Simulation methods in physics

Homework 6
Return before 16.00 on Tue 8.4., to Ari Hietanen's mailbox (C-wing 3rd floor), or by e-mail ari.hietanen@helsinki.fi

1. Let us consider **Ising gauge theory** (or Z_2 gauge theory) in 3 dimensions. The partition function is

$$
\begin{aligned}
Z &= \sum_{s_i(\mathbf{x})} \exp[-\beta H] \\
H &= \sum_{\mathbf{x},i<j} -s_\Box(i,j;\mathbf{x}); \qquad s_\Box(i,j;\mathbf{x}) = s_i(\mathbf{x}) s_j(\mathbf{x}+\hat{i}) s_i(\mathbf{x}+\hat{j}) s_j(\mathbf{x})
\end{aligned}
$$

   where $\mathbf{x}$ is 3-dimensional coordinate (with periodic boundary conditions), directions $i, j = 1, 2, 3$, and the link variable $s_i(\mathbf{x}) = \pm 1$ lives on the link connecting points $\mathbf{x}$ and $\mathbf{x} + \hat{i}$.

   This model can be understood as an extreme limit of a SU(2) gauge theory; it is also used in condensed matter physics as a statistical model of closed loop networks (why?).

   Write a lattice Monte Carlo update program for the Ising gauge theory. The simplest way to do it is probably roughly as follows:

   for each $\mathbf{x}$ and i {

   1. Calculate the part of energy which depends on variable $s_i(\mathbf{x})$: $H_i(\mathbf{x}) = s_i(\mathbf{x}) U_i(\mathbf{x})$, where the sum of *staples* is:
      $U_i(\mathbf{x}) = \sum_{|j| \neq i} s_j(\mathbf{x}+\hat{i}) s_i(\mathbf{x}+\hat{j}) s_j(\mathbf{x})$.
      Note that j here goes over positive and negative directions. Negative direction means that we take the spin variable from the backwards link: $s_{-j}(\mathbf{x}) \equiv s_j(\mathbf{x} - \hat{j})$.
   2. Perform heat bath or Metropolis update on $s_i(\mathbf{x})$, using $H_i(\mathbf{x})$.

   }

   Using a lattice of size $\sim 16^3$, measure the energy H (or average plaquette $s_\Box$) as a function of β at $\gtrsim$12 values of $\beta < 1.2$. Can you locate approximately a phase transition?

   Check how the result compares to leading nontrivial order(s) in low/high-temperature expansions.

   (This program will be used to measure string tension in next week homework!)

2. Bonus problem, for your amusement (points not counted): show that the 3-dimensional Ising gauge model is dual to the 3-dimensional Ising model.