Infrared conformality on the lattice

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- Background: walking, IRFP and all that
- Minimal walking technicolor: SU(2) with 2 adjoint rep fermions
- $O(a)$ improvement
- SU(2) with $N_f = 6$ and 10 fundamental rep. fermions

Background:

- The Standard Model (with Higgs) is phenomenologically extremely successful.
- However: The Higgs field is special:
  - *it is the linchpin of the standard model: provides the mechanism for the electroweak symmetry breaking*
  - *it has not been seen*
  - *it is a scalar*

  $\Rightarrow$ *theoretical problems at very high scales:*
  hierarchy problem, vacuum stability, unitarity bound . . .
- Most BSM models aim to ameliorate these problems by e.g.
  - pairing scalars with fermions (SUSY)
  - introducing a cutoff (extra dimensions)
  - not having scalars at all (Technicolor and many other strongly coupled BSM models)

Chiral symmetry breaking vs. Higgs mechanism

Consider the standard Electroweak symmetry breaking with Higgs and the chiral symmetry breaking (χSB) in QCD:

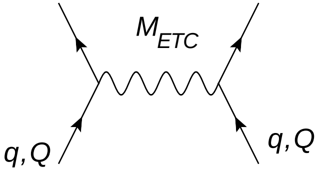
	EWSB	χSB
condensate (Breaks EW):	Higgs vev v	f_π decay constant
Goldstone bosons:	W, Z longitudinal modes	π-mesons
radial excitation:	Higgs particle	scalar meson

Technicolor

- New gauge field (technigauge) + massless fermions (techniquarks) Q.
- Techniquarks have both technicolor and EW charge (exactly like quarks in the SM)
- Chiral symmetry breaking in technicolor $\longrightarrow$ Electroweak symmetry breaking
- Scale: $\Lambda_{\rm TC} \approx \Lambda_{\rm EW}$
- After chiral symmetry breaking:
  - $\Rightarrow$ decay constant $f_{\rm TC} \leftrightarrow$ Higgs expectation value v.
  - $\Rightarrow$ scalar $\bar{Q}Q$ -meson $\leftrightarrow$ Higgs
  - $\Rightarrow$ pseudoscalars $\leftrightarrow$ W,Z -longitudinal modes
  - $\Rightarrow$ exotic technihadrons (observable!)
- Describes well the W, Z+Higgs sector (depending on the model, may have too many Goldstone bosons)
- Elegant, "proven" mechanism in the Standard Model
- Does *not* explain fermion masses (Yukawa). For that, we need additional structure $\rightarrow$ *Extended technicolor*

Extended technicolor

- In addition to the "pure" technicolor, introduce a new higher-energy interaction coupling Standard Model fermions q (quarks, leptons) and techniquarks (Q): extended technicolor (ETC)
  Several options, e.g. massive gauge boson, $M_{\rm ETC}$:



[Eichten,Lane,Holdom,Appelquist,Sannino,Luty. . .]

- $\frac{g^2_{\rm ETC}}{M^2_{\rm ETC}} \bar{Q}Q\bar{q}q \longrightarrow$ SM fermion mass $m_q \propto \frac{1}{M^2_{\rm ETC}} \langle \bar{Q}Q \rangle_{\rm ETC}$
- $\frac{g^2_{\rm ETC}}{M^2_{\rm ETC}} \bar{q}q\bar{q}q \longrightarrow$ extra FCNC's (harmful!)
- $\frac{g^2_{\rm ETC}}{M^2_{\rm ETC}} \bar{Q}Q\bar{Q}Q \longrightarrow$ explicit χSB in the techniquark sector

$\langle \bar{Q}Q \rangle_{\rm ETC}$: condensate evaluated at the ETC scale
$\langle \bar{Q}Q \rangle_{\rm TC}$: condensate at TC (EW) scale

Extended technicolor

I) $\bar{q}q\bar{q}q$ -term leads to unwanted FCNC's. In order to be compatible with precision electroweak tests, we must have $\Lambda_{\rm ETC} \approx M_{\rm ETC} \gtrsim 1000 \times \Lambda_{\rm TC} (\Lambda_{\rm TC} \approx \Lambda_{TC})$

II) For EWSB we must have $\langle \bar{Q}Q \rangle_{\rm TC} \propto \Lambda^3_{\rm TC} \approx \Lambda^3_{\rm EW}$

III) On the other hand, $\langle \bar{Q}Q \rangle_{\rm ETC} \propto m_q \Lambda^2_{\rm ETC}$ (top quark!)

