Higgs inflation with the Nieh–Yan and the Holst term

Licentiate thesis

Juha-Matti Ojanperä

Helsinki institute of Physics
University of Helsinki
Finland

Acknowledgments

First of all I would like to thank my supervisor Syksy Räsanen for giving me a chance to work on such an interesting topic and having the patience to see through the long process of writing this thesis. I also want thank Miklos Långvik for interesting conversations and support. I also would like to thank the people sharing the dungeon of the workroom 313 during my time there and giving invaluable peer support and much needed humor. There are lots of friends who have always been there and new ones who I have come to know along the way. I am grateful to have people like you in my life.

Last and most importantly I want to thank my family and especially my parents who have always supported me in my life.

Contents

Included publications

This thesis is based on the following publication.

Paper I. **Higgs inflation with the Holst and the Nieh–Yan term** *41*

M. Långvik, Juha-Matti Ojanperä, S. Raatikainen and S. Räsanen
Phys. Rev. D **103** 8, 083514 (2021)

The author of thesis did initial calculations for the work and the tetrad solution for the torsion.

Chapter 1

Introduction

The Big Bang model[1] successfully describes the universe's evolution from a hot, dense state to the present-day universe. However, it faces several problems. First of these is the horizon problem: how distant regions of the universe, too far apart to have ever shared light or information since the Big Bang, still ended up with almost exactly the same temperature and conditions. The second is the flatness problem: why the universe's overall geometry is so extremely close to perfectly flat today, when even the tiniest deviation in its early state would have grown over time into a vastly curved universe. The third problem is how we can explain the origin of primordial inhomogeneities: how tiny, random quantum fluctuations in the very early universe grew into the large-scale structures we see today like galaxies and clusters without disrupting the universe's overall smoothness. Cosmic inflation is a paradigm proposing an epoch of accelerated expansion immediately after the initial singularity. It explains the observed universe by separating once causally connected regions beyond the observable horizon, stretching quantum fluctuations to large scales, smoothing initial anisotropies, and driving the cosmos toward spatial flatness.

We do not have one single inflationary model that would fit the observations; we have a huge number of them. While the inflationary era of the universe is widely accepted as a working paradigm, we do not have consensus on the model, mainly due to the restrictions of the observations and also because the models themselves give results that overlap with each other.

Of the many inflationary models, Higgs inflation stands out for its conservative simplicity. Instead of introducing a new, previously unknown scalar field as the inflaton, Higgs inflation is built upon the idea that the Standard Model Higgs boson observed in the Large Hadron Collider (LHC) acts as the inflaton.

However, the dynamics and viability of Higgs inflation have a close connection to how the Higgs field interacts with gravity. This is realized when we consider Higgs inflation in the context of different formulations of Einstein's general relativity. The standard way

[1] Meaning thermal equilibrium of particles in Friedman–Robertson–Lemaître–Walker-background.

(or the simplest way) to model gravity is through the Einstein–Hilbert action, where the existing fields besides gravity are minimally coupled, meaning they do not directly interact with the gravitational part of the action described by the Ricci scalar. However, in quantum field theory in curved spacetime, a non-minimal coupling between the Higgs field and the Ricci scalar is not only permissible but often anticipated. The non-minimal coupling makes Higgs inflation sensitive to differences in different formulations of gravity, potentially yielding distinctive predictions for observable cosmological parameters, such as the spectral index and tensor-to-scalar ratio of primordial fluctuations.

General relativity can be formulated in various mathematical ways, which may have different predictions for Higgs inflation in the case of non-minimal coupling. Beyond the standard metric formulation, where the metric tensor is the sole independent gravitational degree of freedom, alternative approaches like the Palatini formulation treat the metric and the connection (which describes how vectors are transported across spacetime) as independent variables.

Einstein's general relativity is a geometrical theory; it depicts gravitational phenomena as the curvature of spacetime. However, one may also consider theories in which we describe gravity using other geometrical quantities instead of curvature. One example is Einstein–Cartan gravity [1] which mixes non-zero torsion, meaning the failure to parallel transport a vector, and non-zero curvature. In teleparallel gravity [2], the effects of gravity are described by torsion, while curvature is set to zero. Teleparallel gravity also has another version called symmetric teleparallel gravity (STGR) [3] in which the curvature and torsion are set to zero but the non-metricity, meaning the failure of covariant constancy of a vector, is not. The effects of gravity are fully described in STGR with the non-metricity tensor. It is also possible to consider the Einstein–Hilbert action with extra terms. These could be for example, higher-order curvature terms or terms that the non-minimal coupling elevates to dynamical ones, such as terms of a topological nature. In this thesis, we consider terms of the last type, the Holst and Nieh–Yan terms, particularly relevant in the context of Loop Quantum Gravity (LQG), a non-perturbative approach to a theory of quantum gravity. If torsion vanishes, the Holst term will be zero at the classical level. However, it has a role in loop quantum gravity.

In this work, we first go through an overview of cosmology and general relativity in Chapter 2. After this, we take a first glance into inflation in the case of a single scalar field in Chapter 3. We introduce Higgs inflation in detail in Chapter 4 and derive the predictions for the inflationary observables in the metric and in the Palatini case. In Chapter 5 we investigate how the non-minimal coupling affects the predictions of Higgs inflation with the Holst and Nieh–Yan terms and whether we can have successful inflation, meaning that the predictions are in the limits of observations, with different combinations of couplings to the Ricci scalar and to the Holst and Nieh–Yan terms. Conclusions are presented in Chapter 6.

Conventions

- For the metric tensor we use the signature $(-1, 1, 1, 1)$.
- We use unit system where the speed of light c and the reduced Planck constant $\hbar$ are set to one: $c = 1$ and $\hbar = 1$.
- We set the Planck mass $M_{Pl} = \sqrt{\frac{1}{8\pi G}} = 1$.
- The Levi–Civita connection compatible with the metric is

$$\overset{\circ}{\Gamma}{}^{\gamma}_{\alpha\beta} = \frac{1}{2} g^{\sigma\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right) \ . \tag{1.1}$$

  We will denote by "o" the quantities built from the Levi–Civita connection.
- The Riemann tensor is defined as

$$R^{\alpha}{}_{\beta\gamma\delta} \equiv \partial_\gamma \Gamma^{\alpha}_{\delta\beta} - \partial_\delta \Gamma^{\alpha}_{\gamma\beta} + \Gamma^{\alpha}_{\gamma\lambda} \Gamma^{\lambda}_{\delta\beta} - \Gamma^{\alpha}_{\delta\lambda} \Gamma^{\lambda}_{\gamma\beta} \ , \tag{1.2}$$

  and the Ricci scalar is

$$R \equiv \delta^{\gamma}_{\alpha} g^{\beta\delta} R^{\alpha}{}_{\beta\gamma\delta} \ . \tag{1.3}$$

Chapter 2

General Relativity and Cosmology

On the largest scales, the dominant interaction in the universe is gravitation. The theory that we use to describe gravitation is still, today, after passing countless experiments, Albert Einstein's general relativity [4]. It depicts gravitational phenomena as a consequence of the geometric structure of spacetime. More precisely, the gravitational field manifests itself as the curvature of spacetime. The energy and matter content of spacetime dictate how spacetime should curve, and at the same time, the curved spacetime geometry tells matter how to move[1]. The framework for general relativity is set by Einstein's field equations [6]

$$\mathring{G}_{\mu\nu} = \mathring{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathring{R} = T_{\mu\nu} \ . \tag{2.1}$$

The Einstein field equations relate the Einstein tensor $G_{\mu\nu}$, constructed from the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R, which describe the geometry of spacetime, to the matter and energy content of spacetime described by the stress-energy tensor $T_{\mu\nu}$. The Einstein field equations can be obtained from the Einstein–Hilbert action via variation with respect to the metric[2] $g_{\mu\nu}$

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \mathring{R} + S_{\text{matter}} \ , \tag{2.2}$$

where S_{matter} is the action for the matter fields.

In cosmology, we assume, based on observations, that on large scales, the universe is statistically isotropic everywhere. This also leads to the fact that the universe is statistically homogeneous [7]. So all directions and places in the universe are on equal footing. The assumption of an isotropic and homogeneous universe on large scales is also remarkably well supported by observations, and the approximation is even better justified when we go further into the past [7].

[1] This description was given by John Wheeler, found for example in [5].

[2] We will later discuss the Palatini formalism, where the variations are done with respect to the metric and the connection.

2.1 The Friedmann–Robertson–Walker metric and Friedmann equations

An isotropic and homogeneous spacetime describing our universe is represented by the Friedmann–Robertson–Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi\right)\right] , \tag{2.3}$$

where $a(t)$ is the scale factor depending on cosmic time t. The scale factor determines how space expands. The radial coordinate is r, the constant k is related to the curvature of space, and the usual angular spherical coordinates are θ and ϕ.

In order to figure out the dynamics of the universe, we need the Einstein field equations. The energy-momentum tensor is that of a perfect fluid with energy density $\rho = \rho(t)$ and pressure density $p = p(t)$

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu} , \tag{2.4}$$

with U_μ being the four-velocity of the fluid [6] .

By substituting the metric (2.3) into the Einstein field equations, we obtain the Friedmann equations [6] for the scale factor $a(t)$:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3} - \frac{k}{a^2} , \tag{2.5}$$

which can be written with the Hubble parameter $H \equiv \frac{\dot{a}}{a}$ as

$$H^2 = \frac{\rho}{3} - \frac{k}{a^2} . \tag{2.6}$$

The second equation we obtain is

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) . \tag{2.7}$$

The dot refers to a derivative with respect to cosmic time t. In the Friedmann equations (2.5) and (2.7), we have three unknowns: the scale factor $a(t)$, energy density $\rho(t)$, and pressure density $p(t)$, but only two equations. Therefore, we need some extra information in order to find the solutions. This is achieved if we have an equation of state relating pressure and energy density. The simplest scenarios for these in cosmology are matter with $p = 0$, radiation with $p = \frac{\rho}{3}$, and vacuum energy $\rho =$ constant.

For later use, let us define the density parameter $\Omega \equiv \frac{\rho}{\rho_c}$, where $\rho_c = 3H^2$ is the critical density. The critical density corresponds to the energy density of the universe for which the universe is spatially flat in the absence of the cosmological constant. In terms of the density parameter Ω, the equation (2.5) can be written as

$$\Omega - 1 = \frac{k}{a^2 H^2} \ . \tag{2.8}$$

Chapter 3

Inflation

3.1 Shortcomings of the Hot Big Bang Model

Even though the conventional Hot Big Bang model (without inflation) was successful in predicting, for example, the cosmic microwave background [8, 9] and the nucleosynthesis of the light elements in the early universe [10], it does have some problems.

The cosmic microwave background (CMB) temperature spectrum reveals that regions of the sky separated by at least one angular degree have the same temperature with an accuracy of 10^{-4}. However, in the Hot Big Bang model, regions that are separated by more than about one angular degree could not have been in causal contact with each other [11]. How is it then possible to observe a universe that is so isotropic and homogeneous? This is often titled the 'horizon problem'. On the other hand, the universe seems to be statistically isotropic and homogeneous on a large scale, but on a small scale we have inhomogeneities such as galaxies, stars, and voids. One may ask, what is the origin of these small scale variations?

Another problem that arises in the Hot Big Bang model is why we observe a spatially flat universe today. This means that in the Friedmann equation (2.5), the term related to the curvature $-\frac{k}{a^2}$ is either zero or extremely small compared to the value of the other term containing the energy density ρ. This feature is often called the flatness problem, or the oldness problem.

In the case where $\Omega = 1$ the universe is flat, there is no time evolution for the density parameter, and the universe remains flat forever. We can further see that in the equation (2.8) the so-called comoving Hubble length[1]

$$l_{cH} = \frac{1}{aH} \ , \tag{3.1}$$

[1] Naively one might say that the comoving Hubble length gives the comoving distance light travels during one Hubble time. However, this is not the whole picture, since the comoving Hubble length and the Hubble time change with time.

increases as a function of time during matter or radiation domination in the universe [11] driving the universe away from flatness. Furthermore, $|\Omega - 1| \propto t$ in the radiation dominated era. Considering that the observed value today for $\Omega(t_0)$ is very close to one:

$$|\Omega(t_0) - 1| < 10^{-3} ,$$

we can conclude that in the past it must have been even closer to one [12].

We cannot predict the value of Ω from theory, so one may ask why the initial conditions were so finely tuned that the value of Ω is so close to one. Even the slightest deviations in the initial conditions lead either to a universe which collapses shortly after its beginning, or to a universe which cools down and becomes empty way too fast.

3.2 How inflation alleviates the problems

The source of both of the problems can be traced to the behaviour of the comoving Hubble length (3.1), which grows as a function of time. In order to change this behaviour, in inflation [13–16] one postulates an acceleration period of the early universe in which the scale factor of the universe $\ddot{a} > 0$. This means that for $\frac{d}{dt}\frac{1}{aH} < 0$ and so the Hubble radius shrinks. In the equation (2.8) we see that the universe is driven towards flatness in the early times and we have a solution to the flatness problem.

As for the horizon problem, the shrinking comoving Hubble length during inflation means that the causally connected distance between two points gets smaller in comoving units and makes it possible for the presently observable universe to originate from a small, causally connected region.

3.3 The basics of inflation

In order to have the condition for inflation met in other words $\ddot{a} > 0$, we see from equation (2.7) that we must have

$$\rho + 3p < 0 , \tag{3.2}$$

and therefore

$$\frac{p}{\rho} < -\frac{1}{3} . \tag{3.3}$$

The energy density ρ is assumed to be positive for ordinary matter[2], so for the condition of inflation to be met, we need to have pressure that is negative: $p < 0$. A simple example of a matter type that has this property is a slowly rolling scalar field, meaning that the potential of the field dominates the kinetic term, and the friction term dominates the acceleration of the field. For a scalar field, the Lagrangian is

[2] Also pressure is positive for ordinary matter under normal circumstances.

$$\mathcal{L} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right] \ , \tag{3.4}$$

where ϕ is the scalar field, the inflaton and $V(\phi)$ is the potential of the field. We assume that the field ϕ is minimally coupled to gravity, meaning that the spacetime curvature is manifest only in the metric of the kinetic term and in the determinant of the metric. Later on, we will discuss the case of non-minimal coupling in the context of Higgs inflation, where the Higgs field directly couples to the Ricci scalar.

When we have an isotropic and homogeneous universe with the scalar field, the Friedmann equations (2.6) and (2.7) take the form[3]

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \tag{3.5}$$

and

$$\frac{\ddot{a}}{a} = -\frac{1}{3}\left(\dot{\phi}^2 - V(\phi)\right) \ . \tag{3.6}$$

With these, we can also derive an equation of motion for the scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \ , \tag{3.7}$$

where prime denotes the derivative with respect to the scalar field ϕ. The $3H\dot{\phi}$ term acts like a friction term and slows down the evolution of the field.

From (3.5) we see that the condition for inflation $\ddot{a} > 0$ now requires that $\dot{\phi}^2 < V(\phi)$ in other words, the potential dominates the kinetic term of the field, as we mentioned earlier. We also want the second derivative of the field to be small in order to maintain inflation long enough. These matters can be addressed in the so-called slow-roll approximation, where we assume the following slow-roll conditions [6]:

$$\dot{\phi}^2 < V(\phi) \ , \tag{3.8}$$

and

$$|\ddot{\phi}| \ll |3H\dot{\phi}| \ . \tag{3.9}$$

Equivalently, for these conditions to be met, we require that, in terms of the slow-roll parameters ϵ_H

$$\epsilon_H \equiv \frac{\dot{\phi}^2}{2H^2} < 1 \tag{3.10}$$

and η_H

$$\eta_H \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}, \quad |\eta_H| \ll 1 \ , \tag{3.11}$$

where the lower index H refers to the fact that these are written in terms of the Hubble parameter. The slow-roll conditions simplify the equations (3.5) and (3.7) into linear

[3]The curvature term can also be neglected since inflation will smooth down the universe.

first-order differential equations

$$H^2 \approx \frac{1}{3}V(\phi) \tag{3.12}$$

and

$$3H\dot{\phi} \approx -V'(\phi)\ . \tag{3.13}$$

The slow-roll parameters can also be written in terms of the potential [7]

$$\epsilon_H \approx \epsilon_V, \quad \eta_H \approx \eta_V - \epsilon_V\ , \tag{3.14}$$

with

$$\epsilon_V \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2\ , \tag{3.15}$$

and

$$\eta_V \equiv \frac{V''}{V}\ . \tag{3.16}$$

The condition that the slow-roll parameters are small is a necessary condition, but not sufficient for the slow-roll approximation to be applicable. The slow-roll parameters only limit the form of the potential. It is possible to fix the initial conditions in a way that the derivative of the field $|\dot{\phi}|$ is so large that the flatness of the potential will not guarantee successful inflation. However, most initial conditions lead to an inflationary phase if the slow-roll parameters are small [6, 17]. In addition to the above, we also have the higher-order slow-roll parameters [17]

$$\zeta_V \equiv \frac{V'V'''}{V}\frac{1}{V}\ , \ldots \tag{3.17}$$

followed by a new parameter for $n \geq 3$ in the form $(\frac{V'}{V})^{n-1}\frac{1}{V}\frac{d^{n+1}V}{d\phi^{n+1}}$.

We can quantify the amount of inflation during slow roll in e-folds N defined often as

$$N \approx \int_{\phi_2}^{\phi_1} \frac{V}{V'}d\phi\ , \tag{3.18}$$

Where the approximate equality is there to emphasize the fact that this holds only in slow roll. The field starts at some value ϕ_1 and ends up at a value ϕ_2 that corresponds to a situation where inflation ends and the slow roll parameters are of the order of unity. In one e-fold the scale factor a increases by a factor of Euler's number e.

3.4 Testing inflationary models

The remarkable feature of inflationary models of the universe is that they offer an explanation for the observed small anisotropy in the CMB and at the same time they also explain the flatness of the universe. The origin of the observed anisotropy lies in the particle responsible for inflation, the inflaton, and its quantum fluctuations. Inflation stretches the quantum-scale fluctuations of the inflaton field to large scales. The generation and evolution of these perturbations are described by cosmological perturbation theory, which makes it possible to calculate observable quantities in order to test the different inflation models [11, 18].

In general relativistic perturbation theory, tensorial quantities are split into a background part and a small perturbation around it. In cosmological perturbation theory, we add small perturbations on top of the FRW-metric $g_{\mu\nu}$ and the inflaton field ϕ, which leads to deviations from homogeneity. The metric $g_{\mu\nu}$ is divided into a background part $\overline{g}_{\mu\nu}$ and a small perturbation $\delta g_{\mu\nu}(t,\, \mathbf{x})$ so that

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + \delta g_{\mu\nu}(t,\ \mathbf{x}) \ . \tag{3.19}$$

Likewise, for the inflaton field, we have the homogeneous background $\overline{\phi}$ and a small perturbation $\delta\phi(t,\, \mathbf{x})$

$$\phi = \overline{\phi} + \delta\phi(t,\, \mathbf{x}) \ . \tag{3.20}$$

Notice that the perturbation is now a function of space and cosmic time. We only keep the first-order linear terms in the perturbation series.

The basic route following [19] would now be to construct a gauge invariant variable describing both the metric and the scalar perturbations. Since general relativity is diffeomorphism invariant, we have the freedom to choose the coordinate system freely. In cosmological perturbation theory, this complicates the situation since we have a freedom of gauge. This means that a poor choice of coordinates might lead to unphysical degrees of freedom. This means that the physical isotropic homogeneous spacetime could in principle have some inhomogeneities that are just coordinate artifacts, and we must be able to recognize them from the physical perturbations and keep track of them along the way. Another way to address the problem is to build a gauge invariant variable that describes the perturbations. This has been done in various ways and the whole process is beyond our needs. One way is to proceed as in [19, 20]: first build an action characterizing the perturbations. Then the action is expressed in terms of a gauge invariant variable, known as Sasaki–Mukhanov variable Q that represents the scalar perturbations. Then we canonically quantize Q analogously to that of a scalar field with a time-dependent mass. The evolution of the perturbations is described by the Fourier mode functions u_k of Q

$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0 \ , \tag{3.21}$$

with $z = a\frac{\dot{\phi}}{H}$, k being the comoving wave number of the Fourier mode and the prime denote the derivative with respect to the conformal time $\eta = \frac{dt}{a(t)}$. Equation (3.21) can be solved with suitable initial conditions by choosing the vacuum state to be the so-called Bunch-Davies vacuum state [21], meaning a zero-particle state of a comoving observer in the past infinity [22], and using the slow-roll approximation[4]. The solutions to (3.21) are then used to compute the two-point correlation functions such as the power spectrum for the comoving curvature perturbation $\mathcal{P}_{\mathcal{R}}(k)$. The comoving curvature perturbation $\mathcal{R}$ measures the spatial curvature of comoving hypersurfaces induced by the inflaton field. The main advantage of using the comoving curvature perturbation is that it is conserved on super-Hubble scales during inflation. This makes it possible to relate late-time observables such as the galaxy distributions to the initial conditions of inflation [23]. During inflation, the comoving curvature perturbation $\mathcal{R}$ can be written in terms of the Sasaki–Mukhanov variable Q so that

$$\mathcal{R} = \frac{Q}{z}. \tag{3.22}$$

Furthermore, the power spectrum $\mathcal{P}_{\mathcal{R}}(k)$ is described on super-Hubble scales [23]

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\frac{|u_k|^2}{z^2} \approx \frac{V}{24\pi^2\epsilon_V} \ . \tag{3.23}$$

In single-field inflation, the perturbations of the CMB are expected to be very nearly Gaussian in nature; therefore, the power spectrum provides an excellent approximation of their statistical properties [23].

