Monte Carlo simulation methods **Homework 1**

Return the solutions (with program printouts) at the latest at the beginning of the 27.9. excercise session. You can also e-mail the solutions to Ahti Leppänen, <ahtilepp AT mail.student.oulu.fi>.

1. Show that the probability of the needle intersecting one of the lines in Buffon's experiment (see notes) is
$$P = \frac{2\ell}{\pi d}.$$

2. Programming task: Let us compare the performance of the Monte Carlo integration to the regular midpoint method. Consider the integral
$$I = \int_0^1 dx\,\frac{3}{2}(1-x^2) = 1$$
Calculate the integral using
   a) The midpoint method, i.e. divide the integration range into N equal intervals and evaluate the function at the midpoints of the intervals. Give the answers (at least) for $N = 10, 100, 1000, 10000$.
   b) Standard Monte Carlo integration, evaluating the function using the same number of random points as in a). (You can use here, for example, the `drand48()` generator in C-language standard library (see page 26 of the notes for an example of using it) or the "Mersenne twister" generator given in the course web-page.)

   Compare the convergence of the methods towards the correct answer.

   (Extra (not graded): if this was very easy, consider the integral in d-dimensional unit hypercube, with $f(\vec{x}) = \prod_{i=1}^{d} \frac{3}{2}(1-x_i^2)$, with $d \sim 10$. In this case the midpoint method is evaluated at the center of a d-dim. hypercubes.)

3. Estimate the volume of a d-dimensional sphere, with (at least) $d = 2$ and 3, using the hit-and-miss method. Use $N = 10000$ random vectors.