- Using RG evolution

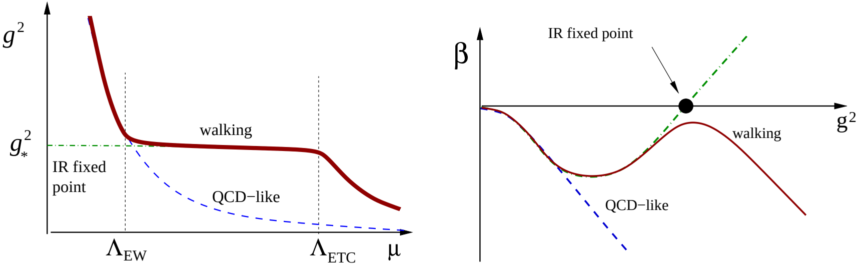
$$\langle \bar{Q}Q \rangle_{\rm ETC} = \langle \bar{Q}Q \rangle_{\rm TC} \exp\left[\int_{\Lambda_{\rm TC}}^{M_{\rm ETC}} \frac{\gamma(g^2)}{\mu} d\mu\right]$$

  where $\gamma(g^2)$ is the mass anomalous dimension.

- In weakly coupled theory $\gamma \sim 0$, and $\langle \bar{Q}Q \rangle$ is $\sim$ constant.
- *Thus, it is not possible to satisfy the constraints I), II), II) in a QCD-like theory, where the coupling is large only on a narrow energy range above χSB.*

Walking coupling

- If the coupling *walks*, i.e. if $g^2 \approx g_*^2$ (constant) over the range from TC to ETC, then $\langle \bar{Q}Q \rangle_{\rm ETC} \approx \left(\frac{\Lambda_{\rm ETC}}{\Lambda_{\rm TC}}\right)^{\gamma(g_*^2)} \langle \bar{Q}Q \rangle_{\rm TC}$ (condensate enhancement)
- $\gamma(g_*^2) \sim 1 - 2$ in order to satisfy the conditions I) – III) (depends on the details).



- In a walking theory the β-function $\beta = \mu \frac{dg}{d\mu}$ reaches almost zero near g_*^2.
- If the β-function hits zero there is an IR fixed point, where the system becomes *conformal*.

Perturbative β-function

2-loop universal β-function for SU(N_c) gauge theory with N_f fermions:

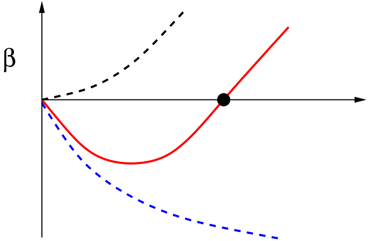
$$\beta(g) = -\mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}$$

where the coefficients are

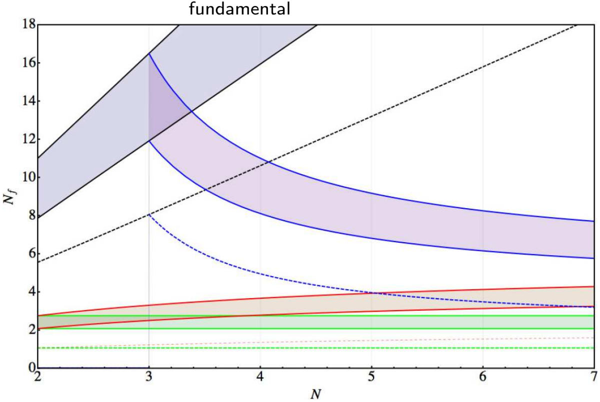
$$\beta_0 = \frac{11}{3} C_r - \frac{4}{3} T_r N_f, \qquad \beta_1 = \frac{34}{3} C_r^2 - \frac{20}{3} C_r T_r N_f - 4 C_r T_r N_f$$

When N_f is varied, generically 3 different behaviours seen:

- confinement and χSB at small N_f
- IR fixed point (conformal window) at medium N_f [Banks,Zaks]
- Asymptotic freedom lost at large N_f



Conformal window in SU(N) gauge



[Appelquist, Lane, Mahanta, Cohen, Georgi, Yamawaki, Schrock, Sannino, Tuominen, Dietrich]

2-index antisymmetric

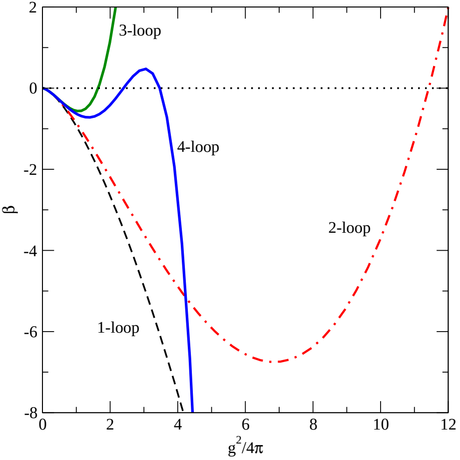
2-index symmetric

adjoint

- Upper edge of band: asymptotic freedom lost
- Lower edge of band: ladder approximation
- Walking can be found near the lower edge of the conformal window: large coupling, non-perturbative - lattice simulations needed!
- In higher reps it is easier to satisfy EW constraints [Sannino,Tuominen,Dietrich] → lot of recent activity!

