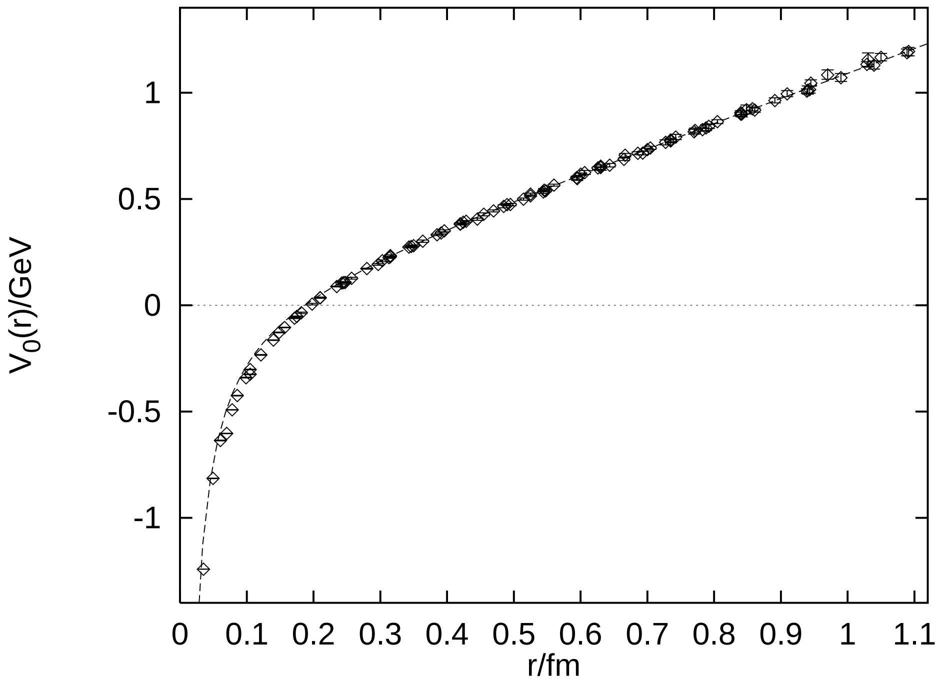
Wilson loop in pure gauge SU(3)

