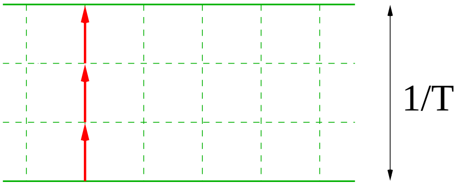
8.15. Deconfinement phase transition

- Non-zero temperature: $1/T$ =extent of the system to imaginary time, with periodic boundary conditions.

- In many gauge theories there is a *phase transition* at finite T, also in SU(3) (QCD).



Order parameter: ***Polyakov loop:***

$$P(\vec{x}) = \text{Tr} \prod_{\tau} U_0(\vec{x}, \tau)$$

- What is the symmetry which gets broken?

- Consider *center* of the gauge group, i.e. elements which commute with all elements U of the group. For SU(N), these are matrices

$$z_n = \mathbf{1} e^{in2\pi/N}, \qquad n = 0, \ldots N-1$$

which are proportional to unit matrix (and naturally belong to SU(N)).

- Take all $U_0(\tau, \vec{x})$ matrices on a fixed τ = const. plane, for example at $\tau = 0$, and multiply these with (a fixed) z_n.

  – plaquettes: constant, thus, this is a symmetry of the action.

  – Polyakov loops: $P \mapsto e^{in2\pi/N} P$; thus, P is an order parameter of the symmetry.

- There is a transition at $T = T_c$, and

- $T < T_c$ (large $L_\tau = 1/T$): $\langle P \rangle = 0$

- $T > T_c$ (small $1/T$): $\langle P \rangle \neq 0$, symmetry broken.

Physical interpretation:

- (Somewhat sloppily:) as with Wilson loop, we can interpret the Polyakov line as a static "test charge", and

$$|\langle P\rangle| \propto e^{-E\Delta\tau} = e^{-E/T}$$

– If $\langle P\rangle = 0$, $E = \infty$: at $T < T_c$ there are no free charged particles (confinement).

– If $\langle P\rangle \neq 0$, E finite: at $T > T_c$, charges are ***deconfined***.

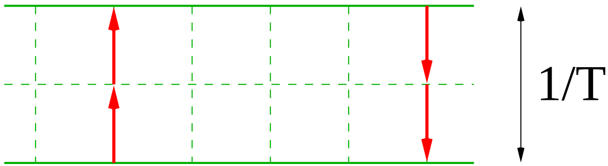
- This transition occurs in QCD (without quarks) at $T \sim 260$MeV!

- Presence of quarks in real QCD modifies the picture somewhat. The transition happens at $T_c \sim 130$ MeV, and the Polyakov line symmetry is only approximate.

$T < T_c$: quarks and gluons are bound tightly into *hadrons* (protons, pions . . .) and glueballs.

$T > T_c$: ***quark-gluon plasma phase***.

- A better way to interpret the Polyakov lines is to consider the ***correlation function*** between two lines:



- The correlation function behaves like a Wilson loop without the top- and bottom legs:

$$\langle P(\vec{x})P^\dagger(\vec{y})\rangle = e^{-V(\vec{x}-\vec{y})/T} \sim \begin{cases} e^{-\sigma R/T} & T < T_c \\ \langle|P|\rangle^2 & T > T_c \end{cases}$$

- Below T_c, the string tension σ can be measured directly from the exponential fall-off of the correlation function

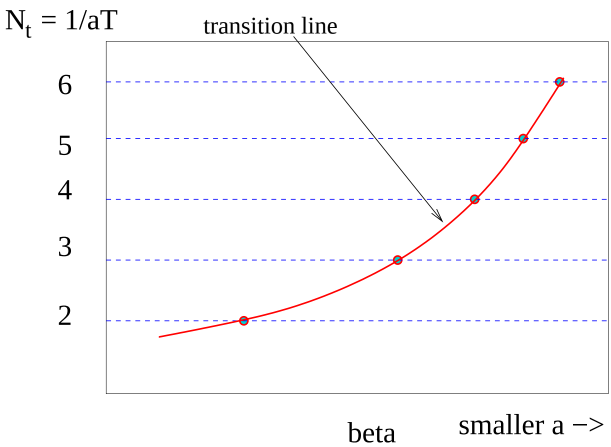
- Above T_c, the correlation function approaches a constant (it has a disconnected part). $\sigma = 0$.