In a similar way that inflation leads to scalar perturbations, it also leads to the production of gravitational waves; in other words tensor perturbations. Analogously to the equation (3.21), for tensor perturbations h_{ij} the mode equation takes the form

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0 \ , \tag{3.24}$$

where now v_k are the mode functions[23]. The power spectrum for the gravitational waves is given by

$$\mathcal{P}_{\mathcal{T}}(k) = \frac{8k^3}{2\pi^2}\frac{|v_k|^2}{z^2} = \frac{2V}{3\pi^2} \ . \tag{3.25}$$

The tensor power spectrum is usually normalized relative to the measured amplitude of scalar perturbations so that we have the so-called tensor-to-scalar ratio

$$r \equiv \frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon_V \ . \tag{3.26}$$

[4]One can also solve the equation (3.21) numerically.

We can also define the scalar spectral index n_s as

$$n_s \equiv 1 + \frac{d\mathcal{P}_\mathcal{R}}{d\ln k} = 1 - 6\epsilon_V + 2\eta_V \tag{3.27}$$

and it's running α_s

$$\alpha_s \equiv \frac{d^2\mathcal{P}_\mathcal{R}}{d(\ln k)^2} = -16\epsilon_V\eta_V + 24\epsilon_V + 2\zeta^2 \ . \tag{3.28}$$

3.5 Observational constraints for inflation

In the last section we defined the power spectrum of the scalar and tensor perturbations. These can be related to cosmological observables such as the CMB temperature. The Planck satellite data [24] with combined measurements from other experiments, gives us the latest observational results for inflation. For scalar perturbations the power spectrum

$$\mathcal{P}_\mathcal{R}(k_*) \equiv A_s = (2.092 \pm 0.034) \times 10^{-9} \tag{3.29}$$

at pivot scale $k_* = 0.05\,\mathrm{Mpc}^{-1}$, where A_s is the amplitude of the spectrum at pivot scale[24]. For the scalar spectral index n_S and for its running α_s we have [24]

$$n_s = 0.9641 \pm 0.0044, \quad \alpha_s = -0.0045 \pm 0.0067 \ . \tag{3.30}$$

For the tensor-to-scalar ratio experiments give an upper limit[25]

$$r < 0.036 \ . \tag{3.31}$$

While this thesis was being prepared, a joint analysis of data from the Atacama Cosmology Telescope (ACT) [26] and the Dark Energy Spectroscopic Instrument (DESI) [27] reported a scalar spectral index of $n_s = 0.974 \pm 0.003$.

Chapter 4

Higgs inflation in the metric and in the Palatini formulation

Since it has been shown that a slowly rolling scalar field leads to successful inflation, it is the starting point of most of the current inflationary models. Many of the existing models require a scalar particle that is outside the realm of the Standard Model of particle physics. In these models, the particle responsible for inflation, the inflaton, can be found from a Grand Unified Theory, supersymmetry, or string theory, etc. (for a review of inflationary models [28]).

From an experimental point of view, the only observed scalar particle is the Higgs boson of the Standard Model of particle physics [29]. It is then quite natural to ask whether the Higgs boson could act as the inflaton. In 2007, Bezrukov and Shaposnikov [30] showed that this is indeed possible. However, Higgs inflation requires a feature that is not evident at first sight. The Higgs field must be non-minimally coupled to the Ricci scalar in order to achieve successful inflation. This is due to the fact that the Higgs self-coupling is too large and results in a spectrum of primordial perturbations that deviate from observations [30].

The gravitational part of the Lagrangian of the Standard Model $\mathcal{L}_{SM}$ non-minimally coupled to gravity is

$$\mathcal{L}_{SM_{n-m}} = \mathcal{L}_{SM} + \frac{1}{2}\mathring{R} + \xi H^{\dagger} H \mathring{R} \, , \tag{4.1}$$

where the H is the Higgs doublet and ξ is a coupling parameter. The presence of the non-minimal coupling to gravity is not just needed for the Higgs inflation to work. It is also required by the renormalization procedure of the energy-momentum tensor in the case of a scalar field in a curved spacetime [21]. Furthermore, when the Standard Model and the Einstein–Hilbert action are combined, the non-minimal coupling term is the only dimension four operator that is allowed to be added to the action.

Taking into account only the radial mode h of the Higgs doublet, we can write the action for the non-minimally coupled Higgs field as

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \left(1 + \xi h^2\right) \mathring{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu h \, \partial_\nu h - V(h) \right] , \tag{4.2}$$

where the tree-level Higgs potential $V(h) = \frac{\lambda}{4}(h^2 - v^2)^2$, with $\lambda \approx 0.129$ being the Higgs self-coupling parameter at the electroweak scale and $v \approx 246\,\mathrm{GeV}$ the expectation value of the Higgs field [31].

4.1 Higgs inflation in the metric formalism

Higgs inflation relies on the known particle content of the Standard Model of particle physics, namely, the Higgs Boson. Therefore, Higgs inflation offers a way to form a link between the experimental physics conducted in particle colliders and the high-energy physics of the early universe. In addition, the presence of the non-minimal coupling between the Higgs field and the Ricci scalar creates the possibility of probing how the formulation of gravity affects Higgs inflation and its predictions.

Higgs inflation was first introduced in [30] in the metric formalism, where it was found to lead to successful inflation. In the metric formalism, the independent gravitational degree of freedom in the action (4.2) is the metric $g_{\alpha\beta}$ and its first derivatives, which are present in the Levi–Civita connection $\mathring{\Gamma}^{\gamma}_{\alpha\beta}$. The Ricci tensor involves second derivatives of the metric, which is a peculiar property of the Einstein–Hilbert action atypical for field theories. Therefore, the variation of the action in the metric formulation results in a situation where we have an ill-defined variational problem if the spacetime has a boundary. This can be dealt with by adding to the original action the Gibbons–Hawking–York term, which cancels the contribution coming from the boundary of the spacetime [32, 33].

Continuing to the analysis of the action (4.2) we perform the conformal transformation

$$\tilde{g}_{\alpha\beta} = \Omega^2 g_{\alpha\beta} \ , \tag{4.3}$$

with the conformal factor

$$\Omega^2 = \left(1 + \xi h^2\right) \ . \tag{4.4}$$

This removes the non-minimal coupling between the Higgs field and the Ricci scalar. In the metric formulation, the determinant transforms as

$$\sqrt{-g} = \Omega^{-4} \sqrt{-\tilde{g}} \tag{4.5}$$

and the Ricci scalar transforms as

$$\mathring{R} = \Omega^2 \tilde{\mathring{R}} + 6\Omega \tilde{g}^{\alpha\beta} \tilde{\nabla}_\alpha \tilde{\nabla}_\beta \Omega - 12 \tilde{g}^{\alpha\beta} \tilde{\nabla}_\alpha \Omega \tilde{\nabla}_\beta \Omega \tag{4.6}$$

The non-minimally coupled version of the action is usually called the Einstein frame, whereas the non-minimally coupled frame is called the Jordan frame. Noting that $\nabla_\alpha \Omega = \partial_\alpha \Omega$ and dropping a total derivative term, the Einstein frame action after the transformation is

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2} \tilde{\mathring{R}} - \frac{1}{2} \frac{1 + \xi h^2 + 6\xi^2 h^2}{(1 + \xi h^2)^2} \tilde{g}^{\mu\nu} \partial_\mu h \, \partial_\nu h - \Omega^{-4} V(h) \right] , \tag{4.7}$$

The conformal transformation has altered the kinetic term of the action. This can be brought back to canonical form with a field redefinition of the form

$$\frac{d\chi}{dh} = \frac{\sqrt{1 + \xi h^2 + 6\xi^2 h^2}}{1 + \xi h^2} , \tag{4.8}$$

and the action becomes

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2} \tilde{\mathring{R}} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) \right] , \tag{4.9}$$

where the potential

$$U(\chi) = \frac{V\left(h(\chi)\right)}{\Omega^4(h(\chi))} = \frac{V\left(h(\chi)\right)}{\left[1 + \xi h^2(\chi)\right]^2} . \tag{4.10}$$

In this form of the action, we can see that the relevant physics has been transferred to the new potential $U(\chi)$ and the standard cosmological perturbation theory and slow-roll approximation are applicable. The potential has been written in terms of the field χ and one has to solve it first from the field redefinition (4.8). An exact analytical solution is possible for χ, yet, for our purposes, we can just approximate the field redefinition in different regimes. For small field values $\chi \ll \frac{1}{\xi}$ we have in the field redefinition $\xi h^2 \ll 6\xi^2 h^2 \ll 1$ and

$$d\chi \simeq dh , \tag{4.11}$$

so in the small field limit

$$\chi \simeq h \tag{4.12}$$

and the potential has the same form in both the non-minimally and minimally coupled frames

$$U(\chi) \approx \frac{\lambda}{4} \left(\chi^2 - v^2\right)^2 . \tag{4.13}$$

In the case of large field values when $h \gg \frac{1}{\sqrt{\xi}}$, we have $\xi h^2 \gg 1$ and (4.8) becomes

$$d\chi \approx \frac{\sqrt{\xi h^2 + 6\xi^2 h^2}}{\xi h^2} dh = \frac{\sqrt{6}}{h} \sqrt{1 + \frac{1}{6\xi}} \, dh \approx \frac{\sqrt{6}}{h} dh . \tag{4.14}$$

So we get

$$\chi = \sqrt{6} \ln \frac{h}{h_0} , \tag{4.15}$$

where h_0 is an integration constant. Solving for h, we get

$$h = h_0 e^{\frac{\chi}{\sqrt{6}}} = \frac{1}{\sqrt{\xi}} e^{\frac{\chi}{\sqrt{6}}} \tag{4.16}$$

where, in the last equality, we set for convenience $h_0 = \frac{1}{\sqrt{\xi}}$. For large field values, we have $h \gg v$, and the potential in the minimally coupled frame becomes

$$U(\chi) \approx \frac{1}{(1+\xi h^2)^2} \frac{\lambda h^4}{4} = \frac{\lambda}{4\xi^2} \left(1 + e^{-\frac{2\chi}{\sqrt{6}}}\right)^{-2} \approx \frac{\lambda}{4\xi^2} \left(1 - 2e^{-\frac{2\chi}{\sqrt{6}}}\right). \tag{4.17}$$

Now we have everything for the calculation of the slow-roll parameters (3.15), (3.16) and (3.17) in terms of the field $h(\chi)$ and we get [30]

$$\epsilon_V = \frac{1}{2}\left(\frac{U'}{U}\right)^2 \approx \frac{4}{3\xi^2 h^4} \ , \tag{4.18}$$

$$\eta_V = \frac{U''}{U} \approx -\frac{4}{3\xi h^2} \tag{4.19}$$

and

$$\zeta_V = \frac{U'}{U}\frac{U'''}{U} \approx \frac{16}{9\xi^2 h^4} \ . \tag{4.20}$$

The prime denotes the derivatives with respect to the χ field $U' = dU/d\chi$. The number of e-folds can be calculated with (3.18)

$$N \approx \int_{\chi_{end}}^{\chi} \frac{U}{U'} d\chi = \frac{3}{4}\left(e^{\frac{2\chi}{\sqrt{6}}} - e^{\frac{2\chi_{end}}{\sqrt{6}}}\right) + \frac{\sqrt{6}}{4}(\chi - \chi_{end}) \approx \frac{3}{4} e^{\frac{2\chi}{\sqrt{6}}} \approx \frac{3\xi h^2}{4} \tag{4.21}$$

In terms of N, we have the slow-roll parameters

$$\epsilon \approx \frac{3}{4N^2}, \quad \eta \approx -\frac{1}{N}, \quad \zeta \approx \frac{1}{N^2} \ . \tag{4.22}$$

We also get the scalar and tensor perturbation spectra

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{U}{24\pi^2 \epsilon_V} \approx \frac{\lambda N^2}{72\pi^2 \xi^2} \ , \tag{4.23}$$

and

$$\mathcal{P}_{\mathcal{T}}(k) = \frac{2U}{3\pi^2} \ . \tag{4.24}$$

The tensor to scalar ratio is

$$r = \frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{R}}} = \frac{12}{N^2} \ . \tag{4.25}$$

In the lowest order of the slow-roll approximation, the scalar spectral index is now

$$n_s = 1 - 6\epsilon_V + 2\eta_V \approx 1 + 2\eta_V = 1 - \frac{2}{N} \tag{4.26}$$

and its running α_s

$$\alpha_s = -16\epsilon_V\eta_V + 24\epsilon_V + 2\zeta^2 \approx 2\zeta = -\frac{2}{N^2}\ . \tag{4.27}$$

We noted earlier that the observed value of the amplitude of the scalar perturbations is $\mathcal{P}_\mathcal{R} = (2.092 \pm 0.034) \times 10^{-9}$ [24] this restricts the value of the coupling constant ξ to be for $N = 50 - 60$ to be $\xi \approx 1.465 - 1.758 \times 10^4$. Now, for $N \approx 60$ we get the following values

$$n_s \approx 0.967\ , \quad r \approx 0.0033\ , \quad \alpha_s = -0.00056\ . \tag{4.28}$$

Comparison with the observed values $n_s = 0.9641 \pm 0.0044$, $r < 0.036$, and $\alpha_s = -0.0045 \pm 0.0067$ shows that they agree with the observations. However, the spectral index for Higgs inflation does not agree with the latest combined results from the ACT telescope and DESI, which is valued at $n_s = 0.974 \pm 0.003$.

Even though Higgs inflation in the metric formalism is a simple and conservative approach to the problem of inflation, it has a problem with perturbative unitarity. This problem arises because the large non-minimal coupling of the Higgs field to gravity leads to a breakdown of perturbative unitarity at energies far below the Planck scale. More precisely, the ultraviolet cutoff of the Higgs interactions and the Hubble rate are of the same magnitude, and this makes the whole inflationary evolution dependent of the unknown UV completion of the Higgs sector.

One can circumvent this problem by considering the Palatini formalism of Einstein's general relativity. In the Palatini formalism the UV cutoff scale is pushed signicantly higher thus eradicating the problem [34].

4.2 Higgs inflation in the Palatini formalism

The fact that we have the non-minimal coupling between the Higgs and the Ricci scalar makes it possible for the action (4.2) behaves differently compared to the metric formalism when we consider different variational principles which use different independent variables to describe gravity. This is the case in the Palatini formalism, also known as first order formalism or the metric affine gravity. In the Palatini formalism, we consider the independent variables to be the metric $g_{\alpha\beta}$ and the connection $\Gamma^\gamma_{\alpha\beta}$, and therefore only first derivatives are present in the action; hence the name first order formalism. Furthermore, there is no need for the Gibbons–Hawking–York-boundary term for the action. In the case of the standard form of the action, meaning the Einstein–Hilbert form with a symmetric connection or metric compatibility with a matter part that does not depend on the connection:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,R(\Gamma,\partial\Gamma) + S_{matter}\ , \tag{4.29}$$

the variation leads to an equation of motion that forces the connection to be the Levi–Civita connection. Usually, we assume that the connection is torsionless: $\Gamma^{\gamma}_{\alpha\beta} = \Gamma^{\gamma}_{\beta\alpha}$; however, one can use an arbitrary connection in the Palatini variation principle. This, however, leads to an underdetermined set of equations for the connection [35].

In the case of Higgs inflation in the Palatini formulation, we again consider the action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(1+\xi h^2\right)R - \frac{1}{2}g^{\mu\nu}\partial_\mu h\,\partial_\nu h - V(h)\right]\ , \tag{4.30}$$

but we take the metric and the connection as independent variables. The analysis is now simpler since the Ricci scalar does not depend on the metric. We can bring the action (4.30) to minimally coupled form by considering again the same conformal transformation as before, leading to an action

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{g}^{\mu\nu}R_{\mu\nu} - \frac{1}{2}\frac{1}{(1+\xi h^2)}\tilde{g}^{\mu\nu}\partial_\mu h\,\partial_\nu h - \frac{1}{(1+\xi h^2)^2}V(h)\right]\ , \tag{4.31}$$

To bring the action (4.30) to canonical form, it suffices to consider the field redefinition

$$d\chi = \frac{1}{\sqrt{1+\xi h^2}}\,dh \tag{4.32}$$

to get

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{g}^{\mu\nu}R_{\mu\nu} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu h\,\partial_\nu h - U(\chi)\right]\ , \tag{4.33}$$

where the potential $U(\chi)$ again is

$$U(\chi) = \frac{V\left(h(\chi)\right)}{\Omega^4(h(\chi))} = \frac{V\left(h(\chi)\right)}{\left[1+\xi h^2(\chi)\right]^2}\ . \tag{4.34}$$

From (4.32) we get

$$h = \frac{1}{\sqrt{\xi}}\sinh\left(\sqrt{\xi}\chi\right) \tag{4.35}$$

and the potential is

$$U(\chi) = \frac{1}{4}\frac{\lambda}{(1+\xi h^2)^2}\left[\frac{1}{\xi}\sinh^2\left(\sqrt{\xi}\chi\right) - v^2\right]^2\ . \tag{4.36}$$

Considering again the small field values when $h \ll 1\sqrt{\xi}$ the field $\chi \approx h$ and the potential $U \approx V$. In effect, there is no distinction between the results of the two frames. When the field values are large $\chi \gg \frac{1}{\sqrt{\xi}}$, we have

$$U(\chi) = \frac{1}{4}\frac{\lambda}{\xi^2}\frac{\sinh^4(\sqrt{\xi}\chi)}{\left(1+\sinh^2(\sqrt{\xi}\chi)\right)^2} \approx \frac{1}{4}\frac{\lambda}{\xi^2}\frac{1}{\left(1+8e^{-2\sqrt{\xi}h}+16e^{-4\sqrt{\xi}h}\right)} \ . \tag{4.37}$$

For the slow-roll parameters, we have now

$$\epsilon_V \approx \frac{8}{\xi h^4}, \quad \eta_V \approx -\frac{8}{h^2}, \quad \zeta \approx \frac{64}{h^4} \tag{4.38}$$

Continuing with the number of e-folds

$$N \approx \int_{\chi_{end}}^{\chi} \frac{U}{U'} d\chi \approx \frac{h^2}{8} \tag{4.39}$$

and furthermore

$$\epsilon_V \approx \frac{1}{8\xi N^2}, \quad \eta_V \approx -\frac{1}{N}, \quad \zeta \approx \frac{1}{N^2} \ . \tag{4.40}$$

We also obtain the scalar and tensor perturbation spectra

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{U}{24\pi^2\epsilon_V} \approx \frac{\lambda N^2}{12\pi^2\xi} \ , \tag{4.41}$$

and

$$\mathcal{P}_{\mathcal{T}}(k) = \frac{U}{24\pi^2\epsilon_V} \approx \frac{\lambda N^2}{72\pi^2\xi^2} \ . \tag{4.42}$$

In the lowest order of the slow-roll approximation, the scalar spectral index is now

$$n_S = 1 - 6\epsilon_V + 2\eta_V \approx 1 + 2\eta_V = 1 - \frac{2}{N} \tag{4.43}$$

and its running α_s

$$\alpha_s = -16\epsilon_V\eta_V + 24\epsilon_V + 2\zeta^2 \approx 2\zeta = -\frac{2}{N^2} \ . \tag{4.44}$$

Now, the value of the coupling constant ξ for $N = 50-60$ is restricted by the observed $\mathcal{P}_{\mathcal{R}}$ to be $\xi \approx 1.3 - 1.9 \times 10^9$. For $N \approx 60$ in the Palatini formulation, we obtain the values

$$n_s \approx 0.967, \quad r \approx 3 \times 10^{-13}, \quad \alpha_s = -0.00056 \ . \tag{4.45}$$

For the Palatini case, the required value for the coupling ξ must be larger than that in the metric case. From this, it follows that in the Palatini formulation, the value of r becomes very small compared to the metric case and undetectable even for future experiments such as CMD-HD, CMB-S4 and LiteBIRD [36–38].

