 5 Fall 2024

Due on December 2 by 14.00.

1. **Gaussian window function.**

   a) Show that that the Fourier transform of the Gaussian window function

   $$W(\boldsymbol{x}) = \frac{1}{(2\pi)^{3/2}R^3}e^{-\frac{1}{2}x^2/R^2}$$

   is $W(\boldsymbol{k}) = e^{-\frac{1}{2}(kR)^2}$.

   b) Starting from eqs. (9.88) and (9.89), derive eq. (9.90).

2. **Power spectrum and the mean square density fluctuation.** Show that if the power spectrum of density perturbations is a power law,

   $$\mathcal{P}_\delta(k) = Ak^{n+3} \ ,$$

   and we use the Gaussian window function, then the mean square density fluctuation on scale R is

   $$\sigma^2(R) = \frac{1}{2}\Gamma\left(\frac{n+3}{2}\right)\mathcal{P}_\delta(R^{-1}) \ .$$

   (The convolution theorem may be helpful.)

3. **Timescales of structure formation.** Structures on scale R typically form when $\sigma^2(R) = 1$. What is the age and redshift of the universe when structures with size R form, when R is a) pc, b) Mpc, c) 10 Mpc, d) k_{eq}^{-1}, e) 1000 Mpc?

   Assume that the universe is spatially flat and matter-dominated. Take the primordial spectrum of comoving curvature perturbations to be scale-invariant with the amplitude $A = 5 \times 10^{-5}$. For the transfer function, you can use the approximation

   $$T(k)^2 = \frac{1}{1 + \beta(k/k_{\mathrm{eq}})^4} \ ,$$

   with $\beta = 3 \times 10^{-4}$. As the window function, you can use the step function $W(R) = \theta(R^{-1} - k)$ instead of a Gaussian. Assume $t_{\mathrm{eq}} = 50\,000$ years and $k_{\mathrm{eq}}^{-1} = 100$ Mpc.