Lattice Field Theory 2017 Problem set 6

Discussed on 20.11 and 27.11.

1. *The triviality of the standard model and the upper bound on the Higgs mass:* Let us take the scalar $\lambda\phi^4$ -model to be a toy model for the Higgs sector in the standard model. In this case we are in the broken phase of the scalar theory. Let us write the effective "renormalized potential" $V_{\rm eff}(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4!}\lambda_R\phi^4$ with $\mu^2 < 0$. Identify the minimum location of $V_{\rm eff}$ with the standard model $v = 246\,\mathrm{GeV}$, and the curvature at the minimum as the Higgs mass,

$$m_H^2 = \frac{d^2V}{d\phi^2}|_{|\phi|=v}.$$

   What is now λ_R as a function of m_H^2 and v?

   Using the solution of the RG equation (from notes) for λ_0, and substituting $am_R \to am_H$, what is the smallest a possible? Using $k \sim \pi/a$, what momentum does this correspond to at $m_H = 100 \ldots 1000\,\mathrm{GeV}$?

   (In real world this has to be studied with more realistic 2-loop β-function, or rather with non-perturbative simulations.)

2. *Fermion propagator:* Consider a two spinor field "action"

$$S = \bar\psi_i M_{ij}\psi_j = \bar\psi_1 M_{11}\psi_1 + \bar\psi_1 M_{12}\psi_2 + \bar\psi_2 M_{21}\psi_1 + \bar\psi_2 M_{22}\psi_2$$

   Show, by explicit computation of Grassmann integrals, that

$$\langle\psi_i\bar\psi_j\rangle \equiv \frac{\int d\psi_1 d\psi_2 d\bar\psi_1 d\bar\psi_2 \;\psi_1\bar\psi_2\, e^{-S}}{\int d\psi_1 d\psi_2 d\bar\psi_1 d\bar\psi_2 e^{-S}} = [M^{-1}]_{ij}.$$

   This generalizes to full field theory, with

$$\langle\psi_\alpha(p)\bar\psi_\beta(q)\rangle = \delta(p-q)\frac{[m - i\hat{\not{p}}]_{\alpha\beta}}{\hat p^2 + m^2}$$