- (strong coupling expansion for $\langle P(\vec{x})P^\dagger(\vec{y})\rangle$ works just like for Wilson loops.)

- Polyakov loop phase transition is easy to identify. However, it is difficult to change the temperature by changing the discrete variable N_τ. It is easier to keep N_τ fixed, and change the lattice spacing a:

$$T = 1/(N_\tau a)$$

- The lattice spacing a is changed by adjusting the coupling constant β.



- Actually, locating the transition coupling β_c at fixed N_τ often offers a good way to measure β as a function of the lattice spacing: at the transition, we know that

$$T_c = 1/(N_\tau a) \Rightarrow a = 1/(N_\tau T_c)$$

$\Rightarrow$ Discrete β-function.

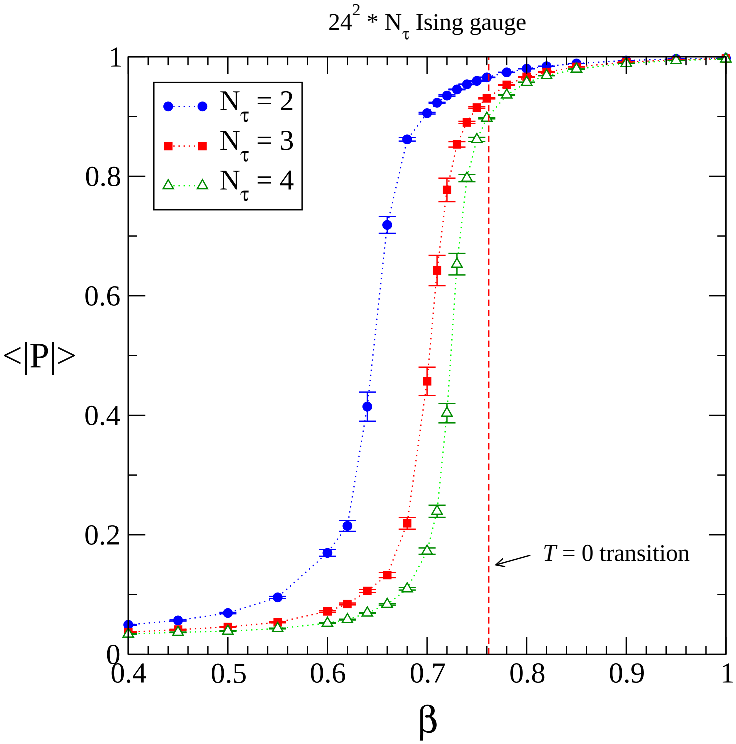
Example: Ising gauge theory

The finite temperature deconfinement transition happens also in the Ising gauge theory. Now

$$P(\vec{x}) = \prod_{\tau} s_0(\vec{x}, \tau)$$

If we fix N_τ, there is a Polyakov loop transition: $\Rightarrow$

When we increase N_τ, the transition $\beta \to \beta_c(T = 0)$, the *bulk* transition point. (Remember: here β is is not $1/T$!)



Clearly, the transition point moves to smaller β as N_τ decreases. We can estimate (very crudely!) that

$$
\begin{array}{ll}
N_\tau = 2: & \beta_c = 0.64 \\
N_\tau = 3: & \beta_c = 0.70 \\
N_\tau = 4: & \beta_c = 0.72 \\
N_\tau = \infty: & \beta_c = 0.76
\end{array}
$$

If we assume that the transition happens at constant ***physical*** temperature T_c, we obtain

$$N_\tau a(\beta_{c,N_\tau}) = 1/T_c \Rightarrow a(\beta_{c,N_\tau}) = 1/T_c \times 1/N_\tau$$

Thus, this would give right away that, for example,

$$a(\beta = 0.64) = 2 \times a(\beta = 0.72)$$

If we would measure the string tension σ at the β_c values above using large symmetric ($T = 0$) lattices, we should observe that the *dimensionless* ratio

$$\frac{\sigma}{T_c^2} = \frac{\sigma a^2}{(T_c a)^2} = (\sigma a^2)(\beta_c) \times N_\tau^2 = \text{const.}$$

(remember that it is σa^2 what you get when you measure σ in lattice units!)

If this is not the case: *scaling violation*, different observables give different lattice spacing. However, if the theory is to have a good continuum limit the scaling violations must go away as $a \to 0$.