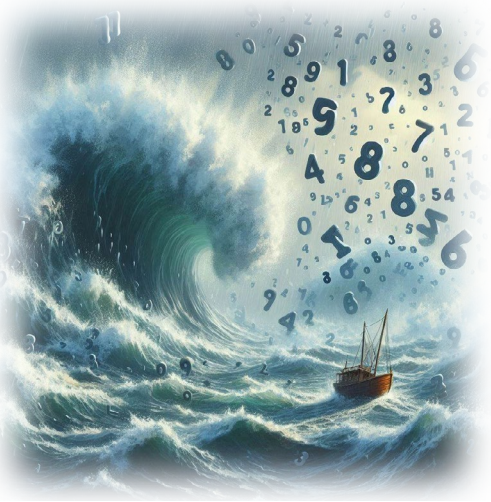
Supported by the Heilbronn institute, Katok Chair and the London Mathematical Society

Dynamical Systems Number Theory and Quantum Chaos

New connections and directions

In recent years many deep connections have been found between Dynamical Systems, Number Theory and Quantum Chaos. The goal of this workshop is to further encourage connections and to cultivate relations between researchers working in these three distinct areas of mathematics.



10 — 13 September 2024 University of Manchester

MANCHESTER 1824 The University of Manchester

Heilbronn Institute for Mathematical Research

LONDON MATHEMATICAL SOCIETY

Contents

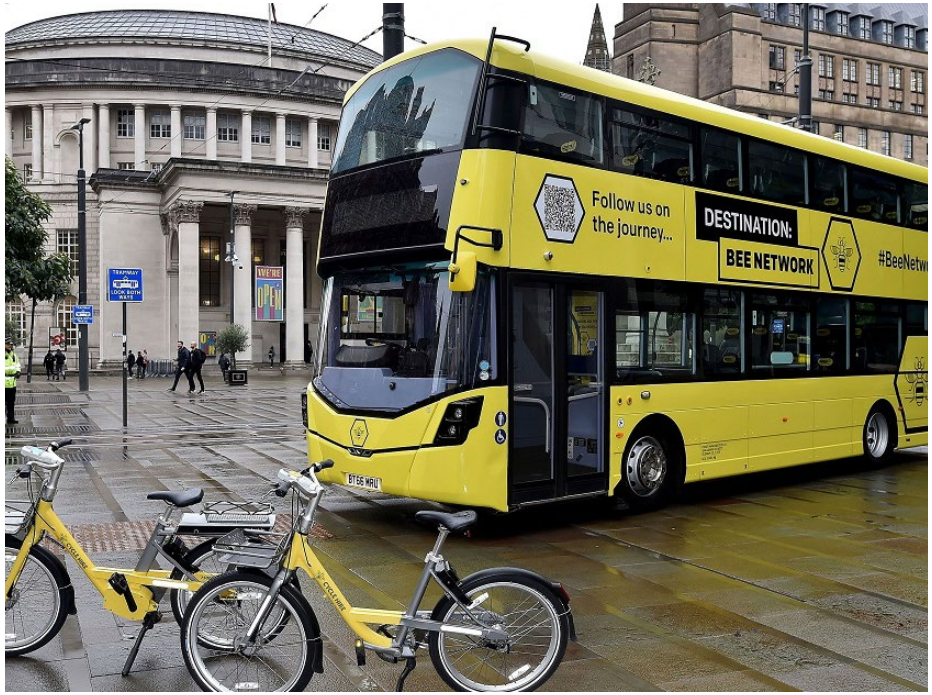
WORKSHOP VENUE

All the talks and refreshments will take place in the Frank Adams Rooms 1 and 2 on the first floor of the Alan Turing Building (the rooms are next to the Atrium Bridge, the area with the blue and green chairs in the picture above) in the Department of Mathematics, University of Manchester, Oxford Road, Manchester, M13 9PY, UK.

TRAVEL TO MANCHESTER

If you are flying to Manchester Airport, you can take a local train to the city center. Both Manchester Piccadilly and Manchester Oxford Road stations are within walking distance of the Alan Turing building. A tram is also possible but it is quite a bit slower. Within Manchester, there are free buses that run in the city center, and a tram that can take you to outside the city.

