Lattice Field Theory 2017 Problem set 4
Discussed on 30.10 and 6.11.

1. Use the strong coupling expansion to get the leading contributions to plaquette-plaquette correlation functions in 4-dimensional U(1) gauge theory. Let us assume that the two plaquettes are oriented along (x, y)-plane, and have the same (x, y, z)-coordinates but are separated by distance at to t-direction. (that is, plaquettes are aligned "on top" of each other, separated by at.)

   The exponential fall-off of the correlator gives the scalar "glueball" mass. What are the masses in U(1) and SU(2) gauge theories at strong coupling?

2. Assuming that M is a symmetric $N \times N$ matrix with all eigenvalues $\lambda_i > 0$. Show that

$$\int \left[\prod_{i=1}^{N} dx_i\right] e^{-\frac{1}{2}x_i M_{ij} x_j} = \frac{(2\pi)^{N/2}}{\sqrt{\mathrm{Det} M}}$$

   Hint: think about diagonalizing M, and changing the integration variable.