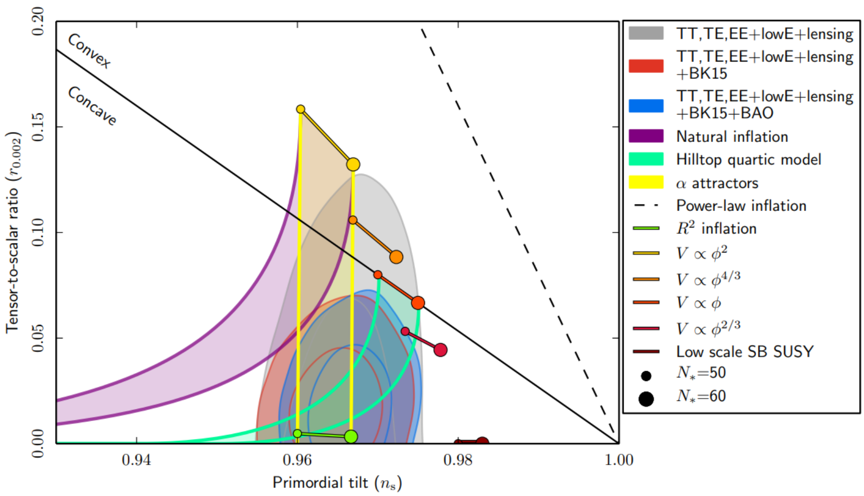

FIGURE 4.1 Here the predictions for n_s and r to first order in the slow-roll approximation for a few inflationary models are presented. Notice that the pivot scale here is $k_* = 0.002\,\text{Mpc}^{-1}$. The metric Higgs inflation case gives the same results as the R^2 inflation. Picture taken from [24]

Chapter 5

Higgs inflation with the Holst and the Nieh–Yan term

This chapter is based on the work the author did in [39]. In addition to considering Higgs inflation with different gravitational degrees of freedom, we can also consider gravitational actions that are more general than the Einstein–Hilbert action. Higgs inflation has been investigated, e.g., with higher order curvature terms that are mainly motivated by effective field theory considerations [40] considering that the Einstein–Hilbert action is a low energy approximation of the full action composed of higher order terms in curvature that become relevant at high energies. These include, e.g., the Ricci scalar squared [41, 42] and the Gauss–Bonnett term [43]. In addition, the non-minimal coupling of the Higgs field can elevate terms that are originally topological in nature, such as the Chern–Simons term or the Nieh–Yan and the Holst terms [44, 45], to dynamical ones.

5.1 Prerequisites: Curvature, non-metricity and torsion

In order to proceed, we introduce some ingredients necessary for the Nieh–Yan and the Holst term in the case of Higgs inflation. We introduce two new quantities: non-metricity and torsion. Along with curvature, non-metricity and torsion fully describe the geometry of spacetime. Non-vanishing torsion is essential for the introduction of the Nieh–Yan and Holst term since otherwise both terms will be zero at the classical level.

As in the Palatini variational principle, the metric $g_{\alpha\beta}$ and the connection $\Gamma^{\gamma}_{\alpha\beta}$ are our independent degrees of freedom. We decompose the general affine connection as

$$\Gamma^{\gamma}_{\alpha\beta} = \mathring{\Gamma}^{\gamma}_{\alpha\beta} + J^{\gamma}{}_{\alpha\beta} + K^{\gamma}{}_{\alpha\beta} \ , \tag{5.1}$$

where $J_{\alpha\beta\gamma}$ is known as disformation and $K_{\alpha\beta\gamma}$ as the contortion defined as

$$J_{\alpha\beta\gamma} \equiv \frac{1}{2}\left(Q_{\alpha\beta\gamma} - Q_{\gamma\alpha\beta} - Q_{\beta\alpha\gamma}\right) \tag{5.2}$$

and

$$K_{\alpha\beta\gamma} \equiv \frac{1}{2}(T_{\alpha\beta\gamma} + T_{\gamma\alpha\beta} + T_{\beta\alpha\gamma}) \ , \tag{5.3}$$

where $Q_{\alpha\beta\gamma}$ is the non-metricity measuring the failure of the connection to preserve the metric under parallel transport, resulting in a change of a vector norm and $T_{\alpha\beta\gamma}$ torsion, describing the failure of parallelograms to close when a vector is parallel transported around an infinitesimal loop. They are defined as follows:

$$Q_{\gamma\alpha\beta} \equiv \nabla_\gamma g_{\alpha\beta} \ , \tag{5.4}$$

and

$$T^{\gamma}{}_{\alpha\beta} \equiv 2\Gamma^{\gamma}_{[\alpha\beta]} \ . \tag{5.5}$$

In the general case, where we allow non-metricity and torsion to be present, the Ricci scalar can be written as

$$R = \mathring{R} + Q + T + \mathring{\nabla}_\alpha(Q^\alpha - \hat{Q}^\alpha + 2T^\alpha) - T_\alpha(Q^\alpha - \hat{Q}^\alpha) + Q_{\alpha\beta\gamma}T^{\gamma\alpha\beta} \ . \tag{5.6}$$

Here the two non-metricity vectors are defined as

$$Q^\gamma \equiv g_{\alpha\beta}Q^{\gamma\alpha\beta} \ , \qquad \hat{Q}^\beta \equiv g_{\alpha\gamma}Q^{\alpha\beta\gamma} \ , \tag{5.7}$$

and the torsion vector and torsion axial vector as

$$T^\beta \equiv g_{\alpha\gamma}T^{\alpha\beta\gamma} \ , \qquad \hat{T}^\alpha \equiv \frac{1}{6}\epsilon^{\alpha\beta\gamma\delta}T_{\beta\gamma\delta} \ . \tag{5.8}$$

The non-metricity scalar is

$$Q \equiv \frac{1}{4}Q_{\alpha\beta\gamma}Q^{\alpha\beta\gamma} - \frac{1}{2}Q_{\alpha\beta\gamma}Q^{\gamma\alpha\beta} - \frac{1}{4}Q_\alpha Q^\alpha + \frac{1}{2}Q_\alpha\hat{Q}^\alpha \tag{5.9}$$

and the torsion scalar

$$T \equiv \frac{1}{4}T_{\alpha\beta\gamma}T^{\alpha\beta\gamma} - \frac{1}{2}T_{\alpha\beta\gamma}T^{\gamma\alpha\beta} - T_\alpha T^\alpha \ . \tag{5.10}$$

Besides the Ricci scalar, one can build another scalar (which is linear in the Riemann tensor and quadratic in the connection) from the Riemann tensor. This is the so-called Holst term which we denote here by

$$\hat{R} \equiv \frac{1}{2}g_{\alpha\mu}\epsilon^{\mu\beta\gamma\delta}R^{\alpha}{}_{\beta\gamma\delta} = -3\mathring{\nabla}_\alpha\hat{T}^\alpha + \frac{1}{4}\epsilon^{\alpha\beta\gamma\delta}T_{\mu\alpha\beta}T^{\mu}{}_{\gamma\delta} + \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}Q_{\alpha\beta\mu}T^{\mu}{}_{\gamma\delta} \ , \tag{5.11}$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is the Levi–Civita tensor.

The Nieh–Yan term is a total derivative and, as such, it does not affect the classical equations of motion. It can be written as

$$\hat{N} = \mathring{\nabla}_\alpha \hat{T}^\alpha = \frac{1}{6}\epsilon^{\alpha\beta\gamma\delta}\mathring{\nabla}_\alpha T_{\beta\gamma\delta} \ . \tag{5.12}$$

5.2 Motivation from loop quantum gravity

Our motivation for considering the Nieh–Yan term and through it, the Holst term in the case of Higgs inflation comes from a quantum gravity approach called loop quantum gravity (LQG). In LQG, a background-independent reformulation of general relativity is constructed in a way that is written in the language of modern gauge theories; after this, the quantum theory is built in a non-perturbative background-independent way. The requirement for background independence and non-perturbative techniques leads to a theory where spacetime is discrete [46].

In LQG one has three different starting points: the traditional canonical approach with real or complex valued connection and the modern, covariant spin foam formalism [46]. The corresponding action for the different formulations is based on the Einstein–Hilbert action plus the Holst action. In the Holst action, the Holst term (5.11) is multiplied by the inverse of the so-called Barbero–Immirzi parameter γ. The value of $\gamma = \pm i$ corresponds to whether one has the self-dual (+) or the anti-self-dual action. A real value is also possible [46].

Our approach in [39] relies on studying LQG with extra terms present in the classical action and adding a non-minimal coupling to the terms and determining whether there are cosmological implications of such alterations. The Holst term does not affect the classical equations of motion gained from the Einstein–Hilbert action if matter is minimally coupled and there is no torsion. With torsion sourced by, for example, a non-minimally coupled scalar field, the Holst term becomes dynamical. The Nieh–Yan term becomes dynamical when the term is non-minimally coupled. In contrast to the usual choice in LQG, we do not set the non-metricity to zero.

5.3 The action and the connection

The action consists of a non-minimal coupling of the scalar field h to the Ricci scalar, the Holst term, and the Nieh–Yan term:

$$\begin{aligned} S &= \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}F(h)R + \frac{1}{2}H(h)\hat{R} + \frac{3}{2}Y(h)\hat{N} - \frac{1}{2}K(h)g^{\alpha\beta}\partial_\alpha h\partial_\beta h - V(h)\right] \\ &= \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}FR + \frac{1}{2}H\hat{R} - \frac{3}{2}\hat{T}^\alpha\partial_\alpha Y - \frac{1}{2}Kg^{\alpha\beta}\partial_\alpha h\partial_\beta h - V(h)\right] \ . \end{aligned} \tag{5.13}$$

We have denoted the coupling functions by $F(h)$, $H(h)$, $Y(h)$ and $K(h)$. In the second line, we did partial integration on the Nieh–Yan term. The standard LQG action can be set up with taking the following coupling functions $F = 1$, $H = 1/\gamma$, $Y = 0$, or alternatively $F = 1$, $H = 0$, $Y = 1/\gamma$.

Variation with respect to the connection $\Gamma^{\gamma}_{\alpha\beta}$ gives the equations of motion:

$$\begin{aligned} &-FQ_{\gamma\alpha\beta} + F\hat{Q}_{\beta}g_{\alpha\gamma} - H\epsilon_{\alpha\beta}{}^{\mu\nu}Q_{\mu\nu\gamma} + Fg_{\alpha[\beta}(Q_{\gamma]} + 2T_{\gamma]}) + FT_{\alpha\beta\gamma} \\ &+H\epsilon_{\alpha\beta\gamma}{}^{\mu}T_{\mu} + \frac{1}{2}H\epsilon_{\alpha\beta}{}^{\mu\nu}T_{\gamma\mu\nu} = -2g_{\alpha[\beta}\partial_{\gamma]}F - \epsilon_{\alpha\beta\gamma}{}^{\mu}\partial_{\mu}(H - Y) \; . \end{aligned} \tag{5.14}$$

This is an algebraic equation and its general solution can be written in the form

$$\begin{aligned} Q_{\gamma\alpha\beta} &= q_1(h)g_{\alpha\beta}\partial_{\gamma}h + 2q_2(h)g_{\gamma(\alpha}\partial_{\beta)}h \\ T_{\alpha\beta\gamma} &= 2t_1(h)g_{\alpha[\beta}\partial_{\gamma]}h + t_2(h)\epsilon_{\alpha\beta\gamma}{}^{\mu}\partial_{\mu}h \; . \end{aligned} \tag{5.15}$$

Substituting this into the equation (5.14) we get

$$\begin{aligned} q_2 &= 0 \\ 2t_1 - q_1 &= \frac{FF' + H(H' - Y')}{F^2 + H^2} \\ t_2 &= \frac{HF' - F(H' - Y')}{F^2 + H^2} \; , \end{aligned} \tag{5.16}$$

where we have denoted the derivative with respect to the scalar field h by a prime. The functions q_1 and t_1 cannot be determined separately based on the equations of motions. We get only the combination $2t_1 - q_1$. This is because the action (5.13) is invariant under a projective transformation, where $\Gamma^{\gamma}_{\alpha\beta} \to \Gamma^{\gamma}_{\alpha\beta} + \delta^{\gamma}{}_{\beta}A_{\alpha}$ with A_{α} being an arbitrary vector [47]. This means that we can fix the projective symmetry and set $q_1 = 0$ without loss of generality and then the non-metricity $Q_{\gamma\alpha\beta}$ vanishes[1].

We can again simplify the action (5.13) by a conformal transformation that removes the non-minimal coupling between the Ricci scalar and the function F. This is achieved with a conformal factor $\Omega = F$. This leads to an action of the form

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}\frac{H(h)}{F(h)}\hat{R} - \frac{3}{2}\hat{T}^{\alpha}\frac{\partial_{\alpha}Y(h)}{F(h)} - \frac{1}{2}\frac{K(h)}{F(h)}g^{\alpha\beta}\partial_{\alpha}h\partial_{\beta}h - U(h)\right] \; , \tag{5.17}$$

with $U \equiv \frac{V}{F^2}$. Variation of the action and substituting the connection back in the action leads to [39]

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left[\frac{1}{2}\mathring{R} - \frac{1}{2}\left\{\frac{K}{F} + \frac{3}{2}\frac{[HF' - F(H' - Y')]^2}{F^2(F^2 + H^2)}\right\}g^{\alpha\beta}\partial_{\alpha}h\partial_{\beta}h - U\right] \\ &\equiv \int d^4x\sqrt{-g}\left[\frac{1}{2}\mathring{R} - \frac{1}{2}\tilde{K}(h)g^{\alpha\beta}\partial_{\alpha}h\partial_{\beta}h - U(h)\right] \; . \end{aligned} \tag{5.18}$$

[1] This is actually the convention in LQG, where the tetrad formulation of general relativity is used and the covariant derivative of the tetrad is assumed to vanish, which means that the non-metricity is zero.

The kinetic function in (5.18) is defined as

$$\tilde{K} = \frac{K}{F} + \frac{3}{2}\frac{\left[HF' - F(H' - Y')\right]^2}{F^2(F^2 + H^2)} \ . \tag{5.19}$$

The form of the coupling functions K, F and H for Higgs inflation are given in the next section. In the action, the Levi-Civita connection $\mathring{R}$ is only present in the gravitational sector, so the effect of torsion has been shifted to the non-canonical kinetic term and the potential. From this action without the Holst and Nieh–Yan term we can recover the Palatini case setting

$$Y' = F(H/F)' \tag{5.20}$$

and the metric case with

$$F' = (Y' - H')(H/F \pm \sqrt{(H/F)^2 + 1}) \ . \tag{5.21}$$

5.4 Higgs inflation in different scenarios

In Higgs inflation we have the following coupling functions considering only terms up to dimension four

$$K = K_0 \ , \quad F = F_0(1 + \xi h^2) \ , \quad H = F_0(H_0 + H_1 h^2) \ , \quad Y = F_0 Y_1 h^2 \ , \tag{5.22}$$

where K_0, F_0, ξ, H_0, H_1 and Y_1 are constants. In LQG with the Holst term, $H_0 = 1/\gamma$. With these the kinetic function $\tilde{K}$ is

$$\begin{aligned} \tilde{K} &= \frac{K}{F} + \frac{3}{2}\frac{[HF' - F(H' - Y')]^2}{F^2(F^2 + H^2)} = \frac{K_0}{F_0(1 + \xi h^2)} \\ &+ 6h^2 \frac{(Y_1 - H_1 + H_0\xi + Y_1\xi h^2)^2}{(1 + \xi h^2)^2[1 + H_0^2 + 2(H_0 H_1 + \xi)h^2 + (H_1^2 + \xi^2)h^4]} \ . \end{aligned} \tag{5.23}$$

We can obtain a canonical kinetic term through the field redefinition

$$\frac{\mathrm{d}\chi}{\mathrm{d}h} = \pm\sqrt{\tilde{K}} \tag{5.24}$$

and the potential at Higgs tree-level

$$U(\chi) = \frac{\lambda}{4F_0^2}\frac{[h(\chi)^2 - v^2]^2}{[1 + \xi h(\chi)^2]^2} \ , \tag{5.25}$$

where λ and v are constants.

5.4.1 Plateau inflation

At large field values $\xi h^2 \gg 1$ the potential becomes asymptotically flat and we have the values for slow-roll observables

$$\begin{aligned} A_s &= \frac{N^2}{12\pi^2}\frac{\lambda}{\xi + \frac{6\xi^2 Y_1^2}{H_1^2+\xi^2}} \\ n_s &= 1 - \frac{2}{N} - \frac{3r}{16} \\ r &= \frac{2}{N^2}\left(\frac{1}{\xi} + \frac{6Y_1^2}{H_1^2+\xi^2}\right) = \frac{\lambda}{6\pi^2 A_s \xi^2}\ . \end{aligned} \tag{5.26}$$

The value of ξ is determined by the observed amplitude of the scalar perturbations A_s only when we fix the value of the Higgs self-coupling λ. At the electroweak scale $\lambda = 0.13$. At the inflationary scale we consider λ to be a free positive parameter. In order to avoid strong coupling we have the limitation $\lambda < 0.1$. We do not consider running for λ. When the Holst and Nieh–Yan terms are zero, we get the same results as in Palatini Higgs inflation for A_s, n_s and r, as one would expect. However, if these are not zero, we see that the dependency of the slow-roll observables on the couplings makes it possible to adjust the values in different ways. It is possible to make the amplitude A_s small without a large coupling constant ξ, since we can instead make the coupling Y_1 with the Nieh–Yan term large.

The observational limit for r and the value of A_s set the value of ξ to be large in any case, unless the Higgs self coupling $\lambda \ll 1$.

In the case where we set the Holst term zero, we can have only plateau inflation. In this situation, we obtain the same results as with Higgs inflation in teleparallel gravity, which has been analyzed in [48]. Other scenarios with different terms present in the action are discussed in the next section.

5.4.2 First case: $Y = 0$

When the Holst term is present in the action but the Nieh–Yan term is not, we can have plateau inflation. However, the contribution from the Holst term is negligible if the coupling is not large. In LQG the value for the Barbero–Immirzi parameter from black hole entropy without chemical potential is $\gamma = 0.274$ [49]. An interesting case for LQG would be to consider taking this value for γ and use it for the coupling $H_0 = \frac{1}{\gamma}$ which results in $H_0 \approx 3.6$. This is too small to have any contribution to the observables. With a large Holst term coupling we can shift n_s down on the plateau.

Another inflationary scenario is also possible if the coupling H_0 is much larger than ξ and H_1. This leads to a situation where the Holst term can dominate the kinetic term in an intermediate regime. If H_0 is larger than other couplings and we have $|\xi|h^2 \gg 1$

we end up in the same result as in Higgs inflation in the metric formulation for plateau inflation. In contrast to the metric case, we can now observe this inflationary behaviour also for $\xi < 0$.

5.4.3 Second case: $\xi = 0$

When we do not have non-minimal coupling to the Ricci scalar, as in standard Higgs inflation, the potential is not asymptotically flat. In an intermediate regime, successful inflation is possible. This requires adjusting the parameters suitably. With $\xi = 0$, the kinetic function (5.23) simplifies to

$$\begin{aligned} \tilde{K} &= 1 + \frac{3}{2}\frac{(H' - Y')^2}{1 + H^2} \\ &= 1 + 6h^2 \frac{(Y_1 - H_1)^2}{1 + (H_0 + H_1 h^2)^2} \ . \end{aligned} \tag{5.27}$$

Successful inflation requires that the second term dominate. We find that this results in a certain limit to

$$\begin{aligned} A_s &\simeq \frac{\lambda \Delta^2 H_0^2}{72\pi^2 H_1^2} N^2 \\ n_s &\simeq 1 - \frac{2}{N} - \frac{r}{4} \\ r &\simeq \frac{12}{\Delta^2 N^2} \simeq \frac{\lambda H_0^2}{6\pi^2 A_s H_1^2} \ . \end{aligned} \tag{5.28}$$

where $\Delta \equiv \left| \frac{H_1}{H_1 - Y_1} \right|$. It is now possible to tweak the tensor-to-scalar ratio up or down with the Holst coupling. Comparison to the plateau inflation case reveals that these are similar in form, with some replacements. Especially for n_s there is a difference in the third term, but it is in any case very small. If we have only the Nieh–Yan term $\xi = H_0 = H_1 = 0$, successful inflationary solutions do not exist since the tensor-to-scalar ratio becomes too large compared to observations.

5.4.4 Third case: Including all three terms

With coupling to the Ricci scalar, the Holst term, and Nieh–Yan term we can have a larger variety of inflationary scenarios. If we consider first the situation $\xi < 0$ we can have an α-attractor solution. However, when $\xi < 0$ we cannot have successful inflation in the case $H_0 = 0$. Another new case is that of an inflection point inflation. When $\xi > 0$ we can again have an inflection point inflation.

The analysis was done numerically over five-dimensional parameter space of the different couplings (h, ξ, H_0, H_1, Y_1) with an adaptive Monte Carlo method. For $\xi < 0$ we have the range $[-10^{10}, 0]$ for ξ, $[-10^{10}, 10^{10}]$ for H_0 and H_1, and $[0, 10^{10}]$ for h and Y_1. The Higgs quartic coupling was limited to the range $[10^{-5}, 10^{-1}]$. One should note that the parameter scan was done before the new observational limit for tensor-to-scalar ratio $r = 0.036$ [25] was given and the old result was $r = 0.067$.

The results are shown in figure 5.1 on the (n_s, r) plane. On the bottom, we have the case $\xi < 0$ and on the top $\xi > 0$. With the coloring we represent the minimal value of the non-minimal coupling $|\xi|$ to the Ricci scalar for which the observational constraints are met. With the solid black line, we have shown the analytical result for plateau inflation. The dashed line represents the results in the case $\xi = 0$. The result of the standard metric formulation is denoted by a star. The values of the tensor-to-scalar ratio can have values from the maximum observationally allowed value all the way down to around 10^{-12}. The values of the spectral index n_s cover the entire current observational range.

