Simulation methods in physics

Examination, spring 2003

Return before **12.00 on Monday 12.5., to Kari Rummukainen's mailbox** (C-wing 3rd floor), and send the codes by e-mail to kari.rummukainen@helsinki.fi.

1. (10 points) Write a program which generates N (parameter) random vectors $\vec{s}$ on a) 2-dimensional and b) 3-dimensional sphere, with probability distribution

$$p(\vec{s}) \propto \exp[\beta \vec{s} \cdot \vec{s}_0] \qquad (1)$$

In 2 and 3 dimensions $\vec{s} = (s_x, s_y)$ and $\vec{s} = (s_x, s_y, s_z)$, respectively, with $\sum_i s_i^2 = 1$. Here $\vec{s}_0$ is a constant unit vector which you can take to be along s_x-direction, and β is a constant.

Write the programs so that they print out N samples of random vectors, with one vector (s_x, s_y or s_x, s_y, s_z) on each line. Keep N and β as easily modified parameters in the program, but fix them in the code to values $N = 1000$ and $\beta = 1$ (so that I can check the output without modifying the program).

These distributions are used in O(2) and O(3) symmetric spin models.

2. (20 points) One of the most important applications for the lattice Monte Carlo method is the location of the phase transition point and determination of the critical exponents. For the sake of coding economy, let us study the **3-dimensional** Ising spin model

$$Z = \sum_{\{s\}} \exp[\beta \sum_{<ij>} s_i s_j] \qquad (2)$$

where $s_i = \pm 1$, and $< ij >$-sum goes over nearest neighbour pairs.

   1) Write i) Heat bath and ii) Wolff single-cluster update codes for the model.
   2) Locate the critical point β_c using finite size scaling. Use the susceptibility $\chi_{|M|}$, where absolute value of magnetization $|M| = 1/V \langle | \sum_i s_i | \rangle$ is the order parameter. Plot the volume dependence of the pseudocritical point $\beta_c(L)$.
   3) Determine the critical exponents γ and γ/ν. Make appropriate plots of the volume dependence.
   4) Determine the dynamical critical exponent z (autocorrelation exponent) $\tau_{\text{int}} \sim L^z$ at $\beta = \beta_c$ for each of the algorithms i) – ii) (using $|M|$ as the observable in the measurement).

e-mail the program(s) to me. Note that 2) and 3) are determined simultaneously: first, scan the β-axis using small volume, and use the results (and pseudocritical $\beta_c(L)$) from smaller volumes as a starting point for the next larger volume simulations.

Note that full results should include error estimates! Error analysis should be done with jackknife or bootstrap.