

Due on December 8 by 24.00. These are the last exercises.

Exam: 18.12 9.00 – 13.00 at Exactum D123. Calculator allowed, help sheet will be distributed.

1. **Gaussian random variables.** Show that if α is a real Gaussian random variable with $\langle \alpha \rangle = 0$, then

$$\langle |\alpha|^4 \rangle = 3 \langle |\alpha|^2 \rangle^2 ,$$

and that, if a is a complex Gaussian random variable, with $\langle a \rangle = 0$, then

$$\langle |a|^4 \rangle = 2 \langle |a|^2 \rangle^2 .$$

2. **Cosmic variance.** Assume that the CMB multipole coefficients a_{lm} are independent Gaussian random variables with zero mean (constrained by $a_{l-m} = a_{lm}^*$). Calculate the cosmic variance $\langle (\hat{C}_l - C_l)^2 \rangle$, where $\hat{C}_l$ is the observed angular power spectrum. (The relations given in the previous problem may be helpful.)

3. **Positions of the acoustic peaks.**

a) Calculate the sound horizon at decoupling,

$$r_s = (1+z)^{-1} \int_0^{t_{\text{dec}}} dt \frac{c_s(t)}{a(t)} ,$$

in terms of z_{dec} , ω_m , and ω_b . Assume constant speed of sound, $c_s = c_s(t_{\text{dec}})$, but include the effect of radiation and matter components in the expansion law. Neglect neutrino masses.

b) What is the separation ℓ_A between the acoustic peaks in the CMB angular power spectrum C_ℓ for the cases $\Omega_\Lambda = 0$ and $\Omega_\Lambda = 1 - \Omega_m$?

c) Give the numerical values of r_s and ℓ_A for $z_{\text{dec}} = 1090$, $h = 0.7$, $\Omega_{m0} = 0.3$ and $\omega_b = 0.02$. Give also the numerical value of the comoving sound horizon.

4. **The effect of varying sound speed.** Same as the previous problem, but now take into account the evolution of the sound speed.