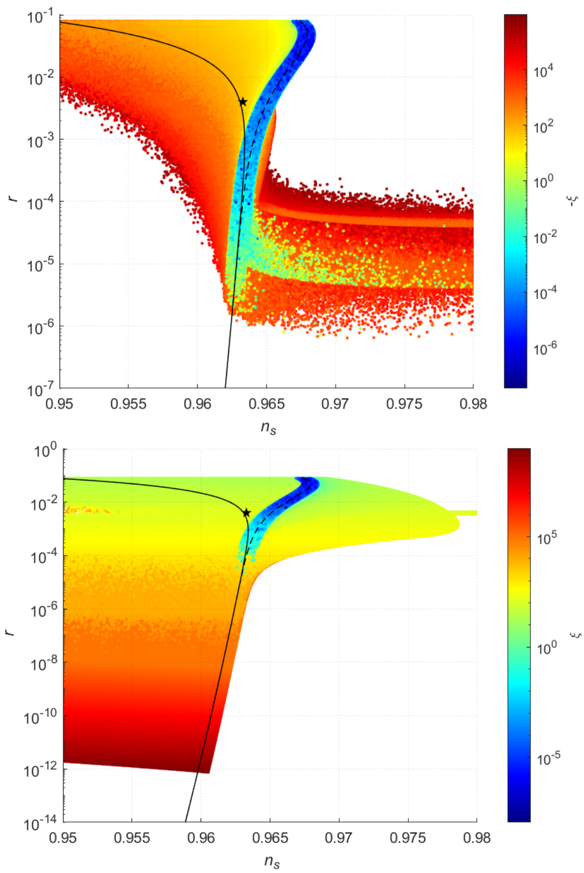

Figure 5.1 Here the spectral index n_s and tensor-to-scalar ratio r are presented for $\xi < 0$ (top) and $\xi > 0$ (bottom). Points in the plane correspond to an successful inflationary model that fulfils all observational CMB constraints. An exception to this is n_s which is allowed to have a wider range of values. With the coloring we represent the minimal value of the non-minimal coupling $|\xi|$ to the Ricci scalar for which the observational constraints are met. The solid line represents the prediction of plateau inflation, and the dashed line has $\xi = 0$. With the star we have shown where the results of standard metric formulation reside.

Chapter 6

Conclusions

In this thesis, our focus has been on Higgs inflation and the non-minimal coupling to the Holst and Nieh–Yan terms with the non-minimal coupling to the Ricci scalar. The core appeal of Higgs inflation lies in its remarkable simplicity: it proposes that the Standard Model Higgs boson, a particle found by the LHC, serves as the inflaton, without relying on new, unobserved particle content. Furthermore, the inherent feature of the non-minimal coupling to gravity makes Higgs inflation a rich inflationary model with a large range of different predictions for observables. Our work continues the mapping of the applicability of Higgs inflation that has been done with different formulations of gravity or with extra terms in the action.

Our main motivation was that, on a classical level, the Holst action works as a starting point for loop quantum gravity. Furthermore, the Holst and the Nieh–Yan terms are dynamical if torsion is non-zero, and the predictions would likely differ from the standard metric or Palatini Higgs inflation case.

We found several cases where we have successful inflation. In one case, our results, derived from a gravitational action where the connection and metric are independent degrees of freedom (as in the Palatini formulation), yield results comparable to those obtained in the metric formulation when incorporating the Holst term. This suggests that the Palatini formulation cannot be definitively excluded, even if the metric formulation achieves observational validation. We can also tune the different couplings in order to widen the possible values of predictions to match future observational limits.

The link between the Higgs field and these additional gravitational terms provides a new way to view LQG cosmology. The non-minimal couplings of the Holst and Nieh-Yan terms could similarly bring quantum gravitational effects, characteristic of LQG, down to the energy scales relevant for inflation and possibly open a possibility to probe these effects via inflationary observations. However, this requires that the couplings to the Holst and Nieh–Yan terms are considerably large.

Higgs inflation remains a highly promising and simple inflationary model. The detailed phenomenology is linked to the underlying gravitational action and the presence of non-minimal couplings. This also makes it a very appealing research topic since there are still different gravitational setups that could be considered in the case of Higgs inflation. The situation for Higgs inflation is the same as for many other inflation models: It is dependent on high precision observations, and it is difficult to rule it out since the observational limits of future experiments can be met by adjusting the model. However, the conservative approach of Higgs inflation is highly appealing when it is compared to more exotic inflationary models.

References

[1] E. Cartan, "Sur une généralisation de la notion de courbure de Riemann et les espaces 'a torsion," C.R. Hebd. Seances Acad. Sci. **174**, 593 (1922).

[2] A. Einstein, "Einheitliche feldtheorie von gravitation und elektrizität," Verlag der Koeniglich-Preussichen Akademie der Wissenschaften **22**, 414 (1925).

[3] J. M. Nester and H.-J. Yo, "Symmetric teleparallel general relativity," Chin. J. Phys. **37**, 113 (1999), `gr-qc/9809049`.

[4] A. Einstein, "Die grundlage der allgemeinen relativitätstheorie," Annalen der Physik **354**, 769 (1916), `https://onlinelibrary.wiley.com/doi/pdf/10.1002/andp.19163540702`.

[5] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, 1973), ISBN 978-0-7167-0344-0, 978-0-691-17779-3.

[6] S. M. Carroll, *Spacetime and Geometry: An Introduction to General Relativity* (Cambridge University Press, 2019), ISBN 978-0-8053-8732-2, 978-1-108-48839-6, 978-1-108-77555-7.

[7] G. F. R. Ellis, R. Maartens, and M. A. H. MacCallum, *Relativistic Cosmology* (Cambridge University Press, 2012).

[8] A. A. Penzias and R. W. Wilson, "A Measurement of Excess Antenna Temperature at 4080 Mc/s.," apj **142**, 419 (1965).

[9] R. H. Dicke, P. J. E. Peebles, P. G. Roll, and D. T. Wilkinson, "Cosmic Black-Body Radiation.," apj **142**, 414 (1965).

[10] R. A. Alpher, H. Bethe, and G. Gamow, "The origin of chemical elements," Phys. Rev. **73**, 803 (1948).

[11] A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure* (Cambridge Univ. Press, Cambridge, 2000), URL `https://cds.cern.ch/record/452061`.

[12] N. Aghanim et al. (Planck), "Planck 2018 results. VI. Cosmological parameters," Astron. Astrophys. **641**, A6 (2020), [Erratum: Astron.Astrophys. 652, C4 (2021)], `1807.06209`.

[13] D. Kazanas, "Dynamics of the Universe and Spontaneous Symmetry Breaking," Astrophys. J. Lett. **241**, L59 (1980).

[14] A. H. Guth, "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," Phys. Rev. D **23**, 347 (1981).

[15] A. D. Linde, "A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems," Phys. Lett. B **108**, 389 (1982).

[16] K. Sato, "Cosmological Baryon Number Domain Structure and the First Order Phase Transition of a Vacuum," Phys. Lett. B **99**, 66 (1981).

[17] A. R. Liddle, P. Parsons, and J. D. Barrow, "Formalizing the slow roll approximation in inflation," Phys. Rev. D **50**, 7222 (1994), `astro-ph/9408015`.

[18] V. Mukhanov, *Physical Foundations of Cosmology* (Cambridge University Press, Oxford, 2005), ISBN 978-0-521-56398-7.

[19] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, "Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions," Phys. Rept. **215**, 203 (1992).

[20] M. Sasaki, "Large Scale Quantum Fluctuations in the Inflationary Universe," Prog. Theor. Phys. **76**, 1036 (1986).

[21] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge Monographs on Mathematical Physics (Cambridge Univ. Press, Cambridge, UK, 1984), ISBN 978-0-521-27858-4, 978-0-521-27858-4.

[22] C. Armendariz-Picon, "Why should primordial perturbations be in a vacuum state?," JCAP **02**, 031 (2007), `astro-ph/0612288`.

[23] D. Baumann, in *Theoretical Advanced Study Institute in Elementary Particle Physics: Physics of the Large and the Small* (2011), pp. 523–686, `0907.5424`.

[24] Y. Akrami et al. (Planck), "Planck 2018 results. X. Constraints on inflation," Astron. Astrophys. **641**, A10 (2020), 1807.06211.

[25] P. A. R. Ade et al. (BICEP, Keck), "Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season," Phys. Rev. Lett. **127**, 151301 (2021), 2110.00483.

[26] E. Calabrese et al. (ACT), "The Atacama Cosmology Telescope: DR6 Constraints on Extended Cosmological Models," (2025), 2503.14454.

[27] A. G. Adame et al. (DESI), "DESI 2024 VI: cosmological constraints from the measurements of baryon acoustic oscillations," JCAP **02**, 021 (2025), 2404.03002.

[28] J. Martin, C. Ringeval, R. Trotta, and V. Vennin, "The Best Inflationary Models After Planck," JCAP **03**, 039 (2014), 1312.3529.

[29] G. Aad et al. (ATLAS), "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC," Phys. Lett. B **716**, 1 (2012), 1207.7214.

[30] F. L. Bezrukov and M. Shaposhnikov, "The Standard Model Higgs boson as the inflaton," Phys. Lett. B **659**, 703 (2008), 0710.3755.

[31] H. Kurkisuonio, "Cosmological perturbation theory lecture notes i-ii," (2025), http:\www.mv.helsinki.fi/home/hkurkisu/cpt/.

[32] G. W. Gibbons and S. W. Hawking, "Action integrals and partition functions in quantum gravity," Phys. Rev. D **15**, 2752 (1977).

[33] J. W. York, Jr., "Role of conformal three geometry in the dynamics of gravitation," Phys. Rev. Lett. **28**, 1082 (1972).

[34] F. Bauer and D. A. Demir, "Higgs-Palatini Inflation and Unitarity," Phys. Lett. B **698**, 425 (2011), 1012.2900.

[35] A. N. Bernal, B. Janssen, A. Jimenez-Cano, J. A. Orejuela, M. Sanchez, and P. Sanchez-Moreno, "On the (non-)uniqueness of the Levi-Civita solution in the Einstein–Hilbert–Palatini formalism," Phys. Lett. B **768**, 280 (2017), 1606.08756.

[36] M. A. et al., in *Bulletin of the American Astronomical Society* (2019), vol. 51, p. 147, 1907.08284.

[37] k. Abazajian et al., in *Bulletin of the American Astronomical Society* (2019), vol. 51, p. 209, 1908.01062.

[38] H. S. et al., "Updated Design of the CMB Polarization Experiment Satellite LiteBIRD," Journal of Low Temperature Physics **199**, 1107 (2020), 2001.01724.

[39] M. Långvik, J.-M. Ojanperä, S. Raatikainen, and S. Rasanen, "Higgs inflation with the Holst and the Nieh–Yan term," Phys. Rev. D **103**, 083514 (2021), 2007.12595.

[40] S. Weinberg, "Effective Field Theory for Inflation," Phys. Rev. D **77**, 123541 (2008), 0804.4291.

[41] X. Calmet and I. Kuntz, "Higgs Starobinsky Inflation," Eur. Phys. J. C **76**, 289 (2016), 1605.02236.

[42] V.-M. Enckell, K. Enqvist, S. Rasanen, and L.-P. Wahlman, "Higgs-R^2 inflation - full slow-roll study at tree-level," JCAP **01**, 041 (2020), 1812.08754.

[43] S. Koh, S. C. Park, and G. Tumurtushaa, "Higgs inflation with a Gauss-Bonnet term," Phys. Rev. D **110**, 023523 (2024), 2308.00897.

[44] H. T. Nieh and M. L. Yan, "An Identity in Riemann-cartan Geometry," J. Math. Phys. **23**, 373 (1982).

[45] S. Holst, "Barbero's Hamiltonian derived from a generalized Hilbert-Palatini action," Phys. Rev. D **53**, 5966 (1996), gr-qc/9511026.

[46] A. Ashtekar and E. Bianchi, "A short review of loop quantum gravity," Rept. Prog. Phys. **84**, 042001 (2021), 2104.04394.

[47] F. W. Hehl and G. D. Kerlick, "Metric-affine variational principles in general relativity. I. Riemannian space-time," Gen. Rel. Grav. **9**, 691 (1978).

[48] S. Raatikainen and S. Rasanen, "Higgs inflation and teleparallel gravity," JCAP **12**, 021 (2019), 1910.03488.

[49] A. Ghosh and P. Mitra, "An Improved lower bound on black hole entropy in the quantum geometry approach," Phys. Lett. B **616**, 114 (2005), gr-qc/0411035.

Paper I

Higgs inflation with the Holst and the Nieh–Yan term

M. Långvik, Juha-Matti Ojanperä, S. Raatikainen and S. Räsanen

Phys. Rev. D **103** 8, 083514 (2021)

I

HIP-2020-22/TH

Higgs inflation with the Holst and the Nieh–Yan term

Miklos Långvik,[a] Juha-Matti Ojanperä,[b] Sami Raatikainen[b] and Syksy Räsänen[b,c]

[a]Åshöjdens grundskola,
Sturegatan 6, 00510 Helsingfors, Finland

[b]University of Helsinki, Department of Physics and Helsinki Institute of Physics,
P.O. Box 64, FIN-00014 University of Helsinki, Finland

[c]Birzeit University, Department of Physics
P.O. Box 14, Birzeit, West Bank, Palestine

E-mail: miklos.langvik@protonmail.com, juha-matti.ojanpera@helsinki.fi,
sami.raatikainen@helsinki.fi, syksy.rasanen@iki.fi

Abstract. The action of loop quantum gravity includes the Holst term and/or the Nieh–Yan term in addition to the Ricci scalar. These terms are expected to couple non-minimally to the Higgs. Thus the Holst and Nieh–Yan terms contribute to the classical equations of motion, and they can have a significant impact on inflation.

We derive inflationary predictions in the parameter space of the non-minimal couplings, including non-minimally coupled terms up to dimension 4. Successful inflation is possible even with zero or negative coupling of the Ricci scalar. Notably, inflation supported by the non-minimally coupled Holst term alone gives almost the same observables as the original metric formulation plateau Higgs inflation. A non-minimally coupled Nieh–Yan term alone cannot give successful inflation. When all three terms are considered, the predictions for the spectral index and tensor-to-scalar ratio span almost the whole range probed by upcoming experiments. This is not true for the running of the spectral index, and many cases are highly tuned.

Contents

1 Introduction

Formulations of general relativity and Higgs inflation. There are several formulations of general relativity, including the metric, the Palatini, the teleparallel and the symmetric teleparallel formulation, among others [1–19]. They are based on different assumptions about spacetime degrees of freedom, in particular about the relation between the metric (or the tetrad) and the connection. When gravity is described by the Einstein–Hilbert action and matter does not couple directly to the connection, these formulations are equivalent. However, for more complicated gravitational actions [20–36] or matter couplings [37–55], different formulations in general lead to different predictions.

Scalar fields couple directly to the connection via the Ricci scalar. Even if such a coupling is not included at tree level, it will be generated by quantum corrections [56]. Even if the coupling is put to zero on some scale, it runs and will thus be non-zero on other scales (although it can be negligible if the running is small). Thus, scalar fields break the equivalence between different formulations of general relativity. In particular, this is true for the Higgs field of the Standard Model of particle physics.

Inflation is the most successful scenario for the early universe, and it is typically driven by a scalar field [57–70]. The non-minimal coupling of the inflaton to gravity can leave an imprint on inflationary perturbations, so observations of the cosmic microwave background and large-scale structure may distinguish between different formulations of general relativity. If the Standard Model Higgs is the inflaton [71] (for reviews, see [72–74]; for an earlier similar model, see [75, 76]), it has (in the simplest cases) a large non-minimal coupling to the Ricci scalar, so different formulations can lead to large observational differences [40, 43–45, 48, 49, 51, 52, 77, 78]. Conclusions regarding perturbative unitarity, a key question on the particle physics side of Higgs inflation, can also be different [41, 77, 79–99]. In the metric formulation, the scale of tree-level unitarity violation is naively below the inflationary scale. In the Palatini formulation, the scale of tree-level unitarity violation is shifted up, and it is possible that inflation may take place below this scale [41, 77, 98, 99].

In addition to the non-minimal coupling of matter, a gravity sector more complicated than the Einstein–Hilbert action can lead to differences between formulations. One example is higher powers of the Ricci scalar, which have to be included because of loop corrections [36, 44, 96, 97, 100–117]. Extended gravitational actions can also be motivated by top-down considerations involving more fundamental theories, such as loop quantum gravity (LQG).

Loop quantum gravity. LQG is a candidate for a non-perturbative background-free theory of quantum gravity. Cosmology in LQG has often been studied in the loop quantum cosmology approach, which involves LQG-quantising[1] a symmetry-reduced model (such as a Friedmann–Lemaître–Robertson–Walker spacetime) and studying the difference to the ordinary "von Neumann representation"-compatible quantisation [118]. We instead consider the cosmological effects of new terms appearing in the LQG action to inflation at the classical level. (See also [119, 120] on inflationary gravitational waves in LQG.)

LQG comes in three main flavours: Hamiltonian form with the Ashtekar SL(2,C)-valued connection; Hamiltonian form with an SU(2)-valued connection; and covariant (or spin foam) form [121–124]. In the first case the Hamiltonian is complex, so reality conditions have to be imposed to obtain real-valued geometry, in the second case the variables are real. In the Hamiltonian forms the action consists of the Einstein–Hilbert action (also called the Palatini action as the connection is an independent variable) plus the Holst action. The Holst term is the contraction of the Riemann tensor with the Levi–Civita tensor, multiplied by a constant whose inverse is called the Barbero–Immirzi parameter γ [125]. As the Holst piece is of the same order in curvature as the Ricci scalar, it is not suppressed by an extra mass scale.

The choice $\gamma = \pm i$ gives the selfdual (or anti-selfdual) SL(2,C) action for LQG, for which all constraints are first class and can be solved [126, 127]. However, as the action is complex, reality conditions have to be imposed, and it is not clear how to handle them when quantising. If γ is real, we get the LQG action for the real-valued SU(2) connection, for which the Hamiltonian constraint is however complicated. In this case the spectrum of the area operator and the volume operator are discrete [128, 129], unlike in the selfdual case when they are continuous [130].

The Holst term is central for black hole entropy. If the Barbero–Immirzi parameter is real, there are two possibilities depending on whether or not there is a chemical potential in the statistical treatment of the black hole entropy. (This depends on the quantisation of the dynamics, which is an open problem.) With no chemical potential, the entropy is inversely proportional to the Barbero–Immirzi parameter, and the semiclassical value of Bekenstein and Hawking [131, 132] is reproduced for $\gamma \approx 0.274$ [133]. When a chemical potential is added, the correct semiclassical value can be obtained independent of the value of γ [134, 135]. Black hole entropy is also independent of the Barbero–Immirzi parameter in the complex selfdual case [136].

[1] By LQG-quantising we mean that the background-free quantisation techniques used in the full LQG theory are mimicked as closely as possible. For example, the holonomy of the connection (rather than the connection) is quantised, the size of plaquettes is not shrunk to zero (as in the usual Wilson loop quantisation) because the minimal area eigenvalue is non-zero, the kinematical space is inequivalent to the one of Wheeler–de Witt quantisation but mimics that of full LQG, and so on.

In the case of pure gravity with the Einstein–Hilbert plus the Holst term, the Holst term does not contribute to the equations of motion at the classical level. The theory thus has a quantisation ambiguity as there is a one-parameter family of quantum theories corresponding to the classical theory. Another term sometimes considered in LQG is the topological Nieh–Yan invariant [137–143]. Like the Holst term, it is dimension 2, and is hence not suppressed by a mass scale compared to the Ricci term. It is obvious that the Nieh–Yan term does not contribute at the classical level, as it is a total derivative. The case of the Holst term is more subtle. It vanishes when there is no torsion, and for minimally coupled matter, the equations of motion for the connection lead to the Levi–Civita connection, for which the torsion is zero[2].

When there is a source for torsion, the Holst term becomes dynamical. One case that has been studied in LQG is fermions whose kinetic term involves the spin connection [139, 140, 144–148]. Substituting the torsion generated by fermions back into the action leads to a four-fermion coupling that depends on the Barbero–Immirzi parameter, breaking the quantisation ambiguity.[3]

Another possibility that has been considered is uplifting the Barbero–Immirzi parameter to a scalar field [138, 142, 149–153], in which case a constant γ is a low-energy approximation for when the field sits at the minimum. This Barbero–Immirzi field will source torsion. Substituting the torsion back into the action generates a free scalar field, and a potential would need to be added by hand for inflation. The coefficient of the Nieh–Yan term has likewise been promoted to a scalar field [138, 141, 142, 154–156].

A third possibility that has been studied is that torsion is generated by the non-minimal coupling of a scalar field to the Ricci scalar [157, 158]. Such a coupling does not spoil the usual LQG quantisation procedure when γ is real [159]. It has been considered both in loop quantum cosmology [160–162] and from the perspective of black hole thermodynamics [163, 164]. Even if the Holst term is minimally coupled, the non-minimal coupling of the Ricci tensor will make it dynamical. If the non-minimally coupled field is the inflaton, the value of the Barbero–Immirzi parameter will be imprinted on the spectrum of perturbations produced during inflation.

We consider non-minimal coupling of a scalar field to the Ricci scalar, Holst term and Nieh–Yan term during inflation, with particular attention to the Higgs case. Unlike for fermions, where the observational signature is negligible because the four-fermion interaction is suppressed by the Planck scale, we find that the scalar-generated torsion can have a significant effect, completely changing the inflationary predictions.

