COSMOLOGY II Homework 1 Fall 2024

Due on November 4 by 14.00. The exercises are returned on the course Moodle page. You will get help for the exercises in the exercise sessions on Thursday at 10-12. The help sheet at the bottom of the course webpage may come in handy. It will also be available in the course exam.

1. **The oldness problem.** Assume that the universe contains only radiation (and has spatial curvature but no cosmological constant) and $g_* = 100$. Suppose that close to the Planck density, $\rho = 10^{-4} M_{\rm Pl}^4$, the density parameter was $\Omega = 0.9$.

   a) What is the age of the universe when Ω becomes smaller than 10^{-2}?
   b) What is the age of the universe when the temperature falls below $T = 2.7$ K?

2. **Free massless scalar field.** Consider a spatially flat universe dominated by a scalar field with $V = 0$. (Again, no cosmological constant.)

   a) Find the scale factor and the scalar field as a function of time.

   b) Compare to a matter-dominated universe: if there is also a small amount of curvature (Ω deviates slightly from 1) initially, which one becomes more easily dominated by curvature?

3. **Slow-roll parameters.**

   a) Demonstrate that $\varepsilon(\varphi) \ll 1$ and $|\eta(\varphi)| \ll 1$ are necessary conditions for the slow-roll approximation to hold.

   b) Explain why these conditions are not sufficient.

4. **E-folds of inflation.**

   a) Derive eq. (7.46) of the lecture notes.

   b) Consider the inflation model with with the potential $V(\varphi) = \frac{1}{2}m^2\varphi^2$. Assume that $m = 2 \times 10^{13}$ GeV. Take inflation to end when $\varepsilon = 1$ and find the value of φ at the end of inflation. Then find the value of ε and η when $N = 50$. What scale k (in units of H_0^{-1}) does this value of N correspond to, assuming instant reheating?

5. **Bonus problem: spatial curvature.** (The points from this question are extra: they only increase your number of points, not the maximum number of points. So it is possible to get more than 100% of the points.) Assume that at the beginning of inflation we have $|\Omega_K| = 0.1$.

   a) Calculate, as a function of the reheating temperature $T_{\rm reh}$, how many e-folds of inflation are required to reduce present-day spatial curvature to $|\Omega_{K0}| < 10^{-2}$. (Assume $h = 0.7$ and that neutrinos are massless.) Approximate that the expansion rate at the beginning of inflation is completely dominated by the inflaton, that the inflaton field value does not change during inflation and that reheating happens instantaneously.

   b) In which directions do the above approximations change the result?

   c) What is the number of e-folds for $T_{\rm reh} = 10^7$ GeV?