[Bali,Schilling, Wachter 1995]
$48^3 \times 64$ quenched lattice, $\beta = 6.8$



For large loops

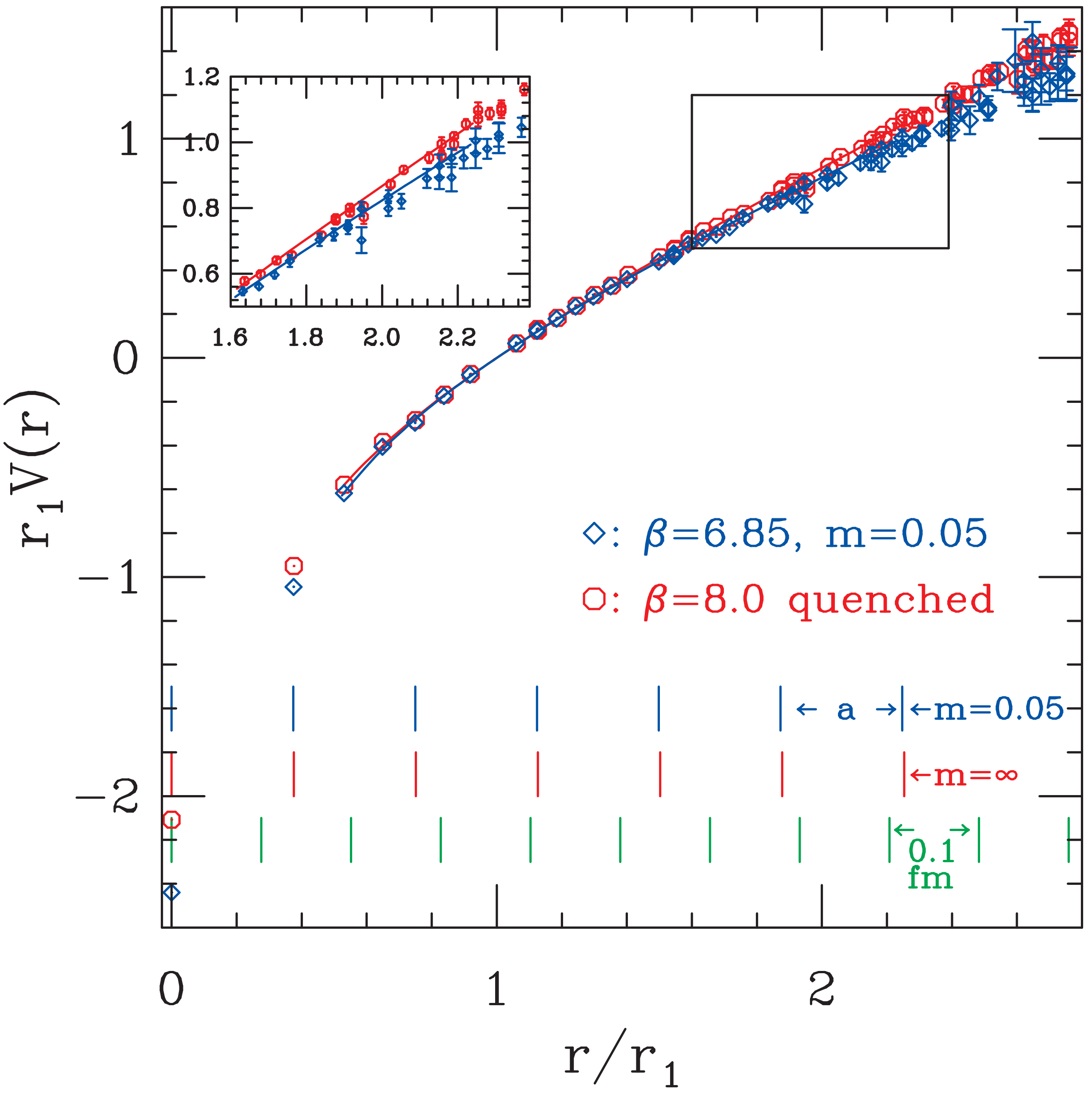
$$\langle W \rangle \sim e^{-V(r)T}$$

Phenomenological form

$$V(r) = V_0 + \sigma r - \frac{e}{r} + f\left[G_L(\vec{r}) - \frac{1}{r}\right]$$

Dynamical quark effects in Wilson loop

KS improved, MILC, Phys.Rev.D 62 (2000)



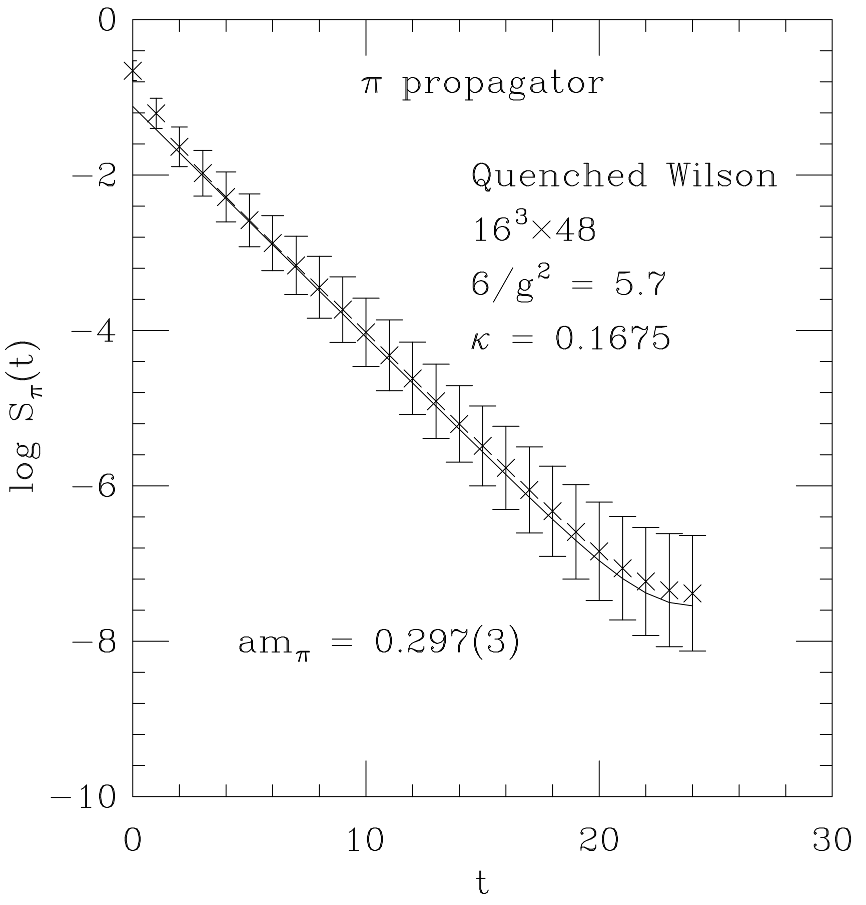
Hadron masses

Pi-meson propagator (Wilson quarks)

$$
\begin{aligned}
G_\pi(z,y) &= \langle \bar\psi(z)\gamma_5\psi(z)\ \bar\psi(y)\gamma_5\psi(y)\rangle \\
&= \langle \mathrm{Tr}\,[Q^{-1}_{z,y}\gamma_5 Q^{-1}_{y,z}\gamma_5]\rangle
\end{aligned}
$$

