1. The standard form of the non-Abelian SU(N) gauge theory lattice action (so-called single plaquette or Wilson gauge action) is

$$S = \beta \sum_{x;\mu<\nu} \left(1 - \frac{1}{N} \mathrm{Re}\,\mathrm{Tr}\,\left[U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu}\right]\right)$$

where $\beta = 2N/g_0^2$, $U_{x,\mu} = e^{ig_0 a A_\mu(x+\frac{1}{2}\hat{\mu})}$ and $\hat{\mu} = a\boldsymbol{e}_\mu$.

Show that in the continuum limit this indeed becomes

$$\int d^4x \frac{1}{2} \mathrm{Tr}\, F_{\mu\nu} F_{\mu\nu}$$

This was shown in the lecture notes for Abelian theory; for non-Abelian one the use of Baker-Campbell-Hausdorff formula

$$e^{aA} e^{aB} = e^{aA+aB+a^2\frac{1}{2}[A,B]+a^3\frac{1}{12}([A,[A,B]]-[B,[A,B]])-a^4\frac{1}{24}[A,[B,[A,B]]]+O(a^5)}$$

may prove to be helpful (you get the right result using only $O(a^2)$ bits above; you should argue higher orders do not contribute).

There is an easy way to show that the discretization error is of order $O(a^2)$; that is, $O(a)$ error is absent. Can you show that? (Hint: consider the analytical structure of S in a).