Existence of the IRFP essentially non-perturbative

Example: Perturbative β-function of SU(2) gauge with $N_f = 6$ fundamental rep fermions



[4-loop MS: Ritbergen, Vermaseren, Larin]

Preliminary results from lattice: IRFP exists, $g_*^2/4\pi \sim 0.8$ [Karavirta et al]

What do we want?

Take SU(N) gauge theory with N_f fermions in some representation.

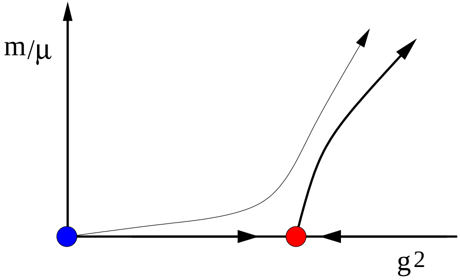
- Measure $\beta(g^2)$-function
- Measure $\gamma(g^2)$
- Classify QCD-like / walking / conformal
- We want to find a theory which
  - is walking or
  - is just within conformal window (easy to deform into walking)
  - has large anomalous exponent γ near FP
  - Compatible with EW precision measurements (S,T,U -parameters) $\rightarrow$ small N_f preferred!
- Favourite candidates: SU(2) or SU(3) gauge theory with $N_f = 2$ adjoint or 2-index symmetric representation fermions. [Sannino,Tuominen,Dietrich]
- "Hadron" spectrum, chiral symmetry breaking pattern

Models studied on the lattice

- SU(3) + $N_f = 8$–16 fundamental rep:
  - $N_f = 8$: χSB [Appelquist et al; Deuzeman et al; Fodor et al; Jin et al]
  - $N_f = 9$: χSB [Fodor et al]
  - $N_f = 10$: unclear [Yamada et al]
  - $N_f = 12$: conflicting results [Hasenfratz; Fodor et al; Appelquist et al; Deuzeman et al]
  - $N_f = 16$: conformal [Damgaard et al; Heller; Hasenfratz; Fodor et al]
- SU(2) + fundamental rep fermions:
  - $N_f = 2$: χSB [many]
  - $N_f = 4$: χSB [Karavirta et al (to be published)]
  - $N_f = 6$: conformal [Del Debbio et al, Karavirta et al (to be published)]
  - $N_f = 8$: conformal [Iwasaki et al]
  - $N_f = 10$: conformal [Karavirta et al (to be published)]
- SU(2) + $N_f = 2$ adjoint rep: (*Minimal walking technicolor*) conformal [Catterall et al; Bursa et al; Hietanen et al]
- SU(3) + $N_f = 2$ 2-index symmetric rep: unclear [de Grand et al; Sinclair and Kogut; Fodor et al]

RG flow in conformal case

- Relevant parameters at UV: g^2 and m_Q



- m_Q is relevant at the IRFP.
- Scaling near IRFP: masses of physical particles $M \propto (m_Q)^{1/(1+\gamma)}$
- New UV fixed point at stronger coupling? [Kaplan et al; Lombardo et al; Hasenfratz]

Case study:

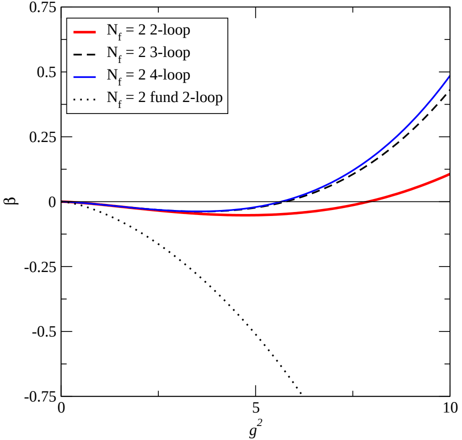
Minimal walking technicolor

Case study: Minimal walking technicolor (MWTC)

- SU(2) with $N_f = 2$ *adjoint* representation techniquarks
- Study on the lattice using (unimproved) Wilson fermions
  [Catterall, Sannino; Del Debbio, Patella, Pica; Hietanen et al, Bursa et al]
- What is studied?
  - Measure the evolution of the coupling directly using the Schrödinger functional method
  - Particle spectrum: do we observe chiral symmetry breaking (QCD) or do all modes become massless as $m_q \to 0$ (no χSB, possibly conformal)
  - Mass anomalous exponent γ (from spectrum or directly using SF methods)
  - Improvement of the lattice action

Minimal technicolor

- Perturbative β-function compared with fundamental rep: very slow evolution!