Extracting the mass:

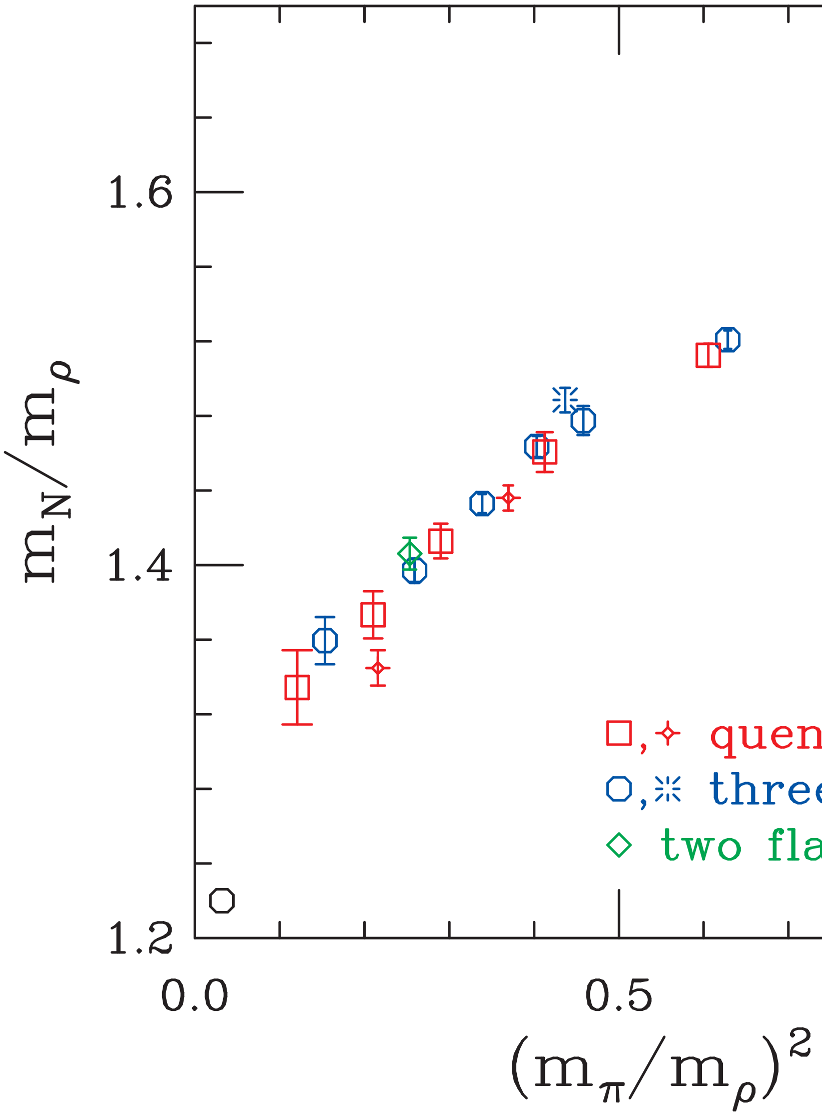
$$\int d^3y d^3z G_\pi(\bar z, t; \bar y, 0) \sim A[e^{-am_\pi t} + e^{-am_\pi(N_t - t)}$$



Mass ratios: APE plot

MILC, hep-lat/0104002

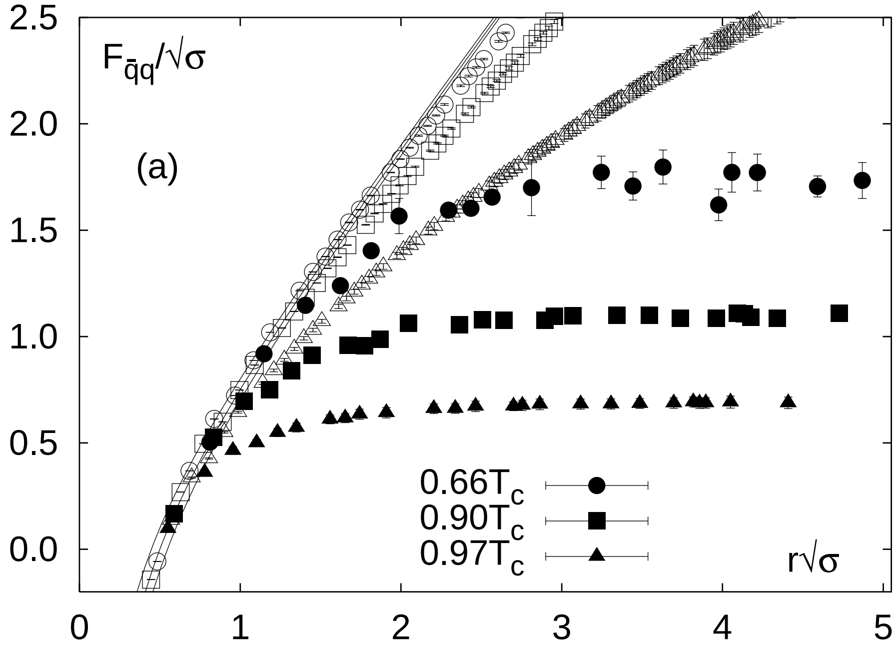
Most points with $a = 0.13$ fm. Left circle: real world. Right circle: $m_q = \infty$ limit.



