Monte Carlo simulation methods, homework 1

1

Examining area $[-\frac{d}{2}, \frac{d}{2}[$ around a line(assuming that the line is thin). Two variables descripe needles position: its direction($\phi : [0, 2\pi[$) and the distance of its center from the line(y: $[-\frac{d}{2}, \frac{d}{2}[$). Cases where the needle crosses a line are such that

$$|sin(\phi)| * \frac{l}{2} > |y|$$

where l ($< d$) is the length of the needle and d is the distance between lines. With assumption that all possible variable values are equally likely, the probability that a needle crosses a line is

$$P = \frac{\int_0^{2\pi} d\phi \int_{-\frac{d}{2}}^{\frac{d}{2}} dy (\frac{l}{2}|sin(\phi)| > |y|)}{\int_0^{2\pi} d\phi \int_{-\frac{d}{2}}^{\frac{d}{2}} dy} = \frac{2\int_0^{\pi} d\phi \int_{-\frac{d}{2}}^{\frac{d}{2}} dy (\frac{l}{2} sin(\phi) > |y|)}{2\pi * d}$$

$$= \frac{2\int_0^{\pi} d\phi \int_{-\frac{l}{2}sin(\phi)}^{\frac{l}{2}sin(\phi)} dy}{2\pi * d} = \frac{2l \int_0^{\pi} sin(\phi) d\phi}{2\pi * d} = \frac{4l}{2\pi * d} = \frac{2l}{\pi d}$$

2

N = 10, MP error = 1.25e-003, MC error = 0.31e-001, 1σ error = 1.5e-001
N = 100, MP error = 1.25e-005, MC error = 5.8e-002, 1σ error = 4.1e-002
N = 1000, MP error = 1.25e-007, MC error = 1.3e-002, 1σ error = 1.4e-002
N = 10000, MP error = 1.25e-009, MC error = 7.7e-003, 1σ error = 4.5e-003

Note: Monte Carlo error values depend on the generated random numbers, so above values are just examples from a calculation. In this case the actual error in MP method seems to be $\propto \frac{1}{N^2}$, and 1σ error $\propto \frac{1}{\sqrt{N}}$ (at least approximately).

2 extra

d = 3, N = 100: MP error: 3.8e-5, MC error: 7.2e-004, 1σ error: 8.5e-004
d = 6, N = 10: MP error: 7.5e-3, MC error: 3.8e-004, 1σ error: 14e-004
d = 10, N = 4: MP error: 8.1e-2, MC error: 9.1e-004, 1σ error: 22e-004
d = 13, N = 3: MP error: 2.0e-1, MC error: 15e-004, 1σ error: 25e-004
(MC integration done with the same number of terms as in MP integration ($= N^d$))

3

One way to do this: Take random vectors from d-dimensional 'box' with every coordinate being within [-r,r], where r is the 'radius' of the d-dimensional sphere. Hit ratio is the same as the ratio of sphere 'volume' and box 'volume',
Sphere volume = $\frac{hits}{total}(2r)^d$. More about d-dimensional spheres can be found for example from http://mathworld.wolfram.com/Ball.html.

d: 1, vol: 2, hits: 10000
d: 2, vol: 3.1456, hits: 7864
d: 4, vol: 4.9888, hits: 3118
d: 6, vol: 5.0176, hits: 784
d: 8, vol: 4.3264, hits: 169
d: 10, vol: 2.4576, hits: 24
d: 12, vol: 0.8192, hits: 2
d: 14, vol: 1.6384, hits: 1
d: 16, vol: 0, hits: 0
d: 18, vol: 0, hits: 0

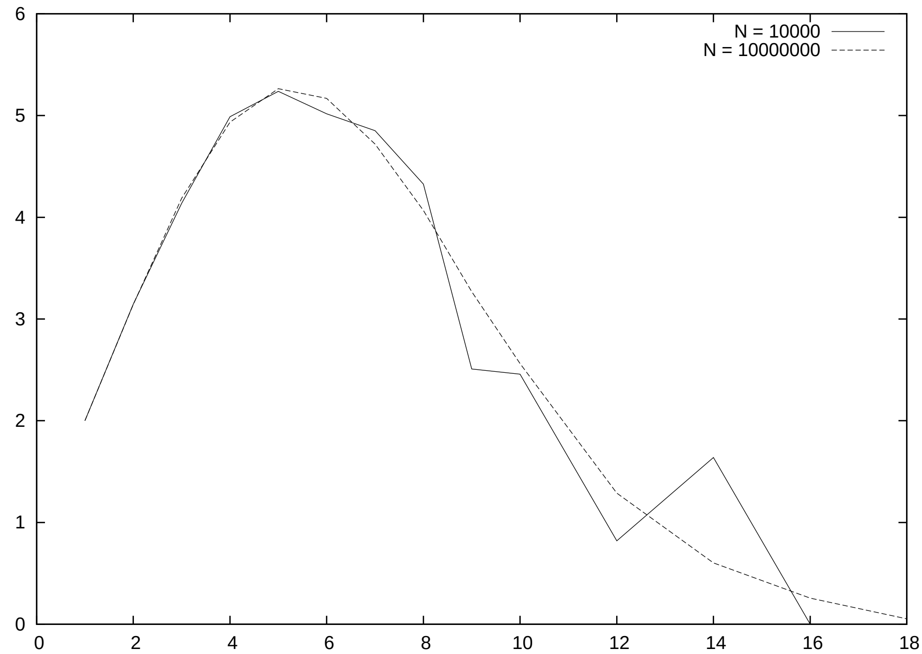

Figure 1: r = 1