In section 2 we present the formalism, give the action where the Ricci scalar, Holst term and Nieh–Yan term are non-minimally coupled to a scalar field, solve the equation of motion of the connection and substitute back into the action. The physics of the non-minimal coupling is thus shifted to the kinetic term and potential of the scalar field. In section 3 we discuss inflationary behaviour in the case of Higgs inflation, including non-minimally coupled terms up to dimension 4, and in section 4 we summarise our results. We use the metric and the connection as the gravitational degrees of freedom, as these are more familiar to cosmologists. In appendix A we present the calculation with tetrads, more familiar to people working on LQG.

[2]Assuming that non-metricity is zero, as usual in LQG.

[3]Fermions can also be coupled to the Levi–Civita spin connection, so that they do not enter the connection equation of motion. Another possibility is to choose a modified kinetic term such that the dependence on the Barbero–Immirzi parameter disappears after solving the equations of motion [140, 144, 145, 147].

2 Non-minimal coupling to Ricci, Holst and Nieh–Yan terms

2.1 Curvature, non-metricity and torsion

We take the metric $g_{\alpha\beta}$ and the connection $\Gamma^{\gamma}_{\alpha\beta}$ to be independent degrees of freedom. The connection, defined with the covariant derivative as $\nabla_\beta A^\alpha = \partial_\beta A^\alpha + \Gamma^{\alpha}_{\beta\gamma} A^\gamma$, can be decomposed as

$$\Gamma^{\gamma}_{\alpha\beta} = \mathring{\Gamma}^{\gamma}_{\alpha\beta} + L^{\gamma}{}_{\alpha\beta} = \mathring{\Gamma}^{\gamma}_{\alpha\beta} + J^{\gamma}{}_{\alpha\beta} + K^{\gamma}{}_{\alpha\beta} \ , \tag{2.1}$$

where $\mathring{\Gamma}^{\gamma}_{\alpha\beta}$ is the Levi–Civita connection defined by the metric $g_{\alpha\beta}$. As the difference of two connections, $L^{\gamma}{}_{\alpha\beta}$ is a tensor, known as the distortion. In the second equality we have decomposed it into the disformation $J_{\alpha\beta\gamma}$ and the contortion $K_{\alpha\beta\gamma}$, defined as

$$J_{\alpha\beta\gamma} \equiv \frac{1}{2}\left(Q_{\alpha\beta\gamma} - Q_{\gamma\alpha\beta} - Q_{\beta\alpha\gamma}\right) \ , \qquad K_{\alpha\beta\gamma} \equiv \frac{1}{2}(T_{\alpha\beta\gamma} + T_{\gamma\alpha\beta} + T_{\beta\alpha\gamma}) \ , \tag{2.2}$$

where $Q_{\alpha\beta\gamma}$ and $T_{\alpha\beta\gamma}$ are the non-metricity and the torsion, respectively, defined as

$$Q_{\gamma\alpha\beta} \equiv \nabla_\gamma g_{\alpha\beta} \ , \qquad T^{\gamma}{}_{\alpha\beta} \equiv 2\Gamma^{\gamma}_{[\alpha\beta]} \ . \tag{2.3}$$

Note that $Q_{\gamma\alpha\beta} = Q_{\gamma(\alpha\beta)}$, $\nabla_\gamma g^{\alpha\beta} = -Q_{\gamma}{}^{\alpha\beta}$, $J_{\alpha\beta\gamma} = J_{\alpha(\beta\gamma)}$ and $K^{\gamma}{}_{\alpha}{}^{\beta} = K^{[\gamma}{}_{\alpha}{}^{\beta]}$.

The two non-metricity vectors are defined as

$$Q^\gamma \equiv g_{\alpha\beta} Q^{\gamma\alpha\beta} \ , \qquad \hat{Q}^\beta \equiv g_{\alpha\gamma} Q^{\alpha\beta\gamma} \ , \tag{2.4}$$

and the torsion vector and torsion axial vector[4] are defined as, respectively,

$$T^\beta \equiv g_{\alpha\gamma} T^{\alpha\beta\gamma} \ , \qquad \hat{T}^\alpha \equiv \frac{1}{6}\epsilon^{\alpha\beta\gamma\delta} T_{\beta\gamma\delta} \ , \tag{2.5}$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is the Levi–Civita tensor. Note that $\nabla_\alpha\sqrt{-g} = \frac{1}{2}\sqrt{-g}Q_\alpha$.

The Riemann tensor can be decomposed into the Levi–Civita and the distortion contribution as

$$R^{\alpha}{}_{\beta\gamma\delta} = \mathring{R}^{\alpha}{}_{\beta\gamma\delta} + 2\mathring{\nabla}_{[\gamma} L^{\alpha}{}_{\delta]\beta} + 2L^{\alpha}{}_{[\gamma|\mu|} L^{\mu}{}_{\delta]\beta} \ , \tag{2.6}$$

where $\mathring{}$ denotes a quantity defined with the Levi–Civita connection. The curvature $R^{\alpha}{}_{\beta\gamma\delta}$, non-metricity $Q_{\alpha\beta\gamma}$ and torsion $T_{\alpha\beta\gamma}$ are the complete set of tensors that characterise the geometry of a manifold.

There are exactly two geometrical scalars that are linear in the Riemann tensor (2.6) and quadratic in the connection: the Ricci scalar and the Holst term. They are defined as, respectively,

$$\begin{aligned} R &\equiv \delta_\alpha{}^\gamma g^{\beta\delta} R^{\alpha}{}_{\beta\gamma\delta} = \mathring{R} + Q + T + \mathring{\nabla}_\alpha(Q^\alpha - \hat{Q}^\alpha + 2T^\alpha) - T_\alpha(Q^\alpha - \hat{Q}^\alpha) + Q_{\alpha\beta\gamma}T^{\gamma\alpha\beta} \\ \hat{R} &\equiv \frac{1}{2} g_{\alpha\mu}\epsilon^{\mu\beta\gamma\delta} R^{\alpha}{}_{\beta\gamma\delta} = -3\mathring{\nabla}_\alpha \hat{T}^\alpha + \frac{1}{4}\epsilon^{\alpha\beta\gamma\delta} T_{\mu\alpha\beta} T^{\mu}{}_{\gamma\delta} + \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta} Q_{\alpha\beta\mu} T^{\mu}{}_{\gamma\delta} \ , \end{aligned} \tag{2.7}$$

where we have used (2.2)–(2.6) to separate the contributions of curvature, non-metricity and torsion. The non-metricity scalar and the torsion scalar are defined as $Q \equiv \frac{1}{4}Q_{\alpha\beta\gamma}Q^{\alpha\beta\gamma} -$

[4] Despite the name, the parity transformation properties of the torsion vector and axial vector are not necessarily those of a vector and pseudovector. How they transform depends on the solution for the torsion tensor.

$\frac{1}{2}Q_{\alpha\beta\gamma}Q^{\gamma\alpha\beta} - \frac{1}{4}Q_\alpha Q^\alpha + \frac{1}{2}Q_\alpha \hat{Q}^\alpha$ and $T \equiv \frac{1}{4}T_{\alpha\beta\gamma}T^{\alpha\beta\gamma} - \frac{1}{2}T_{\alpha\beta\gamma}T^{\gamma\alpha\beta} - T_\alpha T^\alpha$, respectively. We also consider the Nieh–Yan term $\mathring{\nabla}_\alpha \hat{T}^\alpha$, which is equivalent to the Holst term plus a term quadratic in the torsion and a term involving non-metricity. In LQG non-metricity is usually taken to be zero a priori, so this term is absent. Although we are motivated by LQG, our approach is bottom-up, so we keep the non-metricity (although it will turn out it can be set to zero without loss of generality).

2.2 The action and the connection

We consider an action with the Ricci scalar, the Holst term and the Nieh–Yan term coupled to a scalar field h, which we will later identify with the Standard Model Higgs,

$$\begin{aligned} S &= \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}F(h)R + \frac{1}{2}H(h)\hat{R} + \frac{3}{2}Y(h)\mathring{\nabla}_\alpha\hat{T}^\alpha - \frac{1}{2}K(h)g^{\alpha\beta}\partial_\alpha h\partial_\beta h - V(h)\right] \\ &= \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}F(h)R + \frac{1}{2}H(h)\hat{R} - \frac{3}{2}\hat{T}^\alpha\partial_\alpha Y(h) - \frac{1}{2}K(h)g^{\alpha\beta}\partial_\alpha h\partial_\beta h - V(h)\right] , \end{aligned} \quad (2.8)$$

where on the second line we have discarded a boundary term. We neglect fermions. The coefficients have been chosen such that for $Y = H$, the $\mathring{\nabla}_\alpha\hat{T}^\alpha$ parts in the Holst term and the Nieh–Yan term cancel each other. Note the similarity of the Nieh–Yan term to the torsion vector coupling that appears in the teleparallel formulation [52]. The usual LQG case corresponds to $F = 1$, $H = 1/\gamma$, $Y = 0$, or alternatively $F = 1$, $H = 0$, $Y = 1/\gamma$, where γ is the Barbero–Immirzi parameter. (We choose units such that the Planck scale is unity.)

Varying (2.8) with respect to the connection $\Gamma^\gamma_{\alpha\beta}$, we get the equation of motion

$$\begin{aligned} &-FQ_{\gamma\alpha\beta} + F\hat{Q}_\beta g_{\alpha\gamma} - H\epsilon_{\alpha\beta}{}^{\mu\nu}Q_{\mu\nu\gamma} + Fg_{\alpha[\beta}(Q_{\gamma]} + 2T_{\gamma]}) \\ &+FT_{\alpha\beta\gamma} + H\epsilon_{\alpha\beta\gamma}{}^\mu T_\mu + \frac{1}{2}H\epsilon_{\alpha\beta}{}^{\mu\nu}T_{\gamma\mu\nu} = -2g_{\alpha[\beta}\partial_{\gamma]}F - \epsilon_{\alpha\beta\gamma}{}^\mu\partial_\mu(H - Y) . \end{aligned} \quad (2.9)$$

The general solution of (2.9) has the form

$$\begin{aligned} Q_{\gamma\alpha\beta} &= q_1(h)g_{\alpha\beta}\partial_\gamma h + 2q_2(h)g_{\gamma(\alpha}\partial_{\beta)}h \\ T_{\alpha\beta\gamma} &= 2t_1(h)g_{\alpha[\beta}\partial_{\gamma]}h + t_2(h)\epsilon_{\alpha\beta\gamma}{}^\mu\partial_\mu h . \end{aligned} \quad (2.10)$$

The definitions (2.4) and (2.5) give

$$\begin{aligned} Q_\alpha &= (4q_1 + 2q_2)\partial_\alpha h , \quad \hat{Q}_\alpha = (q_1 + 5q_2)\partial_\alpha h \\ T_\alpha &= -3t_1\partial_\alpha h , \qquad\quad \hat{T}_\alpha = t_2\partial_\alpha h . \end{aligned} \quad (2.11)$$

As non-metricity and torsion are only sourced by the scalar field, they can be written in terms of gradients of the scalar field, and reduce to the four vectors (2.11).

Inserting (2.10) into (2.9), we get

$$\begin{aligned} q_2 &= 0 \\ 2t_1 - q_1 &= \frac{FF' + H(H' - Y')}{F^2 + H^2} \\ t_2 &= \frac{HF' - F(H' - Y')}{F^2 + H^2} , \end{aligned} \quad (2.12)$$

where prime denotes derivative with respect to h. If $F = H$, the action has the extra symmetry of invariance under the duality transformation $R_{\alpha\beta\gamma\delta} \to \frac{1}{2}\epsilon_{\gamma\delta}{}^{\alpha\beta}R_{\alpha\beta\alpha\beta}$, which maps

$R \leftrightarrow \hat{R}$. Then the Holst term does not contribute to the equations of motion if $Y' = 0$, as is easily seen by a conformal transformation to the Einstein frame. If $Y' \neq 0$, the Holst term with $H = F$ simply effectively shifts $Y \to Y/2$.

The equations of motion do not fix q_1 and t_1 separately, only the combination $2t_1 - q_1$. This well-known feature is due to invariance of the action (2.8) under the projective transformation $\Gamma^{\gamma}_{\alpha\beta} \to \Gamma^{\gamma}_{\alpha\beta} + \delta^{\gamma}{}_{\beta} A_\alpha$, where A_α is an arbitrary vector [7]. The Riemann tensor transforms as $R_{\alpha\beta\gamma\delta} \to R_{\alpha\beta\gamma\delta} + g_{\alpha\beta}(2\nabla_{[\gamma}A_{\delta]} + T^{\mu}{}_{\gamma\delta}A_\mu)$, so the Ricci scalar is invariant due to the symmetry of its contraction, and the Holst term is invariant due to the antisymmetry of its contraction. The Nieh–Yan term is invariant because $\hat{T}^\alpha$ is invariant. If $A_\alpha = \partial_\alpha A$ for some scalar A, the non-metricity and the torsion transform as $q_1 \to q_1 - 2A$, $t_1 \to t_1 - A$, $q_2 \to q_2$, $t_2 \to t_2$.

The projective symmetry can be explicitly broken in the action [18, 49, 50, 54, 55]. Short of that, the projective invariance is often fixed in the Palatini formulation by assuming a priori that the connection is symmetric, $T_{\alpha\beta\gamma} = 0$. When the Holst term is included, (2.12) shows that this is not possible unless $F'/F = (H' - Y')/H$. (Note the similarity of this condition to the condition for the torsion vector coupling to vanish in the teleparallel formulation [52].) In the tetrad formulation used in LQG, it is instead commonly assumed that the covariant derivative of the tetrad is zero, which goes under the name tetrad postulate, meaning $Q_{\alpha\beta\gamma} = 0$. The scalar field kinetic term and potential are trivially invariant under the projective transformation as they do not depend on the connection. When deriving the equation of motion for the scalar field, the requirement that a total covariant derivative of a scalar field term reduces to a boundary term picks out the Levi–Civita connection, so the full connection does not appear in the scalar field equation of motion.

Following the LQG convention, we fix the projective symmetry by setting $q_1 = 0$, so non-metricity is zero. In fact, we could have put $Q_{\alpha\beta\gamma} = 0$ from the beginning, as the following reasoning shows. We can get rid of the non-minimal coupling F to R by a conformal transformation. As a conformal transformation (see (2.15) below) can only change q_2, not q_1, F cannot generate a q_2 term. And as we can perform a conformal transformation to cancel the source term involving the Holst or the Nieh–Yan term, they also cannot generate q_2. And q_1 can always be transformed into t_1 by the projective transformation. For a different action, setting $Q_{\alpha\beta\gamma} = 0$ may involve loss of generality [49].

2.3 Einstein frame action

The coupled equations of motion for the scalar field, metric and connection can be simplified by choosing suitable coordinates in field space. If we make the conformal transformation

$$g_{\alpha\beta} \to \Omega(h)^{-1} g_{\alpha\beta} \tag{2.13}$$

and absorb the changes in the functions of h in the action, they transform as

$$\begin{aligned} F &\to \Omega^{-1} F \\ H &\to \Omega^{-1} H \\ \partial_\alpha Y &\to \Omega^{-1} \partial_\alpha Y \\ K &\to \Omega^{-1} K \\ V &\to \Omega^{-2} V \ , \end{aligned} \tag{2.14}$$

and the non-metricity transforms as

$$Q_{\gamma\alpha\beta} = \nabla_\gamma g_{\alpha\beta} \to \Omega^{-1}(\nabla_\gamma g_{\alpha\beta} - g_{\alpha\beta}\partial_\gamma \ln \Omega) \ . \tag{2.15}$$

We choose field coordinates where the Ricci scalar is minimally coupled to the scalar field (i.e. the Einstein frame), which corresponds to $\Omega = F$. The action then reads

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{2} \frac{H(h)}{F(h)} \hat{R} - \frac{3}{2} \hat{T}^\alpha \frac{\partial_\alpha Y(h)}{F(h)} - \frac{1}{2} \frac{K(h)}{F(h)} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - U(h) \right] , \tag{2.16}$$

where we have denoted $U \equiv V/F^2$.

Inserting the connection (2.10) (with $F \to 1$, $H \to H/F$ and $Y' \to Y'/F$) back into the action, decomposing R and $\hat{R}$ into their Levi–Civita, non-metricity and torsion parts with (2.7), setting the non-metricity to zero and inserting the torsion (2.12), we get (dropping a boundary term)

$$S = \int \mathrm{d}^4x \sqrt{-g} \bigg[\frac{1}{2} \mathring{R} - \frac{1}{2} \left\{ \frac{K}{F} + 6t_1^2 - \frac{3}{2} t_2^2 - 3t_2 [(H/F)' - Y'/F - 2t_1 H/F] \right\} g^{\alpha\beta} \partial_\alpha h \partial_\beta h \\ - U \bigg] . \tag{2.17}$$

Inserting t_1 and t_2 from (2.12), we arrive at the simple expression

$$\begin{aligned} S &= \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} \mathring{R} - \frac{1}{2} \left\{ \frac{K}{F} + \frac{3}{2} \frac{F^2 + H^2}{F^2} t_2^2 \right\} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - U \right] \\ &= \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} \mathring{R} - \frac{1}{2} \left\{ \frac{K}{F} + \frac{3}{2} \frac{[(H/F)' - Y'/F]^2}{(H/F)^2 + 1} \right\} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - U \right] \\ &= \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} \mathring{R} - \frac{1}{2} \left\{ \frac{K}{F} + \frac{3}{2} \frac{[HF' - F(H' - Y')]^2}{F^2(F^2 + H^2)} \right\} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - U \right] \\ &\equiv \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} \mathring{R} - \frac{1}{2} \tilde{K}(h) g^{\alpha\beta} \partial_\alpha h \partial_\beta h - U(h) \right] . \end{aligned} \tag{2.18}$$

The geometrical contribution of the torsion has been shifted to the scalar kinetic term and the $1/F^2$ modification of the potential, and only the Levi–Civita connection appears. When we vary this action with respect to $g_{\alpha\beta}$ and h, we get equations of motion that are equivalent to those of the original action (2.8), which has a non-trivial gravity part (and hence connection). There is one subtle difference: varying the Einstein frame Levi–Civita action (2.18) leads to boundary terms that depend on the derivative of the variation of the metric. In order to derive the equations of motion, we need to include the York–Gibbons–Hawking boundary term [165, 166] to cancel this contribution. In the original action (2.8), there is no such problem, as the variation of the connection can be taken to vanish on the boundary independently of the metric. From the Palatini perspective, having to add a boundary term to the Einstein–Hilbert action is an artifact of solving part of the equations of motion and inserting the result back into the action. (We discarded boundary terms in the derivation.)

2.4 Recovering the metric and Palatini cases

The action (2.18) reduces to the well-known Palatini case with a non-minimal coupling only to the Ricci scalar [40] when $Y' = F(H/F)'$. Apart from the trivial case $H = Y = 0$, this also happens when $Y = 0$, $H = \alpha F$, where α is an arbitrary constant. If both the Holst and the Nieh–Yan term are non-zero, the condition means that their derivative parts cancel in the action, leaving only a quadratic torsion term.

The results of the metric formulation with a non-minimal coupling to the Ricci scalar are recovered when

$$F' = (Y' - H')(H/F \pm \sqrt{(H/F)^2 + 1}) \ . \tag{2.19}$$

A particularly simple case is $H = 0$, $Y = \pm F$, when there is no Holst term and the coupling functions of the Ricci term and the Nieh–Yan term are identical (possibly up to a sign). Another possibility is $Y = 0$, $F = \pm\alpha^{-1}\sqrt{1 + 2\alpha H}$, where α is an arbitrary non-zero constant.

3 Inflation

3.1 The coupling functions and the potential

Let us now discuss inflation with the Standard Model Higgs. A field-dependent kinetic term corresponds to a monotonic field-dependent remapping of the potential. Including only terms of up to dimension 4 in the action (2.8) and taking into account that only even powers of the field appear, we have (note that Y is defined only up to an additive constant)

$$K = K_0 \ , \quad F = F_0(1 + \xi h^2) \ , \quad H = F_0(H_0 + H_1 h^2) \ , \quad Y = F_0 Y_1 h^2 \ , \tag{3.1}$$

where K_0, F_0, ξ, H_0, H_1 and Y_1 are constants. In LQG with the Holst term, $H_0 = 1/\gamma$. Observational limits on these couplings are very weak, as they effectively only modify the Higgs potential for large field values. In the metric formulation, collider measurements give $|F_0\xi| < 2.6 \times 10^{15}$ [167]. In the Palatini case, non-inflationary constraints on the non-minimal couplings are likewise expected to be so high as not to affect our analysis.