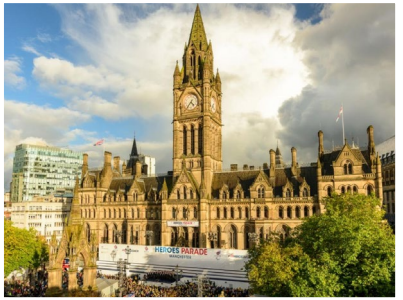
The University campus, where the Alan Turing building is located, is a short walk from the city center but you can also take various buses, e.g. the magic bus numbers 142, 143 and 147 that run along the Princess Street regularly. Please see the guide from the University of Manchester website to get around the campus and maps.

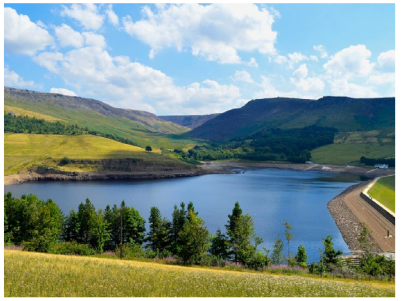


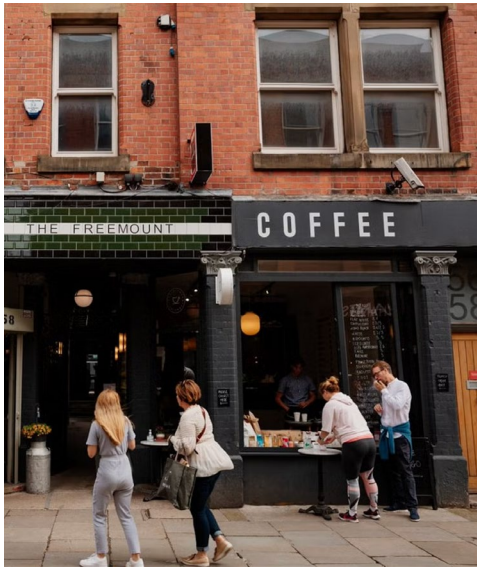
WHAT TO DO IN MANCHESTER?

Some recommendations include a walk near the Castlefield canal area from Deansgate and visiting The Wharf pub there, China Town has good restaurant options. You can also walk around the Northern Quarter area for small shops and restaurants. If are looking for seeing some nature, a recommendation include to take around 20-30min train to Greenfield in Saddleworth to do a hike in the beautiful hills near the Dovestone reservoir and the nearby village of Uppermill.

SEE THE OFFICIAL LOCAL GUIDE FOR MORE THINGS TO DO IN MANCHESTER!







FOOD & DRINK

During the workshop we will offer some refreshments during the coffee breaks. Below is a selected list of lunch and dinner options.

LUNCH

Greenhouse Cafe Wilton St, M13 9PL. Sandwiches, jacket potatoes.

Pret A Manger 34 Oxford St, M1 5EL. Coffee, sandwiches, salads. Limited seating.

Bold Street Cafe 53 Cross St, M2 4JN. Somewhat expensive, some seating. Excellent food and coffee.

Zaytoni (take-away) 127A Oxford Rd, M1 7DY. Falafel shawarma.

Hello Oriental Unit 3B, South Pavilion, 2 Symphony Park, M1 7FS. Food hall, a range of food options.

Brewdog 144 Oxford Rd, M13 9GP. Pub food.

Eighth Day Cafe 111 Oxford Rd, M1 7DU. Vegan food and snacks.

Umami F Tresco House, Oxford Rd, M1 7EE. Japanese food.

There are also several other lunch options in the campus, see from here for example: https://www.foodoncampus.manchester.ac.uk

DINNER

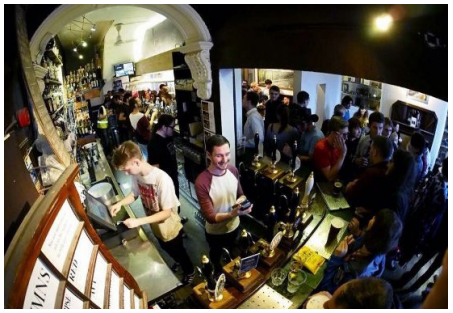
Danish food (and beer)
KroBar 325 Oxford Rd, M13 9PG

