Due on November 25 by 14.00.

1. **CDM density perturbations in the radiation-dominated era.** Consider scales $k \gg aH$ in the radiation-dominated era (and ignore decaying modes), in which case the gravitational potential is given by eq. (9.59). Assume that perturbations are adiabatic.

   a) Show that the solution of eq. (9.60) is

   $$\delta_{c\boldsymbol{k}} = \tilde{A}_{1\boldsymbol{k}} + \tilde{A}_{2\boldsymbol{k}} \ln y + \tilde{A}_{3\boldsymbol{k}} \int_y^\infty \frac{\mathrm{d}y'}{y'} \int_{y'}^\infty \mathrm{d}y'' \frac{\cos y''}{y''} \ ,$$

   where $y = k/(\sqrt{3}aH)$. What are the constants $\tilde{A}_{1\boldsymbol{k}}$ and $\tilde{A}_{2\boldsymbol{k}}$ in terms of $A_{1\boldsymbol{k}}$?

   b) Show that the last term is subdominant. (You can do this either by numerical plotting or analytically.)

2. **Sound waves.** For short wavelength modes with $k \gg k_J$, the Jeans equation for density perturbations in the matter-dominated universe is

   $$\ddot{\delta}_{\boldsymbol{k}} + 2H\dot{\delta}_{\boldsymbol{k}} + c_s^2 \frac{k^2}{a^2} \delta_{\boldsymbol{k}} = 0 \ .$$

   Solve $\delta_{\boldsymbol{k}}(t)$, assuming that matter is the only component present. How do the amplitude and the frequency of the oscillations change with time and scale factor? (Conformal time $\eta = \int \mathrm{d}t/a$ may be helpful.)

3. **Matter perturbations in the dark energy dominated era.** If dark energy is vacuum energy, its fraction of the energy density approaches one without limit. Assuming that vacuum energy dominates completely and the universe expands exponentially, find how matter density perturbations behave on scales $k \gg aH$ (take $c_s^2 \approx v^2 \approx 0$). (Note that the total density contrast and the matter density contrast are different, and vacuum energy density has no perturbations.)

4. **Matter perturbations in an underdense region.** Even if the universe is on average spatially flat (as observations seem to indicate), local underdense regions have negative spatial curvature and overdense regions have positive spatial curvature. A large local underdense region –a void– can be approximated by a negatively curved FLRW region. Find the solution to the Jeans equation for pressureless matter perturbations in a void. Assume there is no radiation or vacuum energy, but $\Omega_0 = \Omega_m \ll 1$, so that you can use the curvature-dominated ($\Omega_0 = 0$) solution for the background.