The kinetic function defined in (2.18) is

$$\begin{aligned} \tilde{K} &= \frac{K}{F} + \frac{3}{2}\frac{[HF' - F(H' - Y')]^2}{F^2(F^2 + H^2)} \\ &= \frac{K_0}{F_0(1 + \xi h^2)} + 6h^2 \frac{(Y_1 - H_1 + H_0\xi + Y_1\xi h^2)^2}{(1 + \xi h^2)^2[1 + H_0^2 + 2(H_0 H_1 + \xi)h^2 + (H_1^2 + \xi^2)h^4]} \ . \end{aligned} \tag{3.2}$$

The kinetic function and thus the physics is invariant under the simultaneous sign change of H_0, H_1 and Y_1. In the small field limit $h \ll 1$ the second term falls off like h^2, and the canonically normalised field is $\chi = \sqrt{K_0/F_0}h$. In general, the transformation between h and the canonical field χ is

$$\frac{\mathrm{d}\chi}{\mathrm{d}h} = \pm\sqrt{\tilde{K}} \ . \tag{3.3}$$

We consider the Higgs tree-level potential, so

$$U(\chi) = \frac{\lambda}{4F_0^2}\frac{[h(\chi)^2 - v^2]^2}{[1 + \xi h(\chi)^2]^2} \ , \tag{3.4}$$

where λ and v are constants. The constants K_0 and F_0 effectively rescale the values of λ and v when we consider the potential in terms of the canonically normalised field [49], and we henceforth take $K_0 = F_0 = 1$. The quartic coupling λ has the value 0.13 at the electroweak scale, and without the non-minimal gravitational couplings it runs down with increasing field value. The running depends on the electroweak scale values of the Higgs mass, top quark mass, and QCD coupling constant. For the measured mean values, λ crosses zero around 10^{11}

GeV $\sim 10^{-7}$, in the case all when non-minimal couplings are zero [168–172]. The running is highly sensitive to the input electroweak scale values, and positivity of λ up to the Planck scale is within the 2σ limits [168–172]. The non-minimal couplings we consider can also change the renormalisation group running. We do not consider running, and take λ at the inflationary scale to be a free positive parameter, limited by $\lambda < 0.1$ to avoid strong coupling. If we used $\lambda < 0.01$ instead, the upper limit for ξ we find would decrease by one order of magnitude, which in the case $\xi > 0$ correspondingly brings the lower limit for r up by one order of magnitude.

The first slow-roll parameters are

$$\epsilon = \frac{1}{2}\left(\frac{U'}{U}\right)^2 , \qquad \eta = \frac{U''}{U} , \qquad \sigma_2 = \frac{U'}{U}\frac{U'''}{U} , \qquad \sigma_3 = \left(\frac{U'}{U}\right)^2 \frac{U''''}{U} , \tag{3.5}$$

where prime denotes derivative with respect to χ.

The amplitude, spectral index, running, running of the running of the scalar perturbations, and the tensor-to-scalar ratio are, respectively,

$$A_s = \frac{1}{24\pi^2}\frac{U}{\epsilon} = 2.099\, e^{\pm 0.014}\, 10^{-9} \tag{3.6}$$
$$n_s = 1 - 6\epsilon + 2\eta = 0.9625 \pm 0.0048 \tag{3.7}$$
$$\alpha_s = -24\epsilon^2 + 16\epsilon\eta - 2\sigma_2 = 0.002 \pm 0.010 \tag{3.8}$$
$$\beta_s = -192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 - 24\epsilon\sigma_2 + 2\eta\sigma_2 + 2\sigma_3 = 0.010 \pm 0.013 \tag{3.9}$$
$$r = 16\epsilon < 0.067 , \tag{3.10}$$

where the observational values with 68% C.L. limits are from Planck and BICEP2/Keck cosmic microwave background (CMB) data at the pivot scale 0.05 Mpc^{-1} [173]. The value for r assumes zero running of the running. The number of e-folds until the end of inflation is

$$N = \int_{\chi_{\rm end}}^{\chi} \frac{d\chi}{\sqrt{2\epsilon}} , \tag{3.11}$$

where $\chi_{\rm end}$ is the field value at the end of inflation (approximating that the field is in slow-roll until the end of inflation). The number of e-folds at the pivot scale is

$$N = 56 - \Delta N - \frac{1}{4}\ln\frac{0.067}{r} , \tag{3.12}$$

where ΔN accounts for the effect of reheating. Reheating is sensitive to the shape of the potential. With a non-minimal coupling only to the Ricci scalar, in the Palatini formulation with a tree-level potential the reheating is almost instant, $\Delta N \ll 1$ [51]. In the metric case it is not clear whether $\Delta N = 4$ or $\Delta N \ll 1$ [174–182]. We assume instant reheating.

3.2 Plateau inflation

Let us first consider inflation on the asymptotically flat plateau, which the potential has when $\xi > 0$. When the Holst and the Nieh–Yan term are zero, this is the only inflationary regime. With either or both non-zero, plateau inflation remains qualitatively the same, and the first slow-roll observables in terms of the number of e-folds are (see e.g. [43] for details)

$$\begin{aligned} A_s &= \frac{N^2}{12\pi^2}\frac{\lambda}{\xi + \frac{6\xi^2 Y_1^2}{H_1^2+\xi^2}} \\ n_s &= 1 - \frac{2}{N} - \frac{3r}{16} \\ r &= \frac{2}{N^2}\left(\frac{1}{\xi} + \frac{6Y_1^2}{H_1^2+\xi^2}\right) = \frac{\lambda}{6\pi^2 A_s \xi^2}\ . \end{aligned} \tag{3.13}$$

The term $\frac{3r}{16}$ in the expression for n_s has sometimes been dropped. While it is negligible for small r, for the maximum value $r = 0.067$ it gives a correction of -0.012. For $N = 56$ [51], we get $n_s = 0.96 - \frac{3r}{16}$, in agreement with observations. In contrast to the cases $H = Y = 0$, the amplitude A_s can be small without a large ξ, if the Nieh–Yan term coupling Y_1 is large instead. However, the observational upper limit (3.10) on r combined with the value (3.6) of A_s anyway requires $\xi > 10^4\sqrt{\lambda}$, so unless $\lambda \ll 1$, we have $\xi \gg 1$. The tensor-to-scalar ratio r can be adjusted up or down from the metric case result $12/N^2$ by shifting the parameters. The minimum value is $r = 5 \times 10^{-13}$ (assuming $\lambda < 0.1$), corresponding to the tree-level Palatini case with a non-minimal coupling only to the Ricci scalar.

If the Holst term is zero, only plateau inflation is possible. In this case the behaviour is identical to the teleparallel case studied in [52]. However, if $H_1 \neq 0$, we can get qualitatively different inflationary behaviour. Let us first look at some interesting subcases. We have verified all results by numerically scanning the parameter space.

3.3 $Y = 0$

Let us consider the case when the Nieh–Yan term is zero, but not the Holst term. We see from (3.13) that the Holst term plays no role in plateau inflation, unless its coupling is large. This is because the Holst contribution to the kinetic function (3.2) decreases like $1/h^6$ for large h, in contrast to the $1/h^2$ suppression of the ξ term. So even though the Holst term is non-zero because F generates torsion, its numerical contribution is negligible. In particular, this is the case if we take $H_0 = 1/\gamma \approx 3.6$, where $\gamma = 0.274$ is the value determined from black hole entropy in LQG without chemical potential [133]. If the Holst term coupling is large, n_s can be shifted down on the plateau.

However, if H_0 is much larger than ξ and H_1, there is another inflationary regime in addition to plateau inflation. The contribution of the Holst term can dominate the kinetic function (3.2) in an intermediate regime even though it is subleading in the limit $h \to \infty$. When H_0 dominates over all other terms and $|\xi|h^2 \gg 1$, the kinetic term is $\tilde{K} \simeq 6/h^2$. This agrees with the metric formulation plateau case [71], giving $n_s = 1 - 2/N = 0.96$ and $r = 12/N^2 = 4 \times 10^{-3}$ for $N = 56$. However, now this solution also exists if $\xi < 0$. If the other terms also contribute, the results for r remain the same, but n_s can be adjusted downwards. The running parameters α and β can also take a range of values outside those of plateau inflation driven by a non-minimal coupling to the Ricci scalar. In this inflationary regime, h at the pivot scale can be as small as 2×10^{-3}, in contrast to usual plateau inflation, where $h \approx 0.08$ in the metric formulation and $h \approx 20$ in the Palatini formulation. Interestingly, this case is possible even if $H_1 = 0$, i.e. if the Holst coupling is constant. The non-minimal coupling F generates torsion, making the Holst term dynamical, its effect enhanced by the large value of H_0.

If $\xi = 0$, there is also a third inflationary regime, which gives predictions close to the metric case, as we discuss in the next section.

3.4 $\xi = 0$

If the non-minimal coupling to the Ricci scalar is zero, the potential is not asymptotically flat. Nevertheless, we can have an intermediate flat regime where inflation can be successful (meaning the predictions agree with observations). The kinetic function (3.2) simplifies to

$$\begin{aligned} \tilde{K} &= 1 + \frac{3}{2}\frac{(H' - Y')^2}{1 + H^2} \\ &= 1 + 6h^2 \frac{(Y_1 - H_1)^2}{1 + (H_0 + H_1 h^2)^2} \ . \end{aligned} \tag{3.14}$$

We take $H_0 > 0$. (The case $H_0 = 0$ does not lead to successful inflation, and negative values of H_0 are related by symmetry to positive values.) For successful inflation, the second term has to dominate, in which case the canonical field is

$$\chi = \int \mathrm{d}h \sqrt{\tilde{K}} \simeq \sqrt{\frac{3}{2}} \left| \frac{H_1 - Y_1}{H_1} \right| \mathrm{arsinh}(H_0 + H_1 h^2) + \chi_0 \ , \tag{3.15}$$

which gives

$$h^2 = H_1^{-1} \sinh[\sqrt{\frac{2}{3}} \Delta(\chi + \chi_0)] - H_1^{-1} H_0 \ , \tag{3.16}$$

where $\Delta \equiv \left| \frac{H_1}{H_1 - Y_1} \right|$ and $\chi_0 \equiv \sqrt{\frac{3}{2}} \Delta^{-1} \mathrm{arsinh} H_0$, so that $\chi = 0$ corresponds to $h = 0$. The potential (3.4) reads

$$U = \frac{\lambda}{4H_1^2} \left\{ \sinh\left[\sqrt{\frac{2}{3}} \Delta(\chi + \chi_0) \right] - H_0 \right\}^2 \ . \tag{3.17}$$

In the limit $\sinh[\sqrt{\frac{2}{3}}\Delta(\chi+\chi_0)] \gtrsim 1$ (which is required for inflation satisfying the observational constraints (3.6)–(3.10) and the constraint on the number of e-folds), the amplitude, spectral index, tensor-to-scalar ratio and the number of e-folds from (3.6)–(3.11) are ($\simeq$ indicates dropping corrections of order $1/\sinh[\sqrt{\frac{2}{3}}\Delta(\chi + \chi_0)]^2$)

$$\begin{aligned} A_s &\simeq \frac{\lambda H_0^2}{128\pi^2 \Delta^2 H_1^2} \frac{x^4}{(1+x)^2} \\ n_s &= 1 - \frac{8\Delta^2}{3x} - \frac{r}{4} \\ r &\simeq \frac{64\Delta^2}{3x^2} \\ N &\simeq \frac{3}{4\Delta^2}[x - \ln(1+x)] \ , \end{aligned} \tag{3.18}$$

where we have denoted $1 + x \equiv H_0/\sinh[\sqrt{\frac{2}{3}}\Delta(\chi + \chi_0)]$.

The expressions for the running and the running of the running are also straightforward to write down; they are within the observational ranges (3.8)-(3.9). In the limit $x \gg 1$ we can drop the logarithmic corrections to get

$$\begin{aligned} A_s &\simeq \frac{\lambda \Delta^2 H_0^2}{72\pi^2 H_1^2} N^2 \\ n_s &\simeq 1 - \frac{2}{N} - \frac{r}{4} \\ r &\simeq \frac{12}{\Delta^2 N^2} \simeq \frac{\lambda H_0^2}{6\pi^2 A_s H_1^2} \ . \end{aligned} \tag{3.19}$$

These equations are almost identical to those in the plateau inflation case with the replacement $\Delta^{-2} \to \frac{1}{6\xi} + \frac{Y_1^2}{H_1^2+\xi^2}$, $\frac{H_1^2}{H_0^2} \to \xi^2$. The tensor-to-scalar ratio can be as large as the observational upper limit and as small as desired. The only difference is the last term for n_s is $-\frac{r}{4}$ instead of $-\frac{3r}{16}$, but the difference is 4×10^{-3} even for the maximum observationally allowed value of r.

In the pure Holst case, $Y_1 = 0$, we have $\Delta = 1$, and the predictions are identical to the metric plateau case, as mentioned above. This can be seen from (2.19): if $Y' = F' = 0$ and $H \gg F$, the action is the same as in the metric case.

In the pure Nieh–Yan case, $\xi = H_0 = H_1 = 0$, there are no inflationary solutions that agree with observations. (The case with $\xi = H_1 = 0$ but $H_0 \neq 0$ is equivalent to this case with the change $Y_1^2/(1+H_0^2) \to Y_1^2$.) In this case the kinetic function (3.14) grows like h^2 for large h, mapping the potential $\frac{1}{4}\lambda h^4$ to the potential $\frac{\lambda}{6Y_1}\chi^2$ at large field values. Adjusting Y_1 interpolates between the quartic and the quadratic potential. While the spectral index of the quadratic potential (unlike the quartic potential) agrees with the data, the tensor-to-scalar ratio r is too large in both cases [173]. (A similar situation arises in the teleparallel formulation [52].) The value of r can be decreased by including a R^2 term in the action [36].

3.5 $\xi < 0$

Including all three coupling terms (to the Ricci scalar, the Holst term and the Nieh–Yan term) makes it possible to have inflationary behaviour beyond the plateau and the above subcases. Let us first discuss the case $\xi < 0$.

Given that we can have successful inflation when $\xi = 0$, by continuity we expect this to be possible also for small negative values of ξ. However, there are also successful inflationary models for large negative values of ξ. If $\xi < 0$, the non-minimal coupling F goes to zero at $h = 1/\sqrt{|\xi|}$. The kinetic function (3.2) correspondingly diverges, so we have an α-attractor [183], found for Higgs inflation for another action in [49]. (Plateau Higgs inflation can also be viewed in terms of an α-attractor [74, 184, 185].) However, the α-attractor behaviour in the limit $F \to 0$ does not give successful inflation, as n_s and/or r are wrong. Nevertheless, there are other kinds of successful inflationary models with $\xi < 0$.

In the case $H_0 = 0$ there are no viable inflationary models. In the case $Y = 0$ there are no viable models if also $H_1 = 0$. If $Y = 0$ and $H_1 \neq 0$, the only viable case is the one discussed in section 3.3. If we allow both $H_0 \neq 0$ and $Y \neq 0$, the range of predictions widens.

One particular new case is inflection point inflation. At an inflection point $\eta = 0$, so the spectral index there is $n_s = 1 - 3r/8$. The observational limit $r < 0.067$ in (3.10) then gives $n_s > 0.97$, which is at the upper end of the observational range (3.7). So if there is an inflection point close to the pivot scale, the amplitude of inflationary gravitational waves is close to the observational upper limit. This is the reason an inflection point due to quantum corrections [43, 48, 102, 186–195] was highlighted after the claimed detection of gravitational waves by the BICEP2 instrument (which turned out to be incorrect). We find models with an inflection point exactly at the pivot scale that agree with observations, apart from this tension. Inflection point in Higgs inflation from classical contributions to the action that can generate torsion has been earlier discussed in [49].

We scanned numerically over the five-dimensional parameter space (h, ξ, H_0, H_1, Y_1) with an adaptive Monte Carlo method. We take the range $[-10^{10}, 0]$ for ξ, $[-10^{10}, 10^{10}]$ for H_0 and H_1, and $[0, 10^{10}]$ for h and Y_1. (We can fix one sign among H_0, H_1 and Y_1 without affecting the physics.) We check that observables at the pivot scale agree with the observational constraints (3.6)–(3.10), except that n_s can have the somewhat wider range $[0.95, 0.98]$, and the number of e-folds until the end of inflation agrees with (3.12) to within ± 1. We restrict the Higgs quartic coupling to the range $[10^{-5}, 10^{-1}]$. Due to loop corrections, λ runs to smaller values with increasing scale and can even cross zero. Therefore, it can be arbitrarily small at the pivot scale, but very small values require tuning, and the running can spoil the flatness of the potential. (This happens in the minimally coupled case [196–198].)

In figure 1 (left) we show the results on the (n_s, r) plane. The colour indicates the smallest value of $|\xi|$ (it is not single-valued on this plane). The solid line is the analytical result (3.13) for plateau inflation, and the dashed line is the result (3.18) for $\xi = 0$. (It lies in the middle of the blue region corresponding to the limit $\xi \to 0$ because in the numerical scan we allow a variation ± 1 in N.) The star marks the metric case.

The tensor-to-scalar ratio extends from the maximum observationally allowed value down to around 10^{-6}, and n_s covers the entire current observational range. The running is in the small range $-1 \times 10^{-3} \lesssim \alpha \lesssim -5 \times 10^{-4}$. The running of the running also has a small range, $-6 \times 10^{-5} \lesssim \beta \lesssim -2 \times 10^{-5}$. The non-minimal couplings have the ranges $|\xi| < 10^6$, $|H_0| < 10^8$ and $|Y_1| > 3 \times 10^3$; H_1 can take any value in the range we scan.

Smooth, well-defined edges in the figures correspond to true observational constraints for the cosmological observables, while rough edges with individual points scattered about correspond to regions of parameter space that the scan has not fully resolved. In such regions the parameter values for points that satisfy the observational constraints are highly tuned, requiring the precise cancellation of two or more large numbers.

3.6 $\xi > 0$

Finally, let us discuss the case when we include all three coupling functions and $\xi > 0$. As in the case $\xi < 0$, inflection point inflation is possible. Successful inflation is now possible also when $H_0 = 0$. We perform the same kind of numerical scan as in the case $\xi < 0$, except the range of ξ is now $[0, 10^{10}]$. In figure 1 we show the results on the (n_s, r) plane.

The range of the predictions extends to much lower values of r than in the case $\xi < 0$. All of the edges of the allowed region are now well resolved. The non-minimal coupling of the Ricci scalar takes values $\xi < 1 \times 10^9$; H_0, H_1 and Y_1 can take any value in the range we scan. In contrast to the case when one of the three couplings ξ, H and Y is zero, the predictions for n_s and r cover almost all of the range expected to be tested by next generation CMB experiments such the Simons Observatory [199], LiteBIRD [200] and CMB-S4 [201]. However, there are regions on the (n_s, α) and (n_s, β) planes with both positive and negative running within reach of upcoming experiments that the model cannot reproduce.

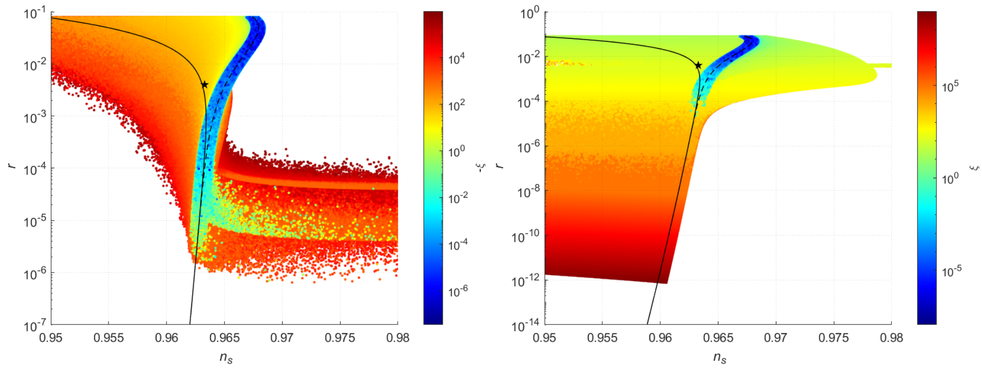

Figure 1. Spectral index n_s and tensor-to-scalar ratio r for $\xi < 0$ (left) and $\xi > 0$ (right). The colour corresponds to the smallest absolute value of the non-minimal coupling $|\xi|$ to the Ricci scalar. The points satisfy all observational CMB constraints, except that the range of n_s is wider. The solid line traces the prediction of plateau inflation, and the dashed line is the case $\xi = 0$. The star marks the metric case.

4 Conclusions

New perspective on inflation in LQG. We have studied the effect of non-minimal coupling of a scalar field to the Holst and Nieh–Yan terms on inflation, in addition to the non-minimal coupling to the Ricci scalar. These terms play a key role in LQG, and are expected to appear in theories where torsion is non-zero. Since the Higgs exists, it will in general couple to these terms, and the couplings have to be taken into account. Motivated by Higgs inflation, we have included terms up to dimension 4 and even in the field.

Non-minimal coupling to the Holst term alone gives inflation with predictions close to those of the metric formulation plateau Higgs inflation for the same amount of e-folds, although reheating and hence the number of e-folds may be different due to the different shape of the effective potential [51, 174–181]. This means that observational verification of the predictions of this simplest metric formulation Higgs inflation [71] would not rule out the Palatini formulation of Higgs inflation. That prediction has been earlier reproduced in the Palatini case with tuned non-metricity terms [49], but the present case shows it can be achieved with a simple Higgs-LQG action with no tuning. Adding a non-minimal coupling ξ to the Ricci scalar recovers the results of Higgs plateau inflation in the Palatini formulation [40] unless the Holst coupling H_0 is much larger than $|\xi|$. If the Holst term coupling dominates but $|\xi|$ also contributes, the spectral index n_s and its running can be adjusted from the metric case. Notably, this form of inflation is possible even when ξ is negative.