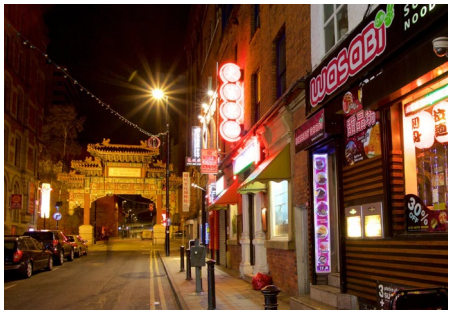
Pizza (and beer)
Sandbar 120 Grosvenor St, M1 7HL

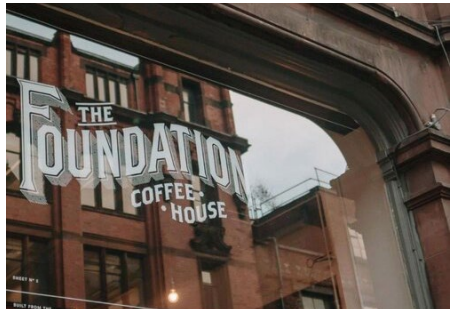
Meat/Veggie pie (and beer)
Great North Pie Co
Aytoun St, M1 3GL (further away, closer to hotels and train station)

The Little Yang Sing:
Chinese and dim sum.

Indian Tiffin Room:
Indian street food.







COFFEE SHOPS

Takk 6 Tariff St
Cool, Icelandic-inspired coffee shop

Mancoco Coffee Bar 85 Hewitt St
Coffee bar & cafe inside a roastery

The Foundation Coffee House
48-50 Whitworth St.
Funky & relaxed coffee house/cafe

Federal Cafe Bar Unit B2 - 2, Circle Square, Oxford Rd, M1 7FS
Coffees, brunches, cakes & cocktails

Moreover Manchester has various food venues to offer (e.g. in the China Town, The Northern Quarter, Spinningfield), see below for a list of various options: https://www.visitmanchester.com/food-and-drink/restaurants/

TUES 10/9

CONFERENCE DAY 1:

9:30-10:30	SANJU VELANI
10:30-11:30	Coffee break
11:30 - 12:30	OSAMA KHALIL
12:30 - 2:30	Lunch
2:30 - 3:30	BRIAN WINN
3:30 - 4:30	Coffee break
4:30 - 5:30	ALEX COHEN

TUES 10/9

DAY 1 SPEAKERS:

SHRINKING TARGETS AND RECURRENCE

- Sanju Velani (University of York)

Abstract Let (X, d) be a compact metric space and (X, A, μ, T) be a probability measure preserving system. Furthermore, given a real, positive function $\psi : N \to R_{\geq 0}$ let $R(\psi) := \{x \in X : d(Tnx,x) < \psi(n)$ for infinitely many $n \in N\}$ denote the associated recurrent set, and given a point $x0 \in X$ let $W(\psi) ::= \{x \in X : d(Tnx,x0) < \psi(n)$ for infinitely many $n \in N\}$ denote the associated shrinking target set. Under certain mixing properties it known that if $Pn \in N\ \psi(n)$ diverges then both the recurrent and shrinking target sets are of full μ-measure.

The purpose of this talk is discuss the potential quantitative strengthening of these full measure statements. This is ongoing work with Bing Li (SCUT) and David Simmons (York)

FLATTENING, MIXING, AND FOURIER DECAY

- Osama Khalil (University of Illinois Chicago)

Abstract: We will discuss a result in additive combinatorics asserting that probability measures on Euclidean space enjoy polynomial Fourier decay outside of a very sparse set of frequencies, unless large subsets of their supports concentrate near proper linear subspaces at many scales.

The talk will focus on dynamical consequences of this statement towards mixing of geodesic flows, and rates of Fourier decay of large classes of dynamically defined measures. Parts of this work are joint with Simon Baker and Tuomas Sahlsten.

QUANTUM CONTINUED FRACTIONS

- Brian Winn (Loughborough University)

Abstract: In order to define a quantisation of rational and real numbers which has interesting combinatorial properties, Morier-Genoud and Ovsienko formulated a procedure based on the regular continued fraction expansion. We review this work and describe some of the main properties of this quantisation. Then we describe some new work based on classifying which numbers have quantisations with largest radius of convergence. This is joint work with S. Evans and A. P. Veselov.