Heavy quark potential: Polyakov loops

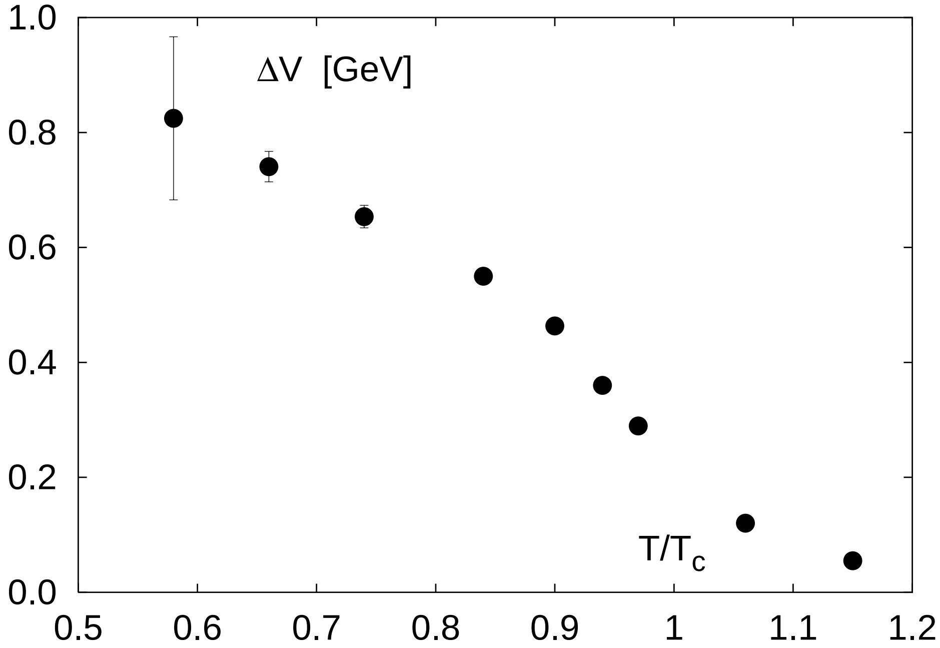
$$e^{V(R,T)/T} = \langle P(0) P^{\dagger}(R) \rangle$$

In 3-flavour QCD vs. quenched:



Quenched: open symbols

Dynamical: filled symbols



Limiting value of $V(r)$ (normalized $V(0.23\,\mathrm{fm}) = 0$).

Symmetries of the quark action

$$\mathcal{L} = \bar{\psi}(\partial_\mu \gamma_m u + m)\psi$$

If $m = 0$: $U_V(1) \times U_A(1) \times SU_V(N_F) \times SU_A(N_F)$

$$\begin{array}{ll} U_V(1): & \psi \to e^{i\theta}\psi \\ U_A(1): & \psi \to e^{i\theta\gamma_5}\psi \\ SU_V(N_F): & \psi \to e^{i\theta^a\lambda^a}\psi \\ SU_A(N_F): & \psi \to e^{i\theta^a\lambda^a\gamma_5}\psi \end{array}$$

Pisarski-Wilczek effective model for chiral symmetry breaking transition:

$$\begin{aligned} \mathcal{L}_{eff} = \quad & -\frac{1}{2}\mathrm{Tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi) - \frac{1}{2}m^2 \mathrm{Tr}(\Phi^\dagger \Phi) + \frac{\pi^2}{3} g_1 \left(\mathrm{Tr}(\Phi^\dagger \Phi)\right)^2 \\ & + \frac{\pi^2}{3} g_2 \mathrm{Tr}\left((\Phi^\dagger \Phi)^2\right) + c\left(\det \Phi + \det \Phi^\dagger\right) \quad , \end{aligned}$$