A non-minimal coupling to the Nieh–Yan term alone does not give successful inflation. If we also have $\xi \neq 0$, plateau inflation can be modified so that it interpolates between the results we get in the Palatini and the metric formulation when only ξ is non-zero, and the tensor-to-scalar ratio r can be even larger than in the metric case. This case is identical to Higgs inflation in the teleparallel formulation [52].

If we include non-minimal coupling to all three terms (Ricci scalar, Holst term and Nieh–Yan term), the range of predictions for n_s and r widens considerably to cover almost all of the values expected to be covered by near-future experiments. However, when we add running or running of the running, not all values to be probed can be reproduced. Also, many of the values correspond to tuned couplings. For example, we can a produce an inflection point, but this requires carefully adjusting the non-minimal couplings, as has been done with quantum corrections [43, 48, 102, 186–195] and classical non-metricity terms [49].

It is interesting that the Higgs field makes the Holst and Nieh–Yan terms dynamical at the classical level, as fermions have been found to do [139, 140, 144–148]. The Higgs generates torsion, which makes the Holst term non-zero. The Holst term can have a large impact on inflation even if it is minimally coupled as long as either the Ricci scalar or the Nieh–Yan term have non-minimal coupling. However, the value for the minimal Holst term coupling (i.e. the Barbero—Immirzi parameter) $1/\gamma \approx 3.6$ determined from black hole entropy in the case with no chemical potential [133], is too small to be discernible from the CMB.

The non-minimal couplings to the Higgs provide a new point of view on LQG cosmology. Just as a large ξ brings the gravity scale down, so that (in the Jordan frame) gravitons violate perturbative unitarity below the Planck scale [41, 41, 77, 79–99], large values of the non-minimal couplings of the Holst and Nieh–Yan terms can bring aspects of LQG down to the scales probed during inflation. They could also help address the issue of apparent violation of unitarity, whose scale is known to be sensitive to the form of the kinetic term [41, 77, 98, 99], which is affected by the Holst and Nieh–Yan terms.

Acknowledgments

ML would like to thank E. Wilson-Ewing for worthwhile suggestions. We thank Eemeli Tomberg for help with renormalisation group equations.

A Solving for torsion in the tetrad formalism

As the tetrad formalism is more familiar to the LQG community, we cover briefly how the results (2.10)–(2.12) for torsion can be elegantly obtained using tetrads. A set of tetrads $\{e^A{}_\alpha\}$ is a linear map from tangent space to spacetime, providing a basis for the tangent space at each point in spacetime; capital Latin indices are associated to the tangent space. We take the basis to be orthonormal with respect to the metric $g_{\alpha\beta}$,

$$g_{\alpha\beta} = \eta_{AB} e^A{}_\alpha e^B{}_\beta \ , \tag{A.1}$$

where $\eta_{AB} = \mathrm{diag}(-1, 1, 1, 1)$ is the Minkowski metric. We also have $g^{\alpha\beta} e^A{}_\alpha e^B{}_\beta = \eta^{AB}$. The inverse tetrad $e_A{}^\alpha$ is defined so that $e_A{}^\alpha e^A{}_\beta = \delta^\alpha{}_\beta$ and $e_A{}^\alpha e^B{}_\alpha = \delta^B{}_A$. We assume from the beginning that the full covariant derivative of the tetrad, acting on spacetime and tangent space indices, vanishes i.e. that the tetrad postulate holds.

In terms of tetrads and the tangent space connection $\omega_\alpha{}^{AB}$ (called the Lorentz connection), the action (2.8) reads

$$S = \int \mathrm{d}^4 x e \bigg[\frac{1}{2} F(h) e_A{}^\alpha e_B{}^\beta F_{\alpha\beta}{}^{AB} + \frac{1}{4} H(h) \epsilon_{AB}{}^{CD} e_C{}^\alpha e_D{}^\beta F_{\alpha\beta}{}^{AB} - \frac{1}{4} Y(h) \epsilon^{\alpha\beta\gamma\delta} \eta_{AB} T^A{}_{\alpha\beta} T^B{}_{\gamma\delta} - \frac{1}{2} K(h) \eta^{AB} e_A{}^\alpha e_B{}^\beta \partial_\alpha h \, \partial_\beta h - V(h) \bigg] \ , \tag{A.2}$$

where $e = \det(e^A{}_\alpha)$, $F_{\alpha\beta}{}^{AB} = 2\partial_{[\alpha}\omega_{\beta]}{}^{AB} + 2\omega_{[\alpha}{}^{AC}\omega_{\beta]}{}^{DB}\eta_{CD}$ is the curvature of the Lorentz connection, related to the Riemann tensor via $e^\mu{}_A e^\nu{}_B F_{\alpha\beta}{}^{AB} = R^{\mu\nu}{}_{\alpha\beta}$. Torsion is defined as $T^A{}_{\alpha\beta} = \mathcal{D}_{[\alpha}e^A{}_{\beta]}$, where the covariant derivative $\mathcal{D}$ acts only on tangent space indices.

The Einstein–Hilbert term plus the Holst term, together known as the Holst action, can be written compactly as

$$S_{\text{Holst}} = \int \mathrm{d}^4x e \, \frac{1}{2} e_A{}^\alpha e_B{}^\beta P^{AB}{}_{CD} \, F_{\alpha\beta}{}^{CD} \ , \tag{A.3}$$

where the projection operator is defined as

$$P^{AB}{}_{CD} = F\delta^{[A}{}_C\delta^{B]}{}_D + \frac{1}{2}H\epsilon^{AB}{}_{CD} \ . \tag{A.4}$$

The inverse of the projection operator is

$$(P^{-1})^{AB}{}_{CD} = \frac{1}{F^2+H^2}\left(F\delta^{[A}{}_C\delta^{B]}{}_D - \frac{1}{2}H\,\epsilon^{AB}{}_{CD}\right). \tag{A.5}$$

Varying the action (A.2) with respect to the Lorentz connection and dropping a boundary term gives the equation of motion

$$\frac{1}{4}\mathcal{D}_\alpha\left(P^{AB}{}_{CD}\,\epsilon_{ABEF}\epsilon^{\alpha\beta\gamma\delta}e^E{}_\gamma e^F{}_\delta\right) + \frac{1}{2}\epsilon_{CDEF}e^{E[\alpha}e^{|F|\beta]}\partial_\alpha Y = 0 \ . \tag{A.6}$$

The solution to (A.6) is obtained straightforwardly, using the definition of torsion and operating with (A.5):

$$T^A{}_{\alpha\beta} = \frac{1}{F^2+H^2}\left\{e^A{}_{[\alpha}\left[(F\partial_{\beta]}F + H(\partial_{\beta]}H - \partial_{\beta]}Y)\right] + e^A{}_\gamma\epsilon^\gamma{}_{\alpha\beta}{}^\delta\left[H\partial_\delta F - F(\partial_\delta H - \partial_\delta Y)\right]\right\} . \tag{A.7}$$

This agrees with the solution for torsion in (2.10)–(2.12).

References

[1] A. Einstein, *Einheitliche feldtheorie von gravitation und elektrizität, Verlag der Koeniglich-Preussichen Akademie der Wissenschaften* **22** (07, 1925) 414–419.

[2] A. Einstein, *Riemann-Geometrie unter Aufrechterhaltung des Begriffes des Fernparallelismus, Sitzungsberichte der Preussischen Akademie der Wissenschaften* (1928) 217–221.

[3] A. Einstein, *Neue Möglichkeit für eine einheitliche Theorie von Gravitation und Elektrizität, Sitzungsberichte der Preussischen Akademie der Wissenschaften* (1928) 224–227.

[4] A. Einstein, *Auf die Riemann-Metrik und den Fern-Parallelismus gegründete einheitliche Feldtheorie*, *Math. Ann.* **102** (1930) 685–697.

[5] M. Krssak, R. J. Van Den Hoogen, J. G. Pereira, C. G. Boehmer and A. A. Coley, *Teleparallel Theories of Gravity: Illuminating a Fully Invariant Approach*, *Class. Quant. Grav.* **36** (2019) 183001, [1810.12932].

[6] F. W. Hehl, P. Von Der Heyde, G. D. Kerlick and J. M. Nester, *General Relativity with Spin and Torsion: Foundations and Prospects*, *Rev. Mod. Phys.* **48** (1976) 393–416.

[7] F. W. Hehl and G. D. Kerlick, *Metric-affine variational principles in general relativity. I - Riemannian space-time*, *General Relativity and Gravitation* **9** (Aug., 1978) 691–710.

[8] A. Papapetrou and J. Stachel, *A new Lagrangian for the vacuum Einstein equations and its tetrad form.*, *General Relativity and Gravitation* **9** (Dec., 1978) 1075–1087.

[9] F. W. Hehl, E. A. Lord and L. L. Smalley, *Metric-affine variational principles in general relativity II. Relaxation of the Riemannian constraint*, *General Relativity and Gravitation* **13** (Nov., 1981) 1037–1056.

[10] R. Percacci, *The Higgs phenomenon in quantum gravity*, *Nucl. Phys.* **B353** (1991) 271–290, [0712.3545].

[11] C. Rovelli, *Ashtekar formulation of general relativity and loop space nonperturbative quantum gravity: A Report*, *Class. Quant. Grav.* **8** (1991) 1613–1676.

[12] J. M. Nester and H.-J. Yo, *Symmetric teleparallel general relativity*, *Chin. J. Phys.* **37** (1999) 113, [gr-qc/9809049].

[13] R. Percacci, *Gravity from a Particle Physicists' perspective*, *PoS* **ISFTG** (2009) 011, [0910.5167].

[14] K. Krasnov and R. Percacci, *Gravity and Unification: A review*, *Class. Quant. Grav.* **35** (2018) 143001, [1712.03061].

[15] S. Gielen, R. d. L. Ardon and R. Percacci, *Gravity with more or less gauging*, 1805.11626.

[16] J. Beltran Jimenez, L. Heisenberg and T. Koivisto, *Coincident General Relativity*, 1710.03116.

[17] M. Ferraris, M. Francaviglia and C. Reina, *Variational formulation of general relativity from 1915 to 1925 "Palatini's method" discovered by Einstein in 1925*, *Gen.Rel.Grav.* **14** (1982) 243–254.

[18] J. Beltrán Jiménez and A. Delhom, *Ghosts in metric-affine higher order curvature gravity*, *Eur. Phys. J. C* **79** (2019) 656, [1901.08988].

[19] R. Percacci, *Towards Metric-Affine Quantum Gravity*, in *25th Current Problems in Theoretical Physics: Aspects of Nonperturbative QFT, Foundations of Quantum Theory, Quantum Spacetime*, 3, 2020, 2003.09486.

[20] H. A. Buchdahl, *Non-linear Lagrangians and Palatine's device*, *Proceedings of the Cambridge Philosophical Society* **56** (1960) 396.

[21] H. A. Buchdahl, *Non-linear Lagrangians and cosmological theory*, *Mon. Not. Roy. Astron. Soc.* **150** (1970) 1.

[22] B. Shahid-Saless, *First-Order Formalism Treatment of R + R**2 Gravity*, *Phys. Rev.* **D35** (1987) 467–470.

[23] E. E. Flanagan, *Palatini form of 1/R gravity*, *Phys. Rev. Lett.* **92** (2004) 071101, [astro-ph/0308111].

[24] E. E. Flanagan, *Higher order gravity theories and scalar tensor theories*, *Class. Quant. Grav.* **21** (2003) 417–426, [gr-qc/0309015].

[25] T. P. Sotiriou and S. Liberati, *Metric-affine f(R) theories of gravity*, *Annals Phys.* **322** (2007) 935–966, [gr-qc/0604006].

[26] T. P. Sotiriou and V. Faraoni, *f(R) Theories Of Gravity*, *Rev. Mod. Phys.* **82** (2010) 451–497, [0805.1726].

[27] G. J. Olmo, *Palatini Approach to Modified Gravity: f(R) Theories and Beyond*, *Int. J. Mod. Phys.* **D20** (2011) 413–462, [1101.3864].

[28] M. Borunda, B. Janssen and M. Bastero-Gil, *Palatini versus metric formulation in higher curvature gravity*, *JCAP* **0811** (2008) 008, [0804.4440].

[29] L. Querella, *Variational principles and cosmological models in higher order gravity*, Ph.D. thesis, Liege U., 1998. gr-qc/9902044.

[30] S. Cotsakis, J. Miritzis and L. Querella, *Variational and conformal structure of nonlinear metric connection gravitational Lagrangians*, *J. Math. Phys.* **40** (1999) 3063–3071, [gr-qc/9712025].

[31] L. Järv, M. Rünkla, M. Saal and O. Vilson, *Nonmetricity formulation of general relativity and its scalar-tensor extension*, *Phys. Rev.* **D97** (2018) 124025, [1802.00492].

[32] A. Conroy and T. Koivisto, *The spectrum of symmetric teleparallel gravity*, 1710.05708.

[33] B. Li, J. D. Barrow and D. F. Mota, *The Cosmology of Ricci-Tensor-Squared gravity in the Palatini variational approach*, *Phys. Rev.* **D76** (2007) 104047, [0707.2664].

[34] B. Li, D. F. Mota and D. J. Shaw, *Microscopic and Macroscopic Behaviors of Palatini Modified Gravity Theories*, *Phys. Rev.* **D78** (2008) 064018, [0805.3428].

[35] Q. Exirifard and M. M. Sheikh-Jabbari, *Lovelock gravity at the crossroads of Palatini and metric formulations*, *Phys. Lett.* **B661** (2008) 158–161, [0705.1879].

[36] V.-M. Enckell, K. Enqvist, S. Räsänen and L.-P. Wahlman, *Inflation with R^2 term in the Palatini formalism*, 1810.05536.

[37] U. Lindstrom, *The Palatini Variational Principle and a Class of Scalar-Tensor Theories*, *Nuovo Cim.* **B35** (1976) 130–136.

[38] U. Lindström, *Comments on the Jordan-Brans-Dicke scalar-field theory of gravitation*, *Nuovo Cimento B Serie* **32** (Apr., 1976) 298–302.

[39] N. Van Den Bergh, *The Palatini variational principle for the general Bergmann–Wagoner–Nordtvedt theory of graviation*, *J. Math Phys.* **22** (1981) 2245.

[40] F. Bauer and D. A. Demir, *Inflation with Non-Minimal Coupling: Metric versus Palatini Formulations*, *Phys. Lett.* **B665** (2008) 222–226, [0803.2664].

[41] F. Bauer and D. A. Demir, *Higgs-Palatini Inflation and Unitarity*, *Phys. Lett.* **B698** (2011) 425–429, [1012.2900].

[42] T. Koivisto and H. Kurki-Suonio, *Cosmological perturbations in the Palatini formulation of modified gravity*, *Class. Quant. Grav.* **23** (2006) 2355–2369, [astro-ph/0509422].

[43] S. Räsänen and P. Wahlman, *Higgs inflation with loop corrections in the Palatini formulation*, 1709.07853.

[44] V.-M. Enckell, K. Enqvist, S. Räsänen and E. Tomberg, *Higgs inflation at the hilltop*, *JCAP* **1806** (2018) 005, [1802.09299].

[45] T. Markkanen, T. Tenkanen, V. Vaskonen and H. Veermäe, *Quantum corrections to quartic inflation with a non-minimal coupling: metric vs. Palatini*, 1712.04874.

[46] L. Järv, A. Racioppi and T. Tenkanen, *Palatini side of inflationary attractors*, *Phys. Rev.* **D97** (2018) 083513, [1712.08471].

[47] D. Iosifidis and T. Koivisto, *Scale transformations in metric-affine geometry*, 1810.12276.

[48] S. Räsänen and E. Tomberg, *Planck scale black hole dark matter from Higgs inflation*, 1810.12608.

[49] S. Räsänen, *Higgs inflation in the Palatini formulation with kinetic terms for the metric*, 1811.09514.

[50] K. Aoki and K. Shimada, *Scalar-metric-affine theories: Can we get ghost-free theories from symmetry?*, *Phys. Rev.* **D100** (2019) 044037, [1904.10175].

[51] J. Rubio and E. S. Tomberg, *Preheating in Palatini Higgs inflation*, *JCAP* **1904** (2019) 021, [1902.10148].

[52] S. Raatikainen and S. Räsänen, *Higgs inflation and teleparallel gravity*, *JCAP* **12** (2019) 021, [1910.03488].

[53] R. Jinno, M. Kubota, K.-y. Oda and S. C. Park, *Higgs inflation in metric and Palatini formalisms: Required suppression of higher dimensional operators*, *JCAP* **03** (2020) 063, [1904.05699].

[54] K. Shimada, K. Aoki and K.-i. Maeda, *Metric-affine Gravity and Inflation*, *Phys. Rev. D* **99** (2019) 104020, [1812.03420].

[55] D. Iosifidis, *Exactly Solvable Connections in Metric-Affine Gravity*, *Class. Quant. Grav.* **36** (2019) 085001, [1812.04031].

[56] C. G. Callan, S. Coleman and R. Jackiw, *A new improved energy-momentum tensor*, *Annals of Physics* **59** (1970) 42 – 73.

[57] A. A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, *Phys. Lett.* **91B** (1980) 99–102.

[58] D. Kazanas, *Dynamics of the Universe and Spontaneous Symmetry Breaking*, *Astrophys. J.* **241** (1980) L59–L63.

[59] A. H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, *Phys. Rev.* **D23** (1981) 347–356.

[60] K. Sato, *First Order Phase Transition of a Vacuum and Expansion of the Universe*, *Mon. Not. Roy. Astron. Soc.* **195** (1981) 467–479.

[61] V. F. Mukhanov and G. V. Chibisov, *Quantum Fluctuations and a Nonsingular Universe*, *JETP Lett.* **33** (1981) 532–535.

[62] A. D. Linde, *A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*, *Phys. Lett.* **108B** (1982) 389–393.

[63] A. Albrecht and P. J. Steinhardt, *Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking*, *Phys. Rev. Lett.* **48** (1982) 1220–1223.

[64] S. W. Hawking and I. G. Moss, *Supercooled Phase Transitions in the Very Early Universe*, *Phys. Lett.* **110B** (1982) 35–38.

[65] G. V. Chibisov and V. F. Mukhanov, *Galaxy formation and phonons*, *Mon. Not. Roy. Astron. Soc.* **200** (1982) 535–550.

[66] S. W. Hawking, *The Development of Irregularities in a Single Bubble Inflationary Universe*, *Phys. Lett.* **115B** (1982) 295.

[67] A. H. Guth and S. Y. Pi, *Fluctuations in the New Inflationary Universe*, *Phys. Rev. Lett.* **49** (1982) 1110–1113.

[68] A. A. Starobinsky, *Dynamics of Phase Transition in the New Inflationary Universe Scenario and Generation of Perturbations*, *Phys. Lett.* **117B** (1982) 175–178.

[69] M. Sasaki, *Large Scale Quantum Fluctuations in the Inflationary Universe*, *Prog. Theor. Phys.* **76** (1986) 1036.

[70] V. F. Mukhanov, *Quantum Theory of Gauge Invariant Cosmological Perturbations*, *Sov. Phys. JETP* **67** (1988) 1297–1302.

[71] F. L. Bezrukov and M. Shaposhnikov, *The Standard Model Higgs boson as the inflaton*, *Phys. Lett.* **B659** (2008) 703–706, [0710.3755].

[72] F. Bezrukov, *The Higgs field as an inflaton*, *Class. Quant. Grav.* **30** (2013) 214001, [1307.0708].

[73] F. Bezrukov and M. Shaposhnikov, *Inflation, LHC and the Higgs boson*, *Comptes Rendus Physique* **16** (2015) 994–1002.

[74] J. Rubio, *Higgs inflation*, 1807.02376.

[75] T. Futamase and K.-i. Maeda, *Chaotic Inflationary Scenario in Models Having Nonminimal Coupling With Curvature*, *Phys. Rev.* **D39** (1989) 399–404.

[76] D. S. Salopek, J. R. Bond and J. M. Bardeen, *Designing Density Fluctuation Spectra in Inflation*, *Phys. Rev.* **D40** (1989) 1753.

[77] M. Shaposhnikov, A. Shkerin and S. Zell, *Quantum Effects in Palatini Higgs Inflation*, 2002.07105.

[78] T. Tenkanen, *Tracing the high energy theory of gravity: an introduction to Palatini inflation*, *Gen. Rel. Grav.* **52** (2020) 33, [2001.10135].

[79] J. L. F. Barbon and J. R. Espinosa, *On the Naturalness of Higgs Inflation*, *Phys. Rev.* **D79** (2009) 081302, [0903.0355].

[80] C. P. Burgess, H. M. Lee and M. Trott, *Power-counting and the Validity of the Classical Approximation During Inflation*, *JHEP* **09** (2009) 103, [0902.4465].

[81] C. P. Burgess, H. M. Lee and M. Trott, *Comment on Higgs Inflation and Naturalness*, *JHEP* **07** (2010) 007, [1002.2730].

[82] R. N. Lerner and J. McDonald, *Higgs Inflation and Naturalness*, *JCAP* **1004** (2010) 015, [0912.5463].