FRACTAL UNCERTAINTY IN HIGHER DIMENSIONS

- Alex Cohen (MIT)

Abstract: A fractal uncertainty principle (FUP) roughly says that a function and its Fourier transform cannot both be concentrated on a fractal set. These were introduced to harmonic analysis in order to prove new results in quantum chaos: if eigenfunctions on hyperbolic manifolds concentrated in unexpected ways, that would contradict the FUP. Bourgain and Dyatlov proved FUP over the real numbers, and in this talk I will discuss an extension to higher dimensions. The bulk of the work is constructing certain plurisubharmonic functions on C^n.

WED 11/9

CONFERENCE DAY 2

9:30-10:30	ZEEV RUDNICK
10:30-11:30	Coffee break
11:30 - 12:30	NATALIA JURGA
12:30 - 2:30	Lunch

Afternoon online session broadcasted in the Frank Adams room (Zoom link will also be provided)

2:30 - 3:15	ANKE POHL
3:15 - 4:00	Discussion and coffee break Zoom link stays open
4:00 - 4:45	MALABIKA PRAMANIK
4:45 - 5:00	Discussion, Zoom link stays open
5:00 - 5:45	SEMYON DYATLOV
5:45 - 6:00	Discussion, Zoom link stays open
6:00 -	Dinner/pub at Sandbar

WED 11/9

DAY 2 SPEAKERS:

SPECTRAL STATISTICS FOR HYPERBOLIC SURFACES OF LARGE GENUS

- Zeev Rudnick (Tel Aviv University)

Abstract: An outstanding conjecture in quantum chaos is that the statistics of the energy levels of ”generic” chaotic systems with time reversal symmetry are described by those of the Gaussian Orthogonal Ensemble (GOE) in Random Matrix Theory. Conjectural examples are the eigenvalues of the Laplacian on a ”generic” hyperbolic surface. This conjecture has proved to be extremely difficult, with no single case being proved, the closest case being some results for the Riemann zeros which seem to have similar statistics, those of the Gaussian Unitary Ensemble.

It has long been desired to improve the situation by averaging over a suitable ensemble of chaotic systems. I will describe a version of such ensemble averaging on the moduli space of compact hyperbolic surfaces, equipped with the Weil-Petersson measure, using the pioneering work of Maryam Mirzakhani. For a suitable quantity, we obtain confirmation of GOE statistics.

STOPPING TIMES IN DYNAMICS

- Natalia Jurga (University of St Andrews)

Abstract: Stopping times play an important role in areas of probability such as random walks on graphs and Markov chains theory. Important examples of stopping times include the hitting time (which records the first time to hit a particular part of the state space), cover time (which records the first time to have visited every state in the space) and blanket time (which records the first time that the empirical distribution of the random walk is, in a specific sense, close to the stationary distribution).

In this talk we will translate these concepts into the language of measure preserving dynamical systems, explore the relationships between them and discuss how they relate to the geometric measure theory properties of the dynamical system.

DIVISOR OF THE SELBERG ZETA FUNCTION WITH UNITARY REPRESENTATIONS

- Anke Pohl (University of Bremen)

Abstract: The classical Selberg zeta function is a mediator between spectral entities and dynamical entities of hyperbolic surfaces, as it is defined by means of the geodesic length spectrum and encodes in its zeros the spectral parameters of the Laplacian of the considered hyperbolic surface. We will consider the Selberg zeta function of infinite-area, geometrically finite hyperbolic orbisurfaces with twists by finite-dimensional unitary representations and hence for vector-valued situations.

We will present a factorization formula in terms of the Weierstrass product of the Laplace resonances, Barnes G-functions, gamma functions and the singularity degrees of the representation. Similar to the classical, untwisted case, this provides a spectral and geometric interpretation of the zeros and poles of the Selberg zeta function, but this time by spectral and geometric entities of the orbisurface and by the representation. We will see that this factorization formula generalizes the factorization result by Borthwick, Judge and Perry to hyperbolic orbisurfaces with orbifold singularities as well as to unitary twists.

We will further see that the presence of orbifold singularities yields a separate, previously unobserved contribution to the factorization formula, even in the untwisted case. This is joint work with Moritz Doll.

NUMBERS - ARE THEY NORMAL?

