Discussed on 27.11 and 4.12.

1. *Chiral condensate* $\Sigma \equiv -\langle\bar{\psi}\psi\rangle$ is an order parameter of chiral symmetry breaking. Clearly, non-zero chiral condensate breaks the axial symmetries of the full QCD (-like) action. In QCD, chiral condensate is broken at low temperatures, restored at high temperatures. The quark masses break the chiral symmetry explicitly, as happens with free fermions. Let us study this below.

   Consider one naive free fermion on the lattice. Show that

   $$\Sigma \equiv -\langle\bar{\psi}\psi\rangle = a^{-3}\int_{-\pi}^{\pi}\frac{d^4k}{(2\pi)^4}\frac{4am}{a^2m^2+\sum_\mu \sin^2 k_\mu}$$

   and which has an expansion

   $$\Sigma = c_1 m a^{-2} + m^3[c_3\ln(am)+c_3'] + O(a)$$

   Hint: use directly the integral on p. 66 on the lecture notes. What are the coefficients c_1 and c_3?

   This shows that the bare chiral condensate on the lattice is UV-divergent and has to be regulated, but for naive fermions it is proportional to m. For Wilson fermions, the leading divergence is $c_0 a^{-3}$. What does this imply about the m-dependence?