Note added to sect. 4.15

This is to be added at the end of 4.15 (autocorrelations)

- Rule of thumb: lag of $2 \times \tau_{\rm int}$ gives new independent configuration. (When there were no autocorrelations, $C(t) = 0$ and $\tau_{\rm int} = 1/2$.)
- How to calculate $\tau_{\rm int}$ in practice? The definitions for $C(t)$ and $\tau_{\rm int}$ above are valid in the limit $N \to \infty$, i.e. infinite statistics. However, in practice finite N modifies the procedure. In practice:
  - We assume that all measurements are written to a file, allowing free post-processing.
  - The optimised autocorrelation function $C(t)$ for lag t now becomes

    $$C(t) = \frac{\frac{1}{N-t}\sum_{i=1}^{N-t} X_i X_{i+t} - \langle X\rangle_1 \langle X\rangle_2}{\langle X^2\rangle - \langle X\rangle^2},$$

    where

    $$\langle X\rangle_1 = \frac{1}{N-t}\sum_{i=1}^{N-t} X_i \quad \text{and} \quad \langle X\rangle_2 = \frac{1}{N-t}\sum_{i=1}^{N-t} X_{i+t}$$

    In the denominator the expectation values can be the usual ones without significant effect.
  - Because of finite statistics, $C(t)$ becomes noisy when t is large, $t \gg \tau_{\rm int}$. This causes the summation in $\tau_{\rm int}$ actually not converge. Thus, the summation over the lag t should be cut self-consistently to a value which is larger than $\tau_{\rm int}$ but not by a large factor; for example, to $6 \times \tau_{\rm int}$. Remember that the real signal comes from the integral of function $\sim \exp[-t/\tau]$; thus, by cutting at 6τ the error in $\tau_{\rm int}$ will be less than 0.5%.

    Now the optimised integrated autocorrelation time can be shown to be

    $$\tau_{\rm int} = \frac{1}{2} + \sum_{t=1}^{6\tau_{\rm int}} C(t)\,\frac{N-t}{N}.$$

    Here the correction factor $(N-t)/N$ corrects the statistical significance of the data. Clearly, as $N \gg \tau_{\rm int}$, the result goes into the one on page 52.
  - Course home page includes a general purpose program `errors.c` which calculates averages and errors (from data in files) using the autocorrelations as described above.