- Malabika Pramanik (The University of British Columbia)

Abstract: They say the only normal people are the ones you don't know very well. What about numbers? Which ones are normal, and how well do we know them? The notion of mathematical normality is related to the occurrence of different digits in a number. Roughly speaking, a normal number is one in which every block of digits appears with the same limiting frequency. For example, 0.12345678910111213.. is normal in base 10, but 0.1212121212... is not. Normality of numbers is connected to many areas of mathematics, like diophantine approximation, ergodic theory, geometric measure theory, analysis, fractal geometry and computer science.

We will discuss a few open problems about normal numbers that lie at the intersection of harmonic analysis and measure theory, and mention some recent progress on them.

CONTROL OF EIGENFUNCTIONS IN HIGHER DIMENSIONS

- Semyon Dyatlov (MIT)

Abstract: Semiclassical measures are a standard object studied in quantum chaos, capturing macroscopic behavior of sequences of eigenfunctions in the high energy limit. In previous work with Jin and Nonnenmacher we showed that for Laplacian eigenfunctions on negatively curved surfaces, semiclassical measures have full support. This was restricted to dimension 2 because the key new ingredient, the fractal uncertainty principle (proved by Bourgain and the speaker), was only known for subsets of the real line.

I will present several recent results on the support of semiclassical measures in higher dimensions, both on manifolds and in the toy model of quantum cat maps, contained in joint work with Jézéquel, joint work with Athreya and Miller, and work in progress by Kim. Some of these use the higher dimensional fractal uncertainty principle recently proved by Cohen. Others rely on separating the stable/unstable directions into fast and slow directions, and only applying the fractal uncertainty principle in the fast directions.

THURS 12/9

CONFERENCE DAY 3

9:30-10:30	VICTOR BERESNEVICH
10:30-11:30	Coffee break
11:30 - 12:30	GAÉTAN LECLERC
12:30 - 2:30	Lunch
2:30 - 3:30	NATTALIE TAMAM
3:30 - 4:30	Coffee break
4:30 - 5:30	LAURITZ STRECK

THURS 12/9

DAY 3 SPEAKERS:

KHINTCHINE'S THEOREM AND MANIFOLDS

- Victor Beresnevich (University of York)

Abstract: Khintchine's Theorem (1924/26) on rational approximations to real points is one of the most beautiful applications of the Borel-Cantelli lemma. In this talk I will discuss generalisations of this classical result to Diophantine approximation on smooth manifolds. The talk will include an overview of known results and problems as well as recent results obtained in a joint work with Shreyasi Datta.

FOURIER DECAY OF EQUILIBRIUM STATES ON 2D BASIC SETS

- Gaétan Leclerc (University of Helsinki)

Abstract: Consider a smooth, area-preserving, Axiom A diffeomorphism on a surface. Fix some equilibrium state on one of its basic sets. It is known that the Fourier decay properties of such equilibrium states are linked with the mixing properties of some suspension flow (namely, the one we get when choosing the roof function to be the geometric potential).

Exponential mixing for this flow can be reduced to finding enough oscillations of the associated "temporal distance function" (this is the usual mechanism behind Dolgopyat's estimates). The goal of this talk is to briefly explain how to find such oscillations in a setting where this temporal distance function is not smooth, following ideas from a recent paper by Tsujii-Zhang (Annals of Maths, 2023).

MARKOFF TRIPLES AND THEIR CONNECTION TO ORBITS OF THE MAPPING CLASS GROUP

- Nattalie Tamam (Imperial College London)

Abstract: The Markoff triples are the integer solutions of the Markoff relation $x^2+y^2+z^2=3xyz$. They were first studied by Markoff who showed that the Vieta transformations act transitively on the non-zero triples. This orbit can be represented as an orbit of a certain mapping class group. We will discuss the possible orbits closures of the general case; the algebraic relations that are preserved under the action, the obstructions to `big' orbits, and the exceptional cases. This is a joint work with Alireza Salehi-Golsefidy.

FOURIER DECAY FOR INHOMOGENEOUS SELF-SIMILAR MEASURES

- Lauritz Streck (University of Edinburgh)

Abstract: In this talk, we calculate the precise speed of the Fourier transform for certain inhomogeneous self-similar measures. An inhonogeneous self-similar measure is defined by $\mu=\sum_{i=1}^m p_i f_i(\mu)$, where p_i are probability weights, $f_i=\lambda_i x+t_i$ with $\lambda_i \in (0,1)$ and $t_i \in \R$ are contracting similarities, and not all λ_i are the same. Similarly to the homogenous case, it is known that for almost all λ_i, the Fourier transform of μ has power decay: That is, $|\widehat{\mu}(\xi)| \lesssim |\xi|^{-\alpha}$ for some $\alpha>0$ (Solomyak).