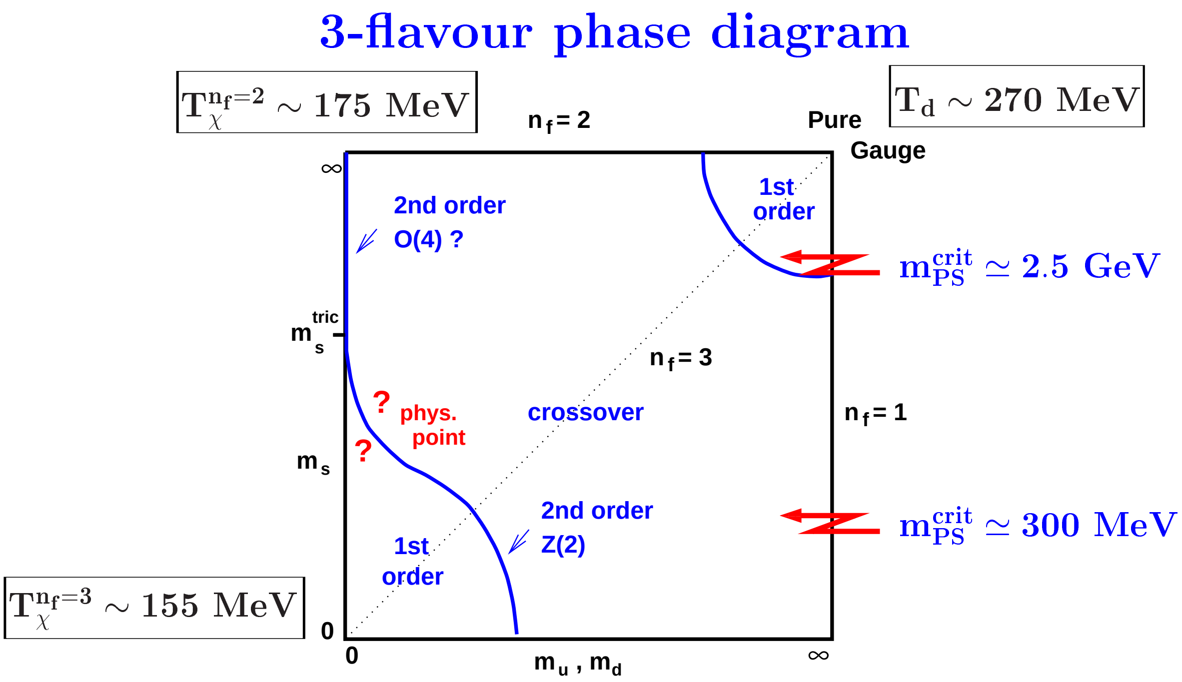
Here $\Phi = \Phi_{ij}$, $i, j = 1 \ldots N_F$.

Symmetry: $\Phi \to A^\dagger \Phi B$, $\quad A, B \in SU(N_F)$ (or $U(1)$).

$N_F = 2$: 2nd order
$N_F \geq 3$: 1st order

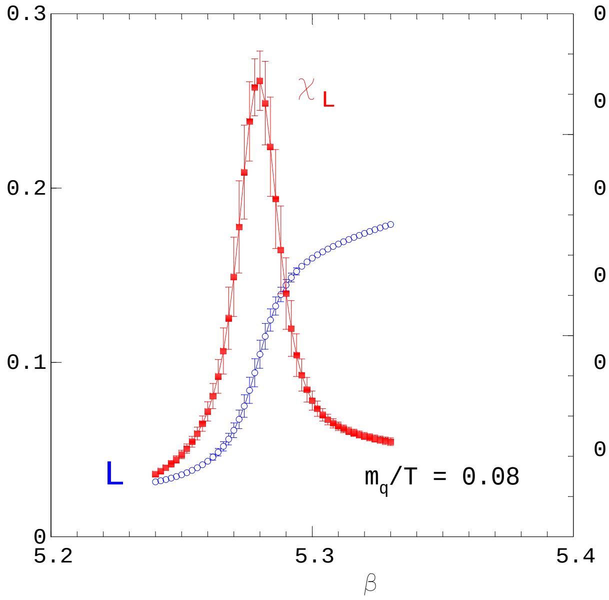
Schematic QCD phase diagram with 2 + 1 quark flavours

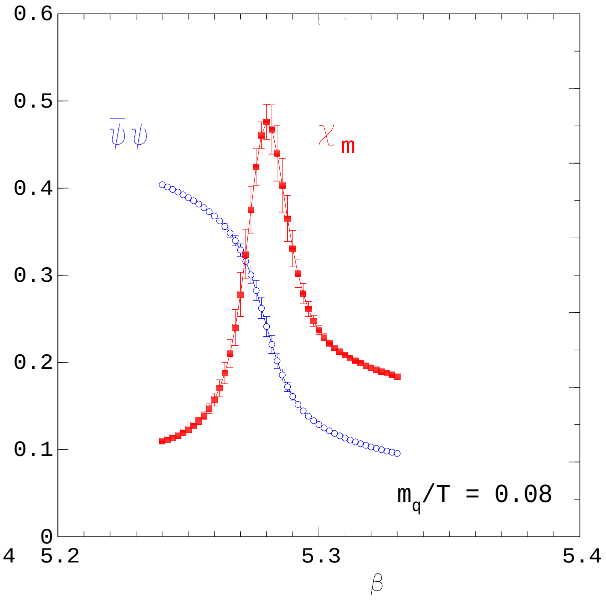
[F. Karsch, QM2001]



Deconfinement vs. chiral symmetry restoration

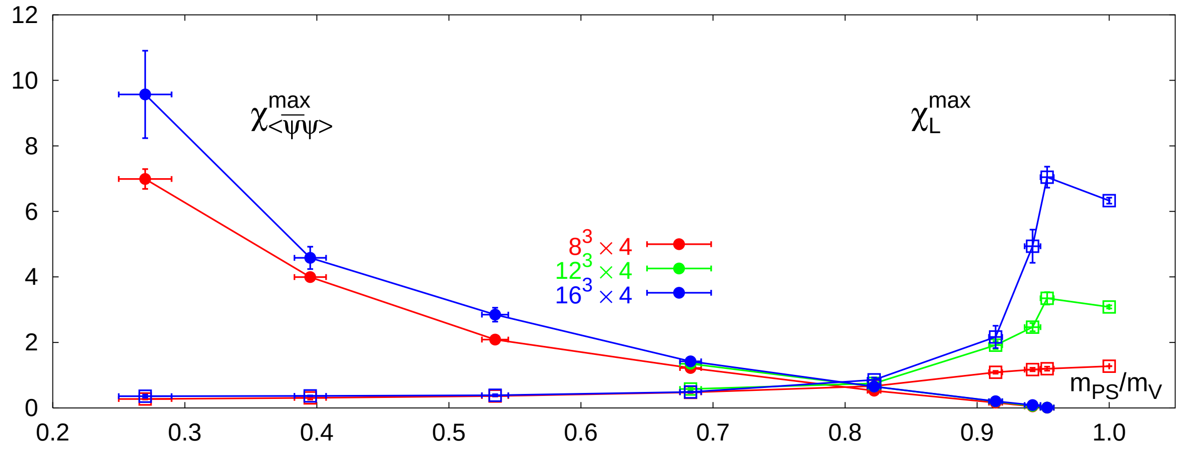
$N_f = 2$ QCD:





Polyakov line $\langle L \rangle$ and susceptibility $\langle (L - \langle L \rangle)^2 \rangle$

Chiral condensate $\langle \bar\psi\psi \rangle$ and susceptibility



Quark mass dependence of the Polyakov loop and chiral susceptibilities versus m_{PS}/m_V for 3-flavour QCD with improved action.
[Karsch, Schladming 2001]

QCD equation of state: energy density and pressure

- Free energy $F = -T \log Z$.
- Pressure in a homogeneous system:
$$p = \frac{\partial F}{\partial V} = -\frac{F}{V}$$
- Cannot measure F (importance sampling!), but differences yes ($\beta = 6/g^2$, lattice coupling, $f = F/V$):
$$\frac{f}{T^4}(\beta_1) - \frac{f}{T^4}(\beta_2) = \int_{\beta_2}^{\beta_1} d\beta \frac{\partial}{\partial \beta}\frac{f}{T^4}$$
$$= 6N_\tau^4 \int_{\beta_1}^{\beta_2} d\beta \langle P \rangle$$
where $P = 1 - 1/3\,\mathrm{Re}\,\mathrm{Tr}\,U_\Box$.
- However, when we change β, we change also $a \to$ cutoff-dependence; this can be canceled by measuring P in $T = 0$ system ($N_\tau \gtrsim N_S$)
$$\frac{f}{T^4}(\beta_1) - \frac{f}{T^4}(\beta_2) = 6N_\tau^4 \int_{\beta_1}^{\beta_2} d\beta [\langle P \rangle_T - \langle P \rangle_0]$$
- Change in $\beta \to$ change in $a \to$ change in $T = 1/(N_\tau a)$.
- *Interaction measure* $\epsilon - 3p$ ($= 0$ for ideal gas)
$$\epsilon = f + sT = f - \frac{\partial f}{\partial T} T = T^2 \frac{\partial}{\partial T} log Z$$
$$p = -\frac{\partial F}{\partial V} = T \frac{\partial}{\partial V} \log Z$$

- Homogeneous change of a:

$$a\frac{\partial}{\partial a} = a[\frac{\partial V}{\partial a}\frac{\partial}{\partial V} + \frac{\partial T}{\partial a}\frac{\partial}{\partial T}] = 3V\frac{\partial}{\partial V} - T\frac{\partial}{\partial T}$$

Thus,

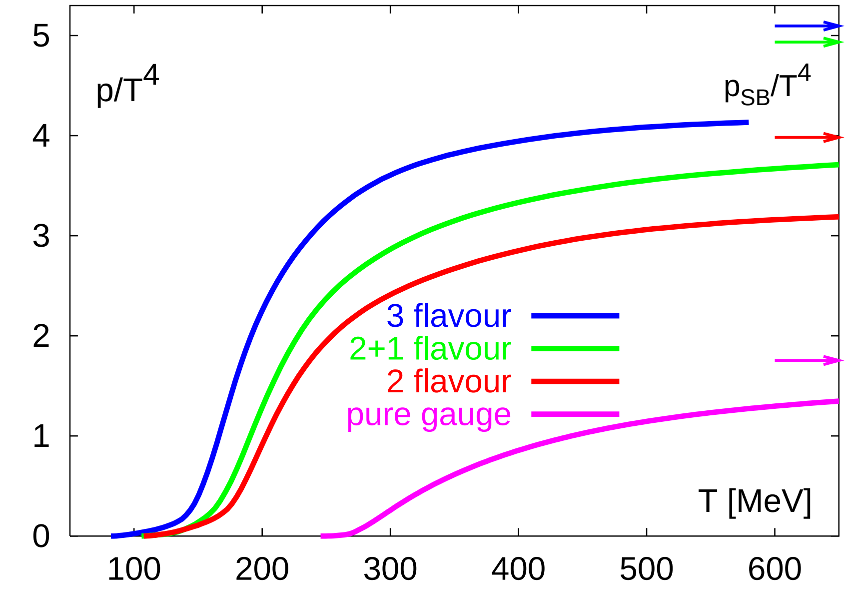
$$\begin{aligned}
3p - \epsilon &= \frac{T}{V}a\frac{\partial}{\partial a}\log Z \\
&= \frac{T}{V}\left[a\frac{\partial(6/g^2)}{\partial a}\frac{\partial}{\partial \beta}a\frac{\partial(am_q)}{\partial a}\frac{\partial}{\partial am_q}\right]\log Z \\
&= T^4N_\tau^4\Big[36g^{-3}\beta_L(g)[\langle P\rangle_T - \langle P\rangle_0] \\
&+ a\frac{\partial(am_q)}{\partial a}\langle\bar{\psi}\psi\rangle\Big]
\end{aligned}$$

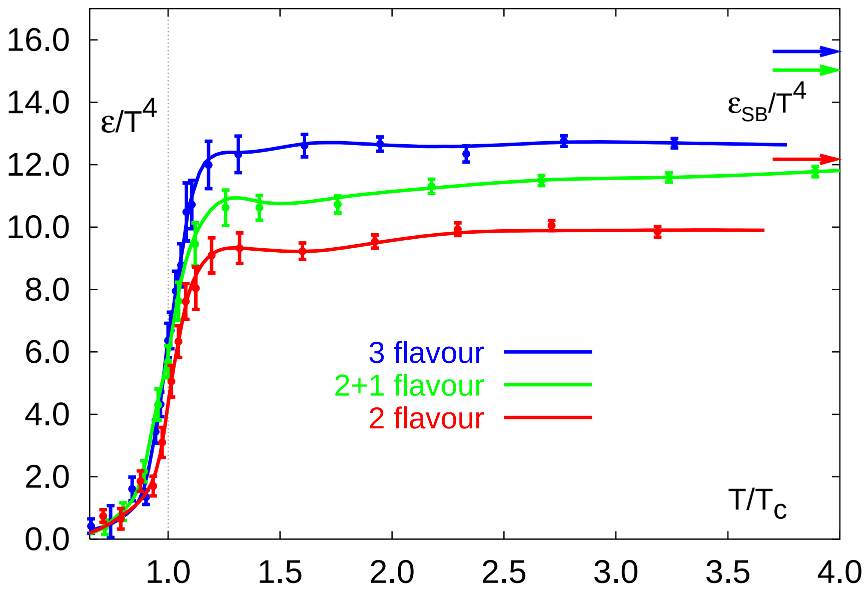
The last derivative does not appear for quenched theory.

- Usually simulations are made at a series of β, both smaller and larger than β_c ($\beta_c \equiv$ the transition coupling for particular N_τ).
- *Reweighting* $\rightarrow$ continuous curves

Pressure and energy density

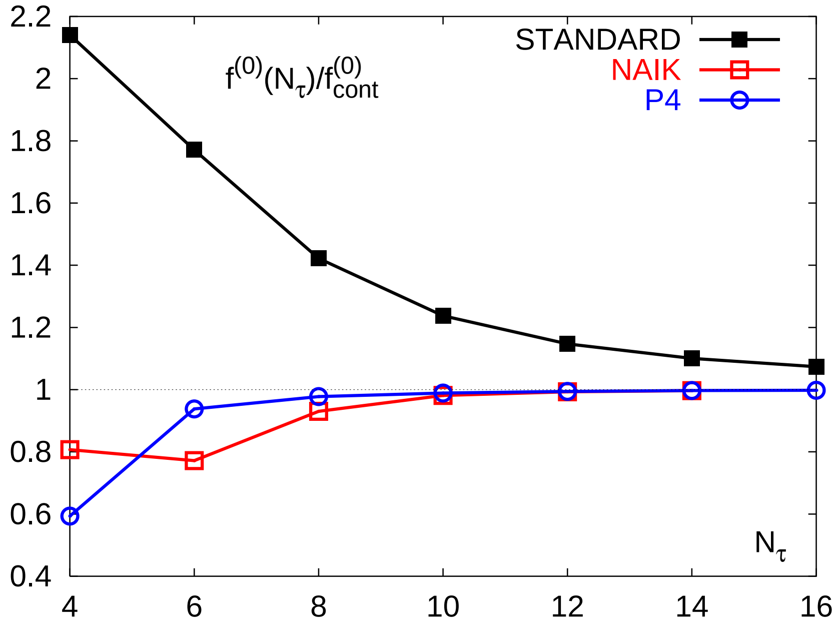
[Karsch, Schladming 2001]





Improvement

Pressure of free (no gauge) staggered quarks (exact):



Improvement is essential for quantitative results!

Standard staggered quark action (error $O(a^2)$):

$$S(m_q) \;=\; \sum_x \bar{\chi}^f_x \; \sum_\mu \; \eta_\mu(x) \; \big(\quad \big) \; \chi^f_y$$

Naik (error $O(a^4), O(g^2a^2)$):

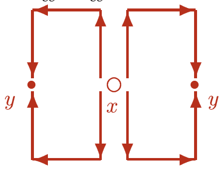
$$S(m_q) \;=\; \sum_x \bar{\chi}^f_x \; \sum_\mu \; \eta_\mu(x) \; \big(\quad + \quad \big) \chi^f_y$$

P4 (error $O(a^4), O(g^2a^2)$:

$$S(m_q) \;=\; c_1^F S_{1-link,fat}(\omega) + c_3^F S_{3-link} + m_q \sum_x \bar{\chi}_x^f \chi_x^f$$

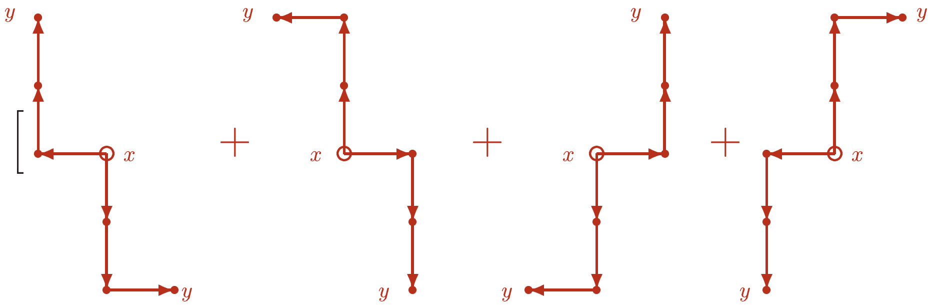
$$\equiv \sum_x \bar{\chi}_x^f \sum_\mu \eta_\mu(x) \Big(\frac{3}{8}\Big[$$

$$+\;\omega \sum_{\nu\neq\mu}$$



$$\Big]$$

$$+\frac{1}{96}\sum_{\nu\neq\mu}\Big[$$



$$\Big]\Big)\chi_y^f$$

$$+m_q \sum_x \;\bar{\chi}_x^f \chi_x^f \quad .$$

Actions become very large!
Coefficients usually "tadpole-corrected"

Wilson fermions: $O(a), O(ag^2)$

Wilson + clover (Sheikholeslami-Wohlert)

$$S_W + C_{SW} i a \sum_x \bar{\psi}_x \sigma_{\mu\nu} F_{\mu\nu} \psi_x$$

Pressure in 2-flavour QCD, with unimproved staggered quarks, “clover-improved” Wilson quarks, and P4-improved staggered quarks.