Lattice model:

- SU(2) gauge action in fundamental rep.
- massless fermions in adjoint rep.

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi} i \gamma_\mu D_\mu \psi$$

- On the lattice:
  - gauge fields U in the fundamental rep.
  - For the fermion action, these transformed into adjoint rep

    $$V^{ab} = 2\,\mathrm{Tr}[U^\dagger \lambda^a U \lambda^b]$$

    $a, b = 1, 2, 3.$
  - We use standard Wilson action (these results); now non-perturbatively O(a) improved Wilson-clover action (future results)
  - For comparison, we also do analysis with $N_f = 2$ fundamental quarks

Evolution of the coupling

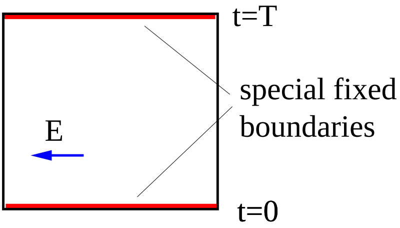
Schrödinger functional: Generate a *background* chromoelectric field using non-trivial fixed boundary conditions, parametrised by a twist angle η

At the classical level, we have

$$\frac{dS_{\text{class.}}}{d\eta} = \frac{A}{g^2}$$

where $A(\eta)$ is a known constant.
At the quantum level, we define the coupling through

$$\begin{aligned}\frac{1}{g^2} &= \frac{1}{A}\frac{dS}{d\eta} \\ &= \text{const.} \times \langle(\text{boundary plaq.})\rangle\end{aligned}$$

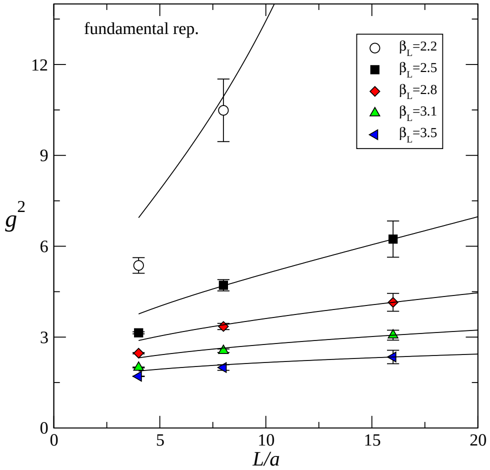


- Evaluates g^2 directly at scale $\mu = 1/L$, the lattice size
- Can use $m_Q = 0$
- Has been used very succesfully in QCD by the Alpha collaboration

Evolution of the coupling: QCD-like

Test with $N_f = 2$ **fundamental representation**, QCD-like test case:

- L/a grows, $k \sim a/L$ decreases, $g^2(L)$ increases: *asymptotic freedom*, OK!
- Large $\beta_L \rightarrow$ small lattice spacing $\rightarrow$ small volume
- Continuous line: coupling evaluated from the 2-loop β-function (integration constant fixed to measurement at $L/a = 16$)
- Not a continuum limit, but shows consistency



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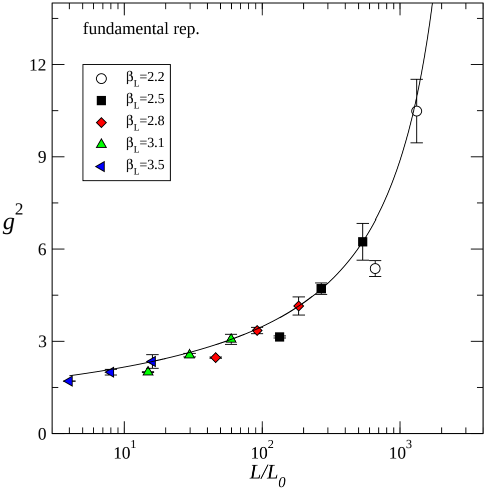
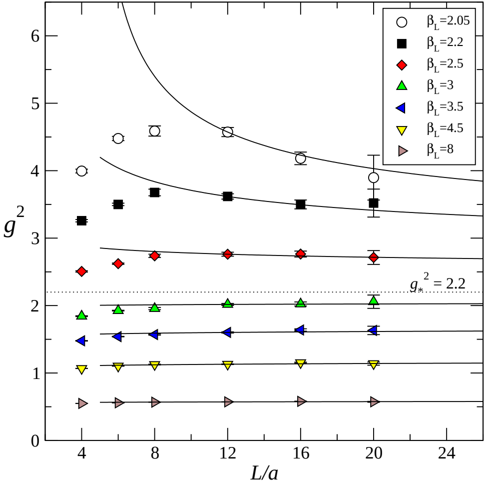