[83] R. N. Lerner and J. McDonald, *A Unitarity-Conserving Higgs Inflation Model*, *Phys. Rev.* **D82** (2010) 103525, [1005.2978].

[84] M. P. Hertzberg, *On Inflation with Non-minimal Coupling*, *JHEP* **11** (2010) 023, [1002.2995].

[85] F. Bezrukov, A. Magnin, M. Shaposhnikov and S. Sibiryakov, *Higgs inflation: consistency and generalisations*, *JHEP* **01** (2011) 016, [1008.5157].

[86] F. Bezrukov, D. Gorbunov and M. Shaposhnikov, *Late and early time phenomenology of Higgs-dependent cutoff*, *JCAP* **1110** (2011) 001, [1106.5019].

[87] X. Calmet and R. Casadio, *Self-healing of unitarity in Higgs inflation*, *Phys. Lett.* **B734** (2014) 17–20, [1310.7410].

[88] J. Weenink and T. Prokopec, *Gauge invariant cosmological perturbations for the nonminimally coupled inflaton field*, *Phys. Rev.* **D82** (2010) 123510, [1007.2133].

[89] R. N. Lerner and J. McDonald, *Unitarity-Violation in Generalized Higgs Inflation Models*, *JCAP* **1211** (2012) 019, [1112.0954].

[90] T. Prokopec and J. Weenink, *Uniqueness of the gauge invariant action for cosmological perturbations*, *JCAP* **1212** (2012) 031, [1209.1701].

[91] Z.-Z. Xianyu, J. Ren and H.-J. He, *Gravitational Interaction of Higgs Boson and Weak Boson Scattering*, *Phys. Rev.* **D88** (2013) 096013, [1305.0251].

[92] T. Prokopec and J. Weenink, *Naturalness in Higgs inflation in a frame independent formalism*, 1403.3219.

[93] J. Ren, Z.-Z. Xianyu and H.-J. He, *Higgs Gravitational Interaction, Weak Boson Scattering, and Higgs Inflation in Jordan and Einstein Frames*, *JCAP* **1406** (2014) 032, [1404.4627].

[94] A. Escrivà and C. Germani, *Beyond dimensional analysis: Higgs and new Higgs inflations do not violate unitarity*, *Phys. Rev.* **D95** (2017) 123526, [1612.06253].

[95] J. Fumagalli, S. Mooij and M. Postma, *Unitarity and predictiveness in new Higgs inflation*, *JHEP* **03** (2018) 038, [1711.08761].

[96] D. Gorbunov and A. Tokareva, *Scalaron the healer: removing the strong-coupling in the Higgs- and Higgs-dilaton inflations*, 1807.02392.

[97] Y. Ema, *Dynamical Emergence of Scalaron in Higgs Inflation*, *JCAP* **1909** (2019) 027, [1907.00993].

[98] J. McDonald, *Does Palatini Higgs Inflation Conserve Unitarity?*, 2007.04111.

[99] V.-M. Enckell, S. Nurmi, S. Rasanen and E. Tomberg, *Critical point Higgs inflation in the Palatini formulation*, 2012.03660.

[100] J. L. F. Barbon, J. A. Casas, J. Elias-Miro and J. R. Espinosa, *Higgs Inflation as a Mirage*, *JHEP* **09** (2015) 027, [1501.02231].

[101] A. Salvio and A. Mazumdar, *Classical and Quantum Initial Conditions for Higgs Inflation*, *Phys. Lett.* **B750** (2015) 194–200, [1506.07520].

[102] A. Salvio, *Initial Conditions for Critical Higgs Inflation*, *Phys. Lett.* **B780** (2018) 111–117, [1712.04477].

[103] S. Kaneda and S. V. Ketov, *Starobinsky-like two-field inflation*, *Eur. Phys. J.* **C76** (2016) 26, [1510.03524].

[104] X. Calmet and I. Kuntz, *Higgs Starobinsky Inflation*, *Eur. Phys. J.* **C76** (2016) 289, [1605.02236].

[105] Y.-C. Wang and T. Wang, *Primordial perturbations generated by Higgs field and R^2 operator*, *Phys. Rev.* **D96** (2017) 123506, [1701.06636].

[106] Y. Ema, *Higgs Scalaron Mixed Inflation*, *Phys. Lett.* **B770** (2017) 403–411, [1701.07665].

[107] S. Pi, Y.-l. Zhang, Q.-G. Huang and M. Sasaki, *Scalaron from R^2-gravity as a heavy field*, *JCAP* **1805** (2018) 042, [1712.09896].

[108] M. He, A. A. Starobinsky and J. Yokoyama, *Inflation in the mixed Higgs-R^2 model*, *JCAP* **1805** (2018) 064, [1804.00409].

[109] D. M. Ghilencea, *Two-loop corrections to Starobinsky-Higgs inflation*, 1807.06900.

[110] S.-J. Wang, *Electroweak relaxation of cosmological hierarchy*, *Phys. Rev. D* **99** (2019) 023529, [1810.06445].

[111] A. Gundhi and C. F. Steinwachs, *Scalaron-Higgs inflation*, 1810.10546.

[112] A. Karam, T. Pappas and K. Tamvakis, *Nonminimal Coleman–Weinberg Inflation with an R^2 term*, 1810.12884.

[113] J. Kubo, M. Lindner, K. Schmitz and M. Yamada, *Planck mass and inflation as consequences of dynamically broken scale invariance*, 1811.05950.

[114] V.-M. Enckell, K. Enqvist, S. Räsänen and L.-P. Wahlman, *Higgs-R^2 inflation - full slow-roll study at tree-level*, 1812.08754.

[115] D. D. Canko, I. D. Gialamas and G. P. Kodaxis, *A simple $F(\mathcal{R}, \phi)$ deformation of Starobinsky inflationary model*, 1901.06296.

[116] M. He, R. Jinno, K. Kamada, A. A. Starobinsky and J. Yokoyama, *Occurrence of Tachyonic Preheating in the Mixed Higgs-R^2 Model*, 2007.10369.

[117] F. Bezrukov and C. Shepherd, *A heatwave affair: mixed Higgs-R^2 preheating on the lattice*, 2007.10978.

[118] A. Ashtekar and P. Singh, *Loop Quantum Cosmology: A Status Report*, *Class. Quant. Grav.* **28** (2011) 213001, [1108.0893].

[119] L. Bethke and J. Magueijo, *Inflationary tensor fluctuations, as viewed by Ashtekar variables and their imaginary friends*, *Phys. Rev. D* **84** (2011) 024014, [1104.1800].

[120] L. Bethke and J. Magueijo, *Chirality of tensor perturbations for complex values of the Immirzi parameter*, *Class. Quant. Grav.* **29** (2012) 052001, [1108.0816].

[121] C. Rovelli, *Quantum gravity*. Cambridge Monographs on Mathematical Physics. Univ. Pr., Cambridge, UK, 2004, 10.1017/CBO9780511755804.

[122] T. Thiemann, *Modern canonical quantum general relativity*, gr-qc/0110034.

[123] R. Gambini and J. Pullin, *A first course in loop quantum gravity*. 2011.

[124] C. Rovelli and F. Vidotto, *Covariant Loop Quantum Gravity*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2014.

[125] S. Holst, *Barbero's Hamiltonian derived from a generalized Hilbert-Palatini action*, *Phys. Rev.* **D53** (1996) 5966–5969, [gr-qc/9511026].

[126] A. Ashtekar, *New Variables for Classical and Quantum Gravity*, *Phys. Rev. Lett.* **57** (1986) 2244–2247.

[127] T. Jacobson and L. Smolin, *Nonperturbative Quantum Geometries*, *Nucl. Phys.* **B299** (1988) 295–345.

[128] E. Bianchi, P. Dona and S. Speziale, *Polyhedra in loop quantum gravity*, *Phys. Rev.* **D83** (2011) 044035, [1009.3402].

[129] C. Rovelli and L. Smolin, *Spin networks and quantum gravity*, *Phys. Rev.* **D52** (1995) 5743–5759, [gr-qc/9505006].

[130] S. Alexandrov and D. Vassilevich, *Area spectrum in Lorentz covariant loop gravity*, *Phys. Rev. D* **64** (2001) 044023, [gr-qc/0103105].

[131] J. D. Bekenstein, *Black holes and the second law*, *Lett. Nuovo Cim.* **4** (1972) 737–740.

[132] S. W. Hawking, *Black hole explosions*, *Nature* **248** (1974) 30–31.

[133] A. Ghosh and P. Mitra, *An Improved lower bound on black hole entropy in the quantum geometry approach*, *Phys. Lett.* **B616** (2005) 114–117, [gr-qc/0411035].

[134] A. Ghosh and A. Perez, *Black hole entropy and isolated horizons thermodynamics*, *Phys. Rev. Lett.* **107** (2011) 241301, [1107.1320].

[135] E. Bianchi, *Entropy of Non-Extremal Black Holes from Loop Gravity*, 1204.5122.

[136] E. Frodden, M. Geiller, K. Noui and A. Perez, *Black Hole Entropy from complex Ashtekar variables*, *EPL* **107** (2014) 10005, [1212.4060].

[137] H. T. Nieh and M. L. Yan, *An Identity in Riemann-cartan Geometry*, *J. Math. Phys.* **23** (1982) 373.

[138] G. Calcagni and S. Mercuri, *The Barbero-Immirzi field in canonical formalism of pure gravity*, *Phys. Rev.* **D79** (2009) 084004, [0902.0957].

[139] S. Mercuri, *Fermions in Ashtekar-Barbero connections formalism for arbitrary values of the Immirzi parameter*, *Phys. Rev.* **D73** (2006) 084016, [gr-qc/0601013].

[140] S. Mercuri, *Nieh-Yan Invariant and Fermions in Ashtekar-Barbero-Immirzi Formalism*, in *Recent developments in theoretical and experimental general relativity, gravitation and relativistic field theories. Proceedings, 11th Marcel Grossmann Meeting, MG11, Berlin, Germany, July 23-29, 2006. Pt. A-C*, pp. 2794–2796, 2006, gr-qc/0610026.

[141] S. Mercuri and V. Taveras, *Interaction of the Barbero-Immirzi Field with Matter and Pseudo-Scalar Perturbations*, *Phys. Rev.* **D80** (2009) 104007, [0903.4407].

[142] S. Mercuri, *Peccei-Quinn mechanism in gravity and the nature of the Barbero-Immirzi parameter*, Phys. Rev. Lett. **103** (2009) 081302, [0902.2764].

[143] G. Date, R. K. Kaul and S. Sengupta, *Topological Interpretation of Barbero-Immirzi Parameter*, Phys. Rev. **D79** (2009) 044008, [0811.4496].

[144] L. Freidel, D. Minic and T. Takeuchi, *Quantum gravity, torsion, parity violation and all that*, Phys. Rev. **D72** (2005) 104002, [hep-th/0507253].

[145] A. Randono, *A Note on parity violation and the Immirzi parameter*, hep-th/0510001.

[146] A. Perez and C. Rovelli, *Physical effects of the Immirzi parameter*, Phys. Rev. **D73** (2006) 044013, [gr-qc/0505081].

[147] M. Bojowald and R. Das, *Canonical gravity with fermions*, Phys. Rev. **D78** (2008) 064009, [0710.5722].

[148] M. Kazmierczak, *Einstein-Cartan gravity with Holst term and fermions*, Phys. Rev. **D79** (2009) 064029, [0812.1298].

[149] L. Castellani, R. D'Auria and P. Fre, *Supergravity and superstrings: A Geometric perspective. Vol. 1: Mathematical foundations*. 1991.

[150] V. Taveras and N. Yunes, *The Barbero-Immirzi Parameter as a Scalar Field: K-Inflation from Loop Quantum Gravity?*, Phys. Rev. **D78** (2008) 064070, [0807.2652].

[151] F. Cianfrani and G. Montani, *The Immirzi parameter from an external scalar field*, Phys. Rev. **D80** (2009) 084040, [0907.1530].

[152] F. Bombacigno, F. Cianfrani and G. Montani, *Big-Bounce cosmology in the presence of Immirzi field*, Phys. Rev. **D94** (2016) 064021, [1607.00910].

[153] A. Torres-Gomez and K. Krasnov, *Remarks on Barbero-Immirzi parameter as a field*, Phys. Rev. **D79** (2009) 104014, [0811.1998].

[154] J. L. Espiro and Y. Vasquez, *Some cosmological models coming from gravitational theories having torsional degrees of freedom*, Gen. Rel. Grav. **48** (2016) 117, [1410.3152].

[155] O. Castillo-Felisola, C. Corral, S. Kovalenko, I. Schmidt and V. E. Lyubovitskij, *Axions in gravity with torsion*, Phys. Rev. D **91** (2015) 085017, [1502.03694].

[156] A. Cisterna, C. Corral and S. del Pino, *Static and rotating black strings in dynamical Chern–Simons modified gravity*, Eur. Phys. J. C **79** (2019) 400, [1809.02903].

[157] R. Capovilla, *Nonminimally coupled scalar field and Ashtekar variables*, Phys. Rev. **D46** (1992) 1450–1452, [gr-qc/9207001].

[158] M. Montesinos, H. A. Morales-Tecotl, L. F. Urrutia and J. D. Vergara, *Real sector of the nonminimally coupled scalar field to selfdual gravity*, J. Math. Phys. **40** (1999) 1504–1517, [gr-qc/9903043].

[159] G. Montani and F. Cianfrani, *Matter in Loop Quantum Gravity without time gauge: a non-minimally coupled scalar field*, Phys. Rev. D **80** (2009) 084045, [0904.4435].

[160] M. Bojowald and M. Kagan, *Loop cosmological implications of a non-minimally coupled scalar field*, Phys. Rev. **D74** (2006) 044033, [gr-qc/0606082].

[161] M. Artymowski, Y. Ma and X. Zhang, *Comparison between Jordan and Einstein frames of Brans-Dicke gravity a la loop quantum cosmology*, Phys. Rev. **D88** (2013) 104010, [1309.3045].

[162] M. Artymowski, A. Dapor and T. Pawlowski, *Inflation from non-minimally coupled scalar field in loop quantum cosmology*, JCAP **1306** (2013) 010, [1207.4353].

[163] A. Ashtekar, A. Corichi and D. Sudarsky, *Nonminimally coupled scalar fields and isolated horizons*, Class. Quant. Grav. **20** (2003) 3413–3426, [gr-qc/0305044].

[164] A. Ashtekar and A. Corichi, *Nonminimal couplings, quantum geometry and black hole entropy*, *Class. Quant. Grav.* **20** (2003) 4473–4484, [gr-qc/0305082].

[165] J. J. W. York, *Role of conformal three geometry in the dynamics of gravitation*, *Phys. Rev. Lett.* **28** (1972) 1082–1085.

[166] G. W. Gibbons and S. W. Hawking, *Action Integrals and Partition Functions in Quantum Gravity*, *Phys. Rev.* **D15** (1977) 2752–2756.

[167] M. Atkins and X. Calmet, *Bounds on the Nonminimal Coupling of the Higgs Boson to Gravity*, *Phys. Rev. Lett.* **110** (2013) 051301, [1211.0281].

[168] J. R. Espinosa, G. F. Giudice, E. Morgante, A. Riotto, L. Senatore, A. Strumia et al., *The cosmological Higgstory of the vacuum instability*, *JHEP* **09** (2015) 174, [1505.04825].

[169] J. R. Espinosa, *Implications of the top (and Higgs) mass for vacuum stability*, *PoS* **TOP2015** (2016) 043, [1512.01222].

[170] G. Iacobellis and I. Masina, *Stationary configurations of the Standard Model Higgs potential: electroweak stability and rising inflection point*, *Phys. Rev.* **D94** (2016) 073005, [1604.06046].

[171] J. R. Espinosa, M. Garny, T. Konstandin and A. Riotto, *Gauge-Independent Scales Related to the Standard Model Vacuum Instability*, *Phys. Rev.* **D95** (2017) 056004, [1608.06765].

[172] A. H. Hoang, *What is the Top Quark Mass?*, 2004.12915.

[173] Planck collaboration, Y. Akrami et al., *Planck 2018 results. X. Constraints on inflation*, 1807.06211.

[174] F. Bezrukov, D. Gorbunov and M. Shaposhnikov, *On initial conditions for the Hot Big Bang*, *JCAP* **0906** (2009) 029, [0812.3622].

[175] J. Garcia-Bellido, D. G. Figueroa and J. Rubio, *Preheating in the Standard Model with the Higgs-Inflaton coupled to gravity*, *Phys. Rev.* **D79** (2009) 063531, [0812.4624].

[176] D. G. Figueroa, *Preheating the Universe from the Standard Model Higgs*, *AIP Conf. Proc.* **1241** (2010) 578–587, [0911.1465].

[177] D. G. Figueroa, J. Garcia-Bellido and F. Torrenti, *Decay of the standard model Higgs field after inflation*, *Phys. Rev.* **D92** (2015) 083511, [1504.04600].

[178] J. Repond and J. Rubio, *Combined Preheating on the lattice with applications to Higgs inflation*, *JCAP* **1607** (2016) 043, [1604.08238].

[179] Y. Ema, R. Jinno, K. Mukaida and K. Nakayama, *Violent Preheating in Inflation with Nonminimal Coupling*, *JCAP* **1702** (2017) 045, [1609.05209].

[180] M. P. DeCross, D. I. Kaiser, A. Prabhu, C. Prescod-Weinstein and E. I. Sfakianakis, *Preheating after multifield inflation with nonminimal couplings, III: Dynamical spacetime results*, *Phys. Rev.* **D97** (2018) 023528, [1610.08916].

[181] E. I. Sfakianakis and J. van de Vis, *Preheating after Higgs Inflation: Self-Resonance and Gauge boson production*, *Phys. Rev.* **D99** (2019) 083519, [1810.01304].

[182] Y. Hamada, K. Kawana and A. Scherlis, *On Preheating in Higgs Inflation*, 2007.04701.

[183] S. Ferrara, R. Kallosh, A. Linde and M. Porrati, *Minimal Supergravity Models of Inflation*, *Phys. Rev.* **D88** (2013) 085038, [1307.7696].

[184] M. Galante, R. Kallosh, A. Linde and D. Roest, *Unity of Cosmological Inflation Attractors*, *Phys. Rev. Lett.* **114** (2015) 141302, [1412.3797].

[185] G. K. Karananas, M. Michel and J. Rubio, *One residue to rule them all: Electroweak symmetry breaking, inflation and field-space geometry*, 2006.11290.

[186] K. Allison, *Higgs xi-inflation for the 125-126 GeV Higgs: a two-loop analysis*, *JHEP* **02** (2014) 040, [1306.6931].

[187] F. Bezrukov and M. Shaposhnikov, *Higgs inflation at the critical point*, *Phys. Lett.* **B734** (2014) 249–254, [1403.6078].

[188] Y. Hamada, H. Kawai, K.-y. Oda and S. C. Park, *Higgs Inflation is Still Alive after the Results from BICEP2*, *Phys. Rev. Lett.* **112** (2014) 241301, [1403.5043].

[189] F. Bezrukov, J. Rubio and M. Shaposhnikov, *Living beyond the edge: Higgs inflation and vacuum metastability*, *Phys. Rev.* **D92** (2015) 083512, [1412.3811].

[190] J. Rubio, *Higgs inflation and vacuum stability*, *J. Phys. Conf. Ser.* **631** (2015) 012032, [1502.07952].

[191] J. Fumagalli and M. Postma, *UV (in)sensitivity of Higgs inflation*, *JHEP* **05** (2016) 049, [1602.07234].

[192] V.-M. Enckell, K. Enqvist and S. Nurmi, *Observational signatures of Higgs inflation*, *JCAP* **1607** (2016) 047, [1603.07572].

[193] F. Bezrukov, M. Pauly and J. Rubio, *On the robustness of the primordial power spectrum in renormalized Higgs inflation*, 1706.05007.

[194] I. Masina, *Ruling out Critical Higgs Inflation?*, *Phys. Rev. D* **98** (2018) 043536, [1805.02160].

[195] J. M. Ezquiaga, J. Garcia-Bellido and E. Ruiz Morales, *Primordial Black Hole production in Critical Higgs Inflation*, *Phys. Lett.* **B776** (2018) 345–349, [1705.04861].

[196] G. Isidori, V. S. Rychkov, A. Strumia and N. Tetradis, *Gravitational corrections to standard model vacuum decay*, *Phys. Rev.* **D77** (2008) 025034, [0712.0242].

[197] Y. Hamada, H. Kawai and K.-y. Oda, *Minimal Higgs inflation*, *PTEP* **2014** (2014) 023B02, [1308.6651].

[198] M. Fairbairn, P. Grothaus and R. Hogan, *The Problem with False Vacuum Higgs Inflation*, *JCAP* **1406** (2014) 039, [1403.7483].

[199] Simons Observatory collaboration, M. H. Abitbol et al., *The Simons Observatory: Astro2020 Decadal Project Whitepaper*, *Bull. Am. Astron. Soc.* **51** (2019) 147, [1907.08284].

[200] H. Sugai et al., *Updated Design of the CMB Polarization Experiment Satellite LiteBIRD*, *J. Low Temp. Phys.* (2020) , [2001.01724].

[201] K. Abazajian et al., *CMB-S4 Decadal Survey APC White Paper*, *Bull. Am. Astron. Soc.* **51** (2019) 209, [1908.01062].