

Due on October 24 by 12.00.

1. **Redshift of non-relativistic particles.** Consider a distribution of non-interacting non-relativistic particles in kinetic equilibrium in an expanding universe. Using the fact that momentum redshifts as $p_2 = (a_1/a_2)p_1$, show that the distribution remains in kinetic equilibrium. How are the temperature and the chemical potential redshifted?
2. **Time and distance scales of the early universe.** Let t denote the age of the universe, t_H the Hubble time, d_{hor} the horizon distance, $d_{pou}(t) = a(t)d_{hor}(t_0)$ the size of the presently observable universe at time t , n_B the baryon density and S the total entropy of the region inside the present-day horizon. Find the values of these quantities at $T = 100$ GeV and 1 MeV. Compare to the density of water and air on Earth and to the density of a nucleus. (Calculate the horizon distance numerically, assuming the spatially flat Λ CDM model with $\Omega_{\Lambda 0} = 0.7$ and $h = 0.7$.)
3. **Transparency of the universe.** We say that the universe is transparent when the photon mean free path λ_γ is larger than the Hubble length $l_H = H^{-1}$, and opaque when $\lambda_\gamma < l_H$. The photon mean free path is determined mainly by the scattering of photons by free electrons, so that $\lambda_\gamma = 1/(\sigma_T n_e)$, where $n_e = xn_e^*$ is the number density of free electrons, n_e^* is the total number density of electrons, and x is the ionization fraction. The cross section for photon-electron scattering is independent of energy for $E_\gamma \ll m_e$ and is then called the Thomson cross section, $\sigma_T = \frac{8\pi}{3}(\alpha/m_e)^2$, where α is the fine-structure constant. In recombination x falls from 1 to 10^{-4} . Assume instant recombination at $1+z = 1300$.
 - a) Show that the universe is opaque before recombination and transparent after recombination. (You can assume a matter-dominated universe; see below for parameter values.)
 - b) Interstellar baryonic matter gets later reionized (to $x \sim 1$) by light from the first stars. What is the earliest redshift when this can happen without making the universe opaque again? (You can assume that most (\sim all) matter has remained interstellar.)

Calculate for $\Omega_{m0} = 1.0$ and $\Omega_{m0} = 0.3$ (note that Ω_m includes nonbaryonic matter). Use $\Omega_\Lambda = 0$, $h = 0.7$ and $\eta = 6 \times 10^{-10}$.