Simulation methods in physics

Homework 7
Return before 16.00 on Tue **22.4.**, (no homework on week 16) to Ari Hietanen's mailbox (C-wing 3rd floor),
or by e-mail ari.hietanen@helsinki.fi

1. Check if the 3-dimensional Ising gauge theory is *confinging* at (at least) $\beta = 0.72$ and $\beta = 0.74$ (check last week's homework for definitions!) by measuring the *string tension* using Wilson loops. Note that these β's are below the transition point $\beta_c = 0.761\ldots$ determined in homework 6.

   For simplicity, measure square $R \times T = n \times n$ Wilson loops

   $$W_{RT} = \prod_{s_i \in \square_{RT}} s_i(x), \tag{1}$$

   where the link variables $s_i(x) = \pm 1$ are along the edge of the square.

   Check whether the area law holds: $\langle W_{RT} \rangle \propto \exp[-\sigma RT]$. If so, what is the string tension σ? Plot $\langle W_{RT} \rangle$ against RT.

   Note that at short distances there are subleading contributions; more precisely, $-\log\langle W_{RT}\rangle = \sigma RT + \alpha(R+T) + TO(1/R) + RO(1/T) + \ldots) \to V(R)T$ when $T \to \infty$. Thus, at very small loop sizes the area law may not hold well.

   Since the magnitude of the loops becomes exponentially smaller as the size increases, you need a lot of measurements to obtain a non-zero signal at large loop sizes (careful about statistical errors!). Probably the largest practical loop sizes at these β's are $\sim 10 \times 10$ (closer to β_c larger loops are possible).

   The loops should be substantially smaller than the box size. Thus, use a volume of order $\sim 24^3$.

   Practical hints: In order to maximize the number of measurements, measure as many squares as is (easily) possible from a given configuration. (However, you don't have to measure *all* possible squares if it is too complicated to program; generation of new configurations is relatively cheap). Naturally, you should measure all sizes simultaneously!

   The construction of all the loops can take a long time. However, in this model, if you construct auxiliary arrays (here $0 \le x < N_x$)

   $$u_x(x,y,z) = s_x(0,y,z)s_x(1,y,z)\ldots s_x(x-1,y,z) \tag{2}$$

   (similarly u_y, u_z) you can do the rectangles just by considering u's at the corners of the rectangles (how?).

2. Compare measured σ to small-β (high-T) expansion. The theory actually has so-called *roughening* transition at $\beta \approx 0.47$. Below this value the small-β expansion converges, above it diverges.

3. Repeat the calculation in 1) at $\beta = 0.8$. Does the area law hold in this case?