Moreover, unless all λ_i are powers of the same number, $\widehat{\mu}(\xi) \to 0$, and in many cases, like for the similarities $1/2 x, 1/3 x+1$, one can show polylog-decay: $|\widehat{\mu}(\xi)| \lesssim (\log |\xi|)^{-\beta}$ for some $\beta>0$ (Li-Sahlsten). However, contrary to the homogenous setting, there are essentially no concrete inhomogeneous self-similar μ for which the speed of the Fourier decay is known.

We argue that for μ defined by similarities like $½ x, 1/3 x+1$, one should expect polylog-decay to be the true answer, and reduce checking showing it to explicit computation. We discuss how these computations are performed and give examples of inhomogeneous measures with polylog-decay. Moreover, we also give examples of inhomogeneous self-similar measures (with the contractions either rational or powers of the same Garsia number) for which power Fourier decay holds. Joint work with S. Paukkonen and T. Sahlsten.

FRIDAY 13/9

CONFERENCE DAY 4

9:30-10:30	JIALUN LI
10:30-11:30	Coffee break
11:30 - 12:30	RAM BAND
12:30 - 4:00	Lunch and discussions with the participants who remain

End of the workshop

FRIDAY 13/9

DAY 4 SPEAKERS

DENSITY OF SHAPES OF PERIODIC TORI IN THE CUBIC CASE - *Jialun Li (CNRS École Polytechnique)*

Abstract: Given a compact orbit of diagonal action on $\SL(3,\R)/\SL (3,\Z)$, the set of periods of the orbit forms a lattice in the diagonal group, which can be identified with $\Z^2$. We refer to this lattice as the shape of the compact orbit, which can be identified as a point in $\SL (2,\R)/\SL(2,\Z)$ after re-scaling to covolume one. We prove that in $\SL(3,\R)/\SL (3,\Z)$ the shapes of periodic tori are dense in $\SL (2,\R)/\SL(2,\Z)$. The dense family of shapes are constructed explicitly from a family of cubic orders and their suborders. The talk is based on an ongoing joint work with Thi Dang and Nihar Gargava.

THE DRY TEN MARTINI PROBLEM FOR STURMIAN SCHRÖDINGER OPERATORS - *Ram Band (Technion)*

Abstract: “Are all gaps there?”, asked Mark Kac in 1981 during a talk at the AMS annual meeting, and offered ten Martinis for the answer. This led Barry Simon to coin the names the Ten Martini Problem (TMP) and the Dry Ten Martini Problem for two related problems concerning the Almost-Mathieu operator. The TMP is about showing that the spectrum of the Almost-Mathieu operator is a Cantor set. Dry TMP is about the values that the integrated density of states (IDS) attains at the spectral gaps. The gap labelling theorem predicts the possible set of values which the IDS may attain at the spectral gaps. The Dry TMP is whether or not all these values are attained, or equivalently, “are all gaps there?”. We present an affirmative solution to the Dry Ten Martini Problem for Sturmian Hamiltonians. Concretely, it is proved that all spectral gaps are there for Schrödinger operators with Sturmian potentials and non-vanishing coupling constant. The talk is based on a joint work with Siegfried Beckus and Raphael Loewy.

ORGANISERS:

Amir Algom, The University of Haifa
email: amir.algom@math.haifa.ac.il

Simon Baker, Loughborough University
email: simonbaker412@gmail.com

Thomas Jordan, University of Bristol
email: thomas.jordan@bristol.ac.uk

Tuomas Sahlsten, University of Helsinki
email: tuomas.sahlsten@helsinki.fi

Roman Schubert, University of Bristol
email: roman.schubert@bristol.ac.uk

Yotam Smilansky, University of Manchester
email: yotam.smilansky @manchester.ac.uk

CONTACT EMAIL:
DSNTQC-workshop2024@manchester.ac.uk

MANCHESTER 1824
The University of Manchester

Heilbronn Institute for Mathematical Research

LONDON MATHEMATICAL SOCIETY