Lattice Field Theory 2017 Problem set 1

These will discussed on 18.9 and 23.9, 14-16. Problem 2 requires mathematica, try to solve it beforehand.

1. Let

$$Z = \int [d\phi] e^{-H[\phi] + \int dx J_x \phi_x}$$

be a partition function of a statistical system, where H is some dimensionless Hamiltonian and J_x is a x-dependent source. Now the expectation value of an operator $O(\phi)$ is given by

$$\langle O(\phi) \rangle = \frac{1}{Z} \int [d\phi] O(\phi) e^{-H[\phi]}$$

where the source field J has been set to zero. Show that the connected 2-point function is generated by the derivative

$$\frac{\partial}{\partial J_x} \frac{\partial}{\partial J_y} \ln Z\Big|_{J=0} = \langle (\phi_x - \langle \phi \rangle)(\phi_y - \langle \phi \rangle) \rangle = \langle \phi_x \phi_y \rangle - \langle \phi \rangle^2$$

Assuming now that $\langle \phi \rangle = 0$, what is the form of the 4-point functions

$$\frac{1}{Z} \frac{\partial}{\partial J_x} \frac{\partial}{\partial J_y} \frac{\partial}{\partial J_z} \frac{\partial}{\partial J_w} Z\Big|_{J=0}$$

$$\frac{\partial}{\partial J_x} \frac{\partial}{\partial J_y} \frac{\partial}{\partial J_z} \frac{\partial}{\partial J_w} \ln Z\Big|_{J=0}$$

How would you represent these using Feynman diagrams? In general, it can be shown that $\ln Z$ generates all connected $n-$point functions.

2. Consider the 3-state Potts model in 1 dimensions:

$$Z = \sum_{s_x=0,1,2} \exp[\beta \sum_x \delta(s_x, s_{x+1}) + h \sum_x \delta(s_x, 0)]$$

where $x = 0, 1, 2, \ldots (N-1)$, and the boundary is periodic: $x_N \equiv x_0$. The field has been chosen to "0"-direction. What is the transfer matrix of the theory, and what are its eigenvalues? Calculate the partition function and magnetizaton density as $N \to \infty$. Use mathematica or somesuch.