Evolution of the coupling: MWTC

In **adjoint representation**:

- At small $g^2(L)$: increases with L (asymptotic freedom)
- At large $g^2(L)$: *decreases* as L increases
  $\Rightarrow \beta$-function positive here!
- Large cutoff effects at small L/a – discard
- As $L/a \to \infty$, apparently $g^2(L) \to g_*^2 \approx 2 \ldots 3$.
  $\Rightarrow$ conformal behaviour!?
- Continuous line: coupling evaluated with fitted β-function ansatz (to be described)

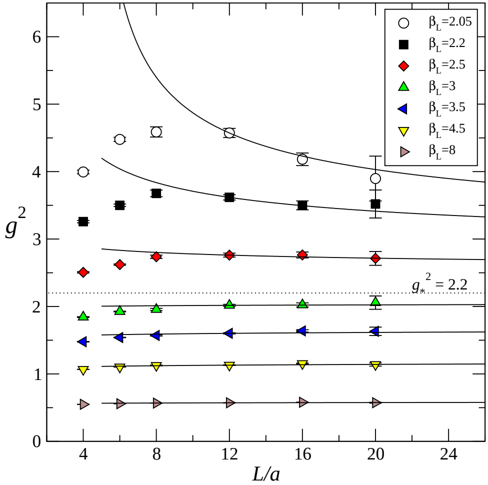


β-function

Assuming that the lattice effects on the large-volume data are small, we can describe the features of the β-function by fitting an ansatz:

$$\beta = -L\frac{dg}{dL} = -b_1 g^3 - b_2 g^5 - b_3 g^\delta$$

Here b_1, b_2 are perturbative constants and b_3 and δ are fit parameters. (Parametrising the location of the fixed point and the slope of the β-function there).
The ansatz is fitted to the data at $L/a = 12, 16, 20$:



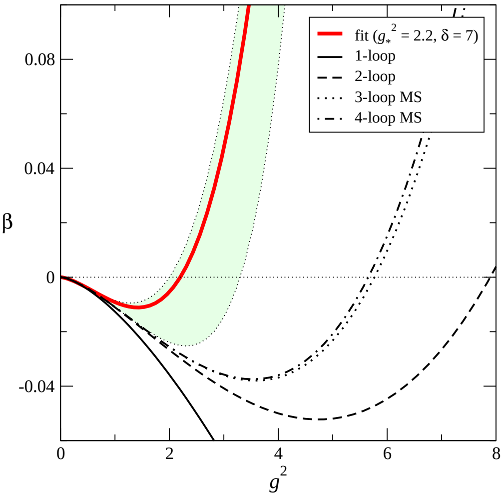
β-function

Fit result:

$$\beta = -L\frac{dg}{dL} = -b_1 g^3 - b_2 g^5 - b_3 g^\delta$$

FP is at substantially smaller coupling than indicated by 2-loop P.T.

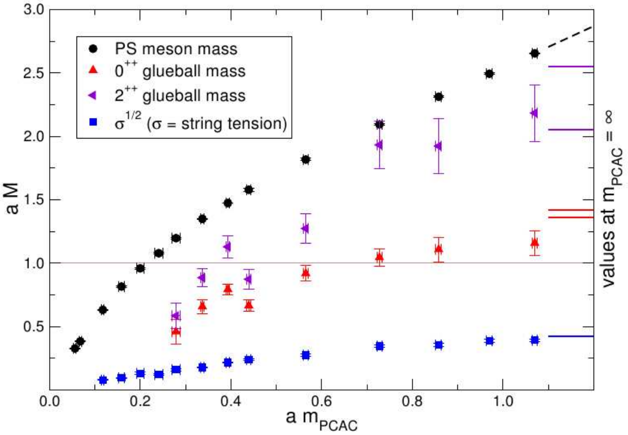
In MS-schema, β-function is known to 4-loop order: [Ritbergen, Vermaseren, Larin]
Not directly comparable to lattice (beyond 2 loops), because of different schema! But quantifies perturbative uncertainty.



Particle spectrum:

- If QCD-like χSB: as $m_Q a \to 0$,
  - $m_\pi \propto m_Q^{1/2}$
  - other states have finite mass.
- If IR fixed conformal point: when $m_Q a \to 0$, all states become massless with the same exponent.
- If walking behaviour: at high energy $\sim$ conformal, at small χSB.

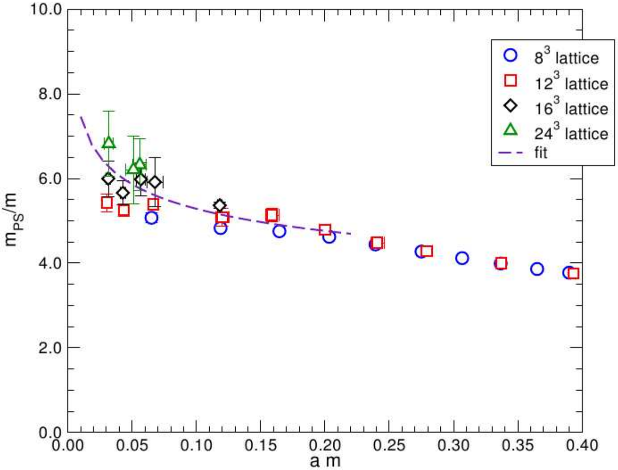
Mass spectrum at $\beta_L = 2.25$



[Del Debbio et al]

Spectrum becomes massless, inverted hierarchy when compared with QCD [Miransky]

Testing scaling of the mass: γ



$$\log m_{PS} = \frac{1}{1+\gamma} \log m + C = 0.85 \log m + C$$

The mass anomalous exponent $\gamma \sim 0.2$

γ can be measured directly using Schrödinger functional scheme, with comparable

Minimal walking technicolor

O(a) improvement

$O(a)$ improvement of the action

- Wilson fermions have large $O(a)$ cutoff-effects. These are cancelled by adding a irrelevant "clover term" with a fine-tuned coefficient $c_{\rm SW}$.
- In the Schrödinger functional scheme also boundary term improvement must be computed

Schrödinger functional scheme action

$$S_i = S_u + \delta S_V + \delta S_{G,b} + \delta S_{F,b}$$

$$\begin{aligned}
\delta S_V &= \frac{ia^5}{4} c_{sw} \sum_{x_0=a}^{L-a} \sum_{\vec{x}} \bar{\psi}(x)\sigma_{\mu\nu}\hat{F}_{\mu\nu}(x)\psi(x) \\
\delta S_{G,b} &= \frac{1}{2g_0^2}(c_s - 1) \sum_{p_s} \mathrm{Tr}[1 - U(p_s)] \\
&\quad + \frac{1}{g_0^2}(c_t - 1) \sum_{p_t} \mathrm{Tr}[1 - U(p_t)] \\
\delta S_{F,b} &= a^4(\tilde{c}_s - 1) \sum_{\vec{x}} [\hat{O}_s(\vec{x}) + \hat{O}'_s(\vec{x})] \\
&\quad + a^4(\tilde{c}_t - 1) \sum_{\vec{x}} [\hat{O}_t(\vec{x}) - \hat{O}'_t(\vec{x})]
\end{aligned}$$

Boundary terms

- The clover coefficient $c_{\rm SW}$ is determined non-perturbatively
- The boundary coefficients c_t, $\tilde{c}_t$ perturbatively
- c_s, $\tilde{c}_s$ are not needed

We obtain [Karavirta et al, for fundamental rep Lüscher, Weisz]

$$\tilde{c}_t = 1 - 0.0135(1) \times C_R g_0^2 + O(g_0^4)$$

Write $c_t = 1 + g_0^2 (c_t^{(1,0)} + N_F * c_t^{(1,1)}) + O(g_0^4)$

N_c	rep.	$c_t^{(1,0)}$	$c_t^{(1,1)}$
2	**2**	−0.0543(5)	0.0192(2)
2	**3**	−0.0543(5)	0.075(1)
3	**3**	−0.08900(5)	0.0192(4)
3	**8**	−0.08900(5)	0.113(1)
3	**6**	−0.08900(5)	0.0946(9)
4	**4**		0.0192(5)

[Sint et al, Karavirta et al]

These are in agreement with $c_t^{(1,1)} = 0.019141 \times (2T_R)$

Boundary conditions for the clover coefficient

- Matche $c_{\rm SW}$ using Schrödinger functional method to generate a background chromoelectric field and "optimizing" the fermion mass defined through axial Ward identity:

$$M(x_0) = \frac{1}{2}\frac{\frac{1}{2}(\partial_0^* + \partial_0) f_A(x_0) + c_A a \partial_0^* \partial_0 f_P(x_0)}{f_P(x_0)}$$

- However: the standard diagonal ("Abelian") boundary matrices are not quite sufficient for higher reps:

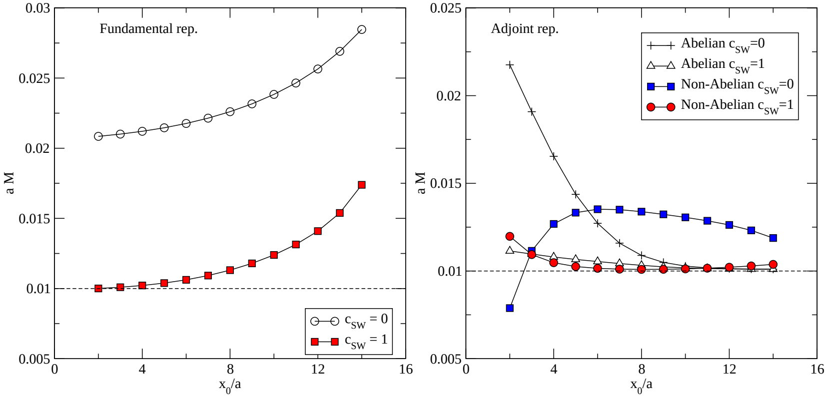
$$U = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \Rightarrow V^{ab} = 2\,\mathrm{Tr}[U^\dagger \lambda^a U \lambda^b] \Rightarrow V = \begin{pmatrix} \cdot & \cdot & 0 \\ \cdot & \cdot & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- For adjoint fermion, there is a color component which does not see the background field: problem at long distances
- We maximise the asymmetry between the boundaries using the following "non-Abelian" boundary conditions:

$$U_i(t=0) = 1, \quad U_i(t=T) = \exp[i\theta\sigma_i]$$

Boundary conditions: demonstrate at the classical level

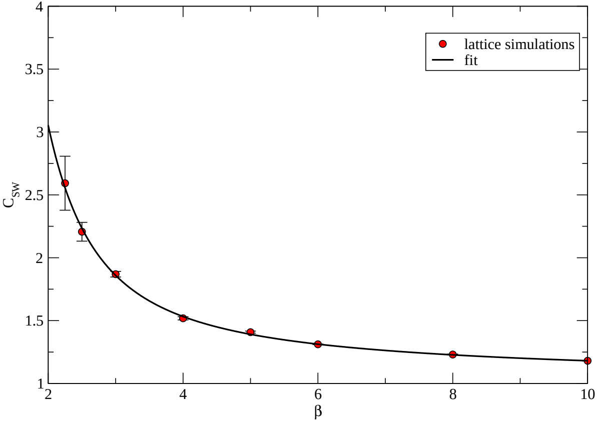
$8^3 \times 16$ lattice, $m_0 a = 0.01$: Axial Ward identity against a classical background in SU(2) fundamental and adjoint rep:



With "Abelian" boundary conditions, no lever-arm to determine the value of c_{SW}.

Clover coefficient: result

$c_{\rm SW}$ coefficient (w. adjoint fermions): [Karavirta et al]



- $c_{\rm SW}$ increases rapidly as β becomes smaller
- Cannot reach small enough β for studying IRFP?

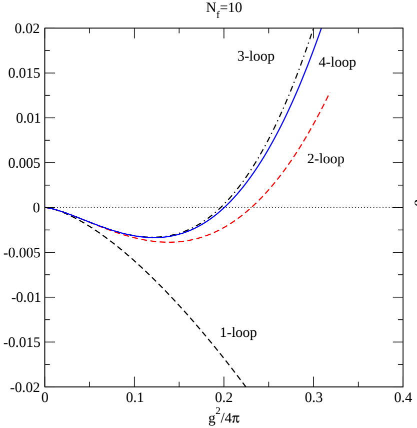
SU(2) fundamental representation at N_f =6–10

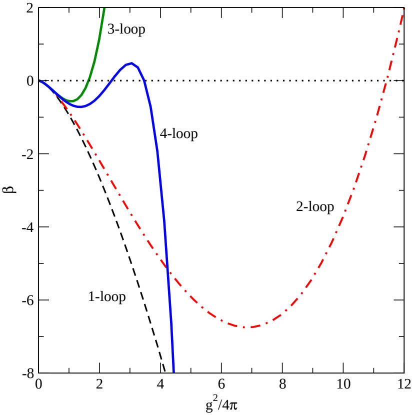
Fudamental rep SU(2) with $N_f = 6$ and 10

- Measure coupling using SF in fundamental representation SU(2)
- Choose:
  - $N_f = 6$: $\sim$ lower edge of conformal window
  - $N_f = 10$: upper edge of conformal window
- We use 1-loop perturbative c_{SW}

Fudamental rep: perturbation theory

Perturbative β-function w. $N_f = 10$ and $N_f = 6$ [3,4-loop MS: Ritbergen, Vermaseren, Larin]

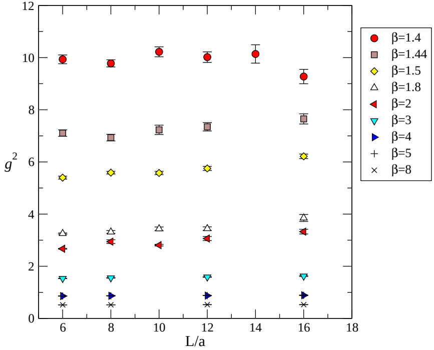


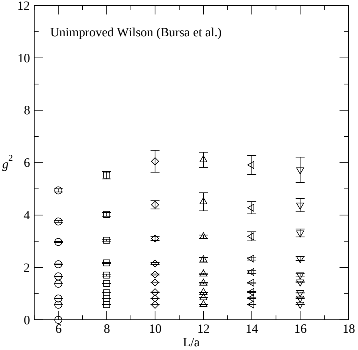


Very preliminary results: at $N_f = 10$, IRFP on the lattice at $g^2/4\pi \sim 0.2$. [Karavirta et al.]

$N_f = 6$: compare clover/Wilson

[Unimproved Wilson: Bursa et al.]

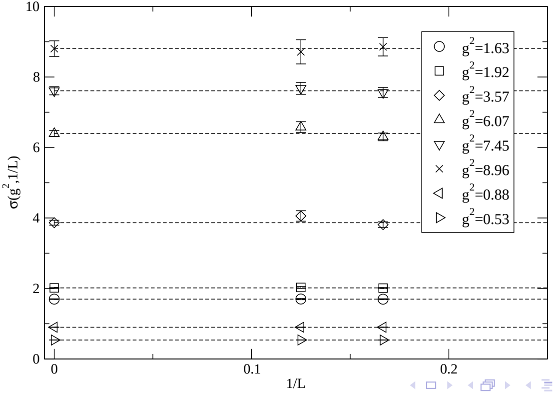




Improved data: very preliminary, under progress.
Unimproved Wilson points towards smaller value of g^2 at IRFP?

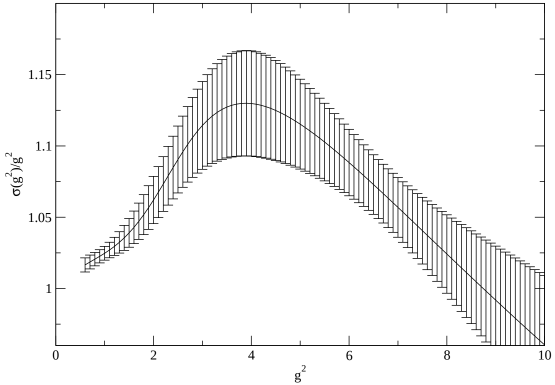
$N_f = 6$: step scaling function

- Interpolate $\frac{4}{g^2} = \beta \sum_i \frac{c_i}{\beta}$
- Construct step scaling using pairs L=(6,12) and (8,16)
- $\sigma(u, L/a) = g^2(2L/a)_{g^2(L/a)=u}$
- Use constant or linear continuum interpolation



$N_f = 6$: continuum limit of step scaling

- Scaled $\sigma(g^2)/g^2$: becomes $= 1$ at IRFP



- *Very preliminary!*
- IRFP at $g^2 \sim$8–12 : $g_*^2/4\pi \sim$0.6–0.95.

What do the results imply?

- Improvement is important for reliable results. For SU(2)+Adjoint rep fermions, the situation must be revisited with improved action.
- Fundamental rep. SU(2) appears to be under control: $N_f = 6$ is apparently within the conformal window.
- Lattice action must be reliable at large bare coupling (small cutoff effects).

$\Rightarrow$ highly improved fermions? Smeared action [HYP-improved clover]?

- Strong systematic effects have been observed e.g. in simulations of SU(3) with 2 sextet fermions. [De Grand et al.]
- *Must have very careful control of the systematics and the continuum limit*

Conclusions

Lattice technicolor and conformality:

- Lot of work has been done, signs of IRFP found in several theories.
- No clear sign of proper walking found in any (massless) theory
- Anomalous dimensions appear to be small (bad for TC)
- Methods still under development; e.g. there are several methods used to measure the evolution of the coupling. Good for cross-checking.
- Need to live at strong coupling: use an action which minimizes lattice effects there. Improvement!
- Cross-checking needed!

Solid results obtainable with present-day resources and methods:

- improved computational methods
- $\sim$ teraflops-year numerical effort

Lattice